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Higher-order kinematic analyses of a planar parallel robot based on screw theory

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Abstract. This paper presents the higher-order kinematic analyses of a planar parallel robot, addressed by means of the theory of screws. The reduced velocity, acceleration, jerk and hyper-jerk state for the end-effector of robot was developed as a spatial vector by applying the concept of Lie algebra and helicoidal vector field. In order to verify the effectiveness of this theoretical development, the kinematics models obtained was solved and simulated in MATLAB environment, using Freeth's Nephroid trajectory as reference path for tracking with the end-effector. The simulation results proved that this type of spatial notation is convenient, because it allows us to quickly develop equations of motion and express them succinctly in symbolic form, reducing the volume of algebra, simplifying the modeling tasks, implementation and execution the algorithms used to solve kinematic problems in parallel robots. The major contribution of this work is the possibility of extended the classical kinematic analysis to a high order system; where the application of screw theory becomes a safe and reliable mathematical tool, which may be successfully used on parallel planar robots with singular configurations, represented with helicoidal vector field.

1. Introduction

In robotics, the formal representation for spatial movements of a rigid body during a kinematic analysis plays a central role in geometric calculations using standard three dimensional Euclidean space [1]. Because the links and payload of a robot are usually modeled as a rigid body. Therefore, according to Mozzi-Chasles' theorem, the displacement in the Euclidean space of a rigid body can be represented by a rotation along a unique axis and a translation parallel to the axis [2]; that is, through a spatial movement known as a screw [3].

The infinitesimal version of this movement is an element of Lie algebra $SE(3)$, known as a twist and it is isomorphic to screw theory [4]. A literature review show papers dedicated to the study of screw theory and Lie algebras: Zheng *et al.*, present a Newton-Euler methodology based on screw theory to analyze the dynamics of the cooperative robot of six degrees of freedom with good results that verify the accuracy of the dynamic model [5]; Guiju *et al.*, show the topographical variation of a robot with 5-DOF spin exponential product kinematics model of the leveling mechanism of Orchard work platform Based on screw Theory. The results show the kinematics model is accurate and reliable, and provides a theoretical basis for real-time control of the leveling mechanism [6]. Wang *et al.*, proposed a 4U spatial quadrilateral balancing



mechanism which is based on screw theory. As a result, a prototype is manufactured and shown to have the ability to roll straight and turn through the corresponding trajectory [7].

Currently, this knowledge field is trend in parallel robots with applications in advanced dynamic modeling [8], flexible robot [9], rehabilitation therapies [10] and exoskeletons [11]; however, the kinematic performance of this type of robots in presence of singularities or disturbances, it is research material. Therefore, in this paper the higher-order kinematic analyses were implemented to the planar 3-RRR parallel robot, for tracking a Freeth's Nephroid trajectory, as an alternative methodology for kinematic analysis to provide a robust system in the presence of challenging trajectories. This paper is organized as follows: section 2, show a description of 3-RRR planar parallel robot; section 3 provides velocity, acceleration, jerk and hyper-jerk analysis using screw theory. The simulation and results are presented in section 4; the section 5 summarizes the significant conclusions.

2. Description of 3-RRR planar parallel robot

Figure 1 illustrates the schematic of the parallel robot 3-RRR planar, with the nomenclature to be used for the analysis. This robot is composed of a fixed base, three independent serial kinematic chains, which are attached to a mobile platform or end effector. Each kinematic chain has two links and three rotational joints, RRR, with one degree of freedom. The mobile platform is joined to the links with a passive joint, and the fixed base is connected to an active joint or motor, allowing to reduce the weight of the structure and the load on the mobile platform. The world reference system is located in O_1 , the angles α , β , and γ , define the rotation of the links with respect to the x axis, positive in counterclockwise. The points O_i , A_i and B_i , specify the position of the rotational joints, where $i = 1, 2, 3$. The point P, defines the desired position of the mechanism, that is, the position of the end effector. In each rotational joint the presence of the corresponding screw or twist, \hat{S} , can be appreciated for the velocity analysis. Likewise, the reciprocal screws, \hat{S}_r , have been incorporated, with the purpose of analytically deriving the expression of the screw-coordinate Jacobian matrix of the system.

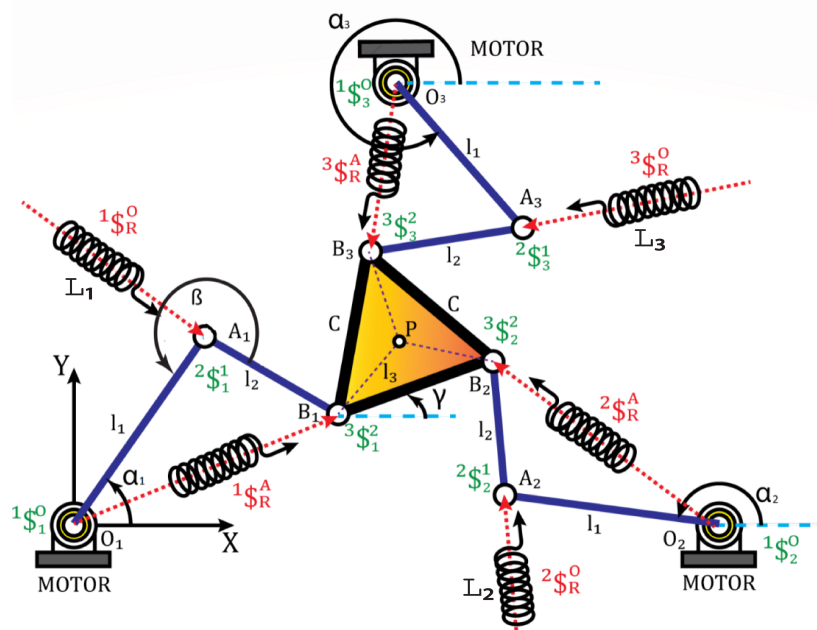


Figure 1. Geometric description of 3RRR planar parallel robot.

3. Kinematic analysis

The kinematics of manipulators refers to geometric and temporal properties of movement to analyses the position, velocity, acceleration, and all time derivatives of a higher degree of the position variables, like jerk, hyper-jerk (snap) and crackle [12].

3.1. Position analysis

The position analysis provides information about the position and orientation of the final effector based on the articular and dimensional parameters of the robot, with respect to the inertial and absolute reference system. The 3-RRR has a wide review in the literature about the position analysis, mainly from inverse and direct kinematics [13]. For the development of this paper the authors recommend to [14].

3.2. Velocity analysis

Velocity analysis consists of formulating the equations for an arbitrary point on the rigid body which describe the change in angular and linear position as an inseparable entity represented by a coordinated vector 6D, known as the spatial velocity, and consisting of a concatenation of the two vectors [15]. The first vector is called the primal vector, while the second vector is referred to as the dual part, which is described by Plücker's coordinates, and given by Equation (1).

$$V_p = \begin{bmatrix} \omega \\ v_p \end{bmatrix}. \quad (1)$$

The velocity state, V_p , of the of the end-effector at point P , may be written in screw form through any kinematic chain O_i , A_i and B_i , as follows in Equation (2).

$${}^0V_p^3 = {}_0\omega_1^{i0}\$}_i^1 + {}_1\omega_2^{i1}\$}_i^2 + {}_2\omega_3^{i2}\$}_i^3, \quad i = 1, 2, 3. \quad (2)$$

Equation (2), represents a necessary and sufficient condition for the twist, $\$$, and the joint velocities, ω , through the screw axes, \hat{S} , to be feasible for all motion configurations. Where, ${}_1\omega_2^{i1} = \dot{q}$, are the active joints rate of the 3-RRR. The screws in Equation (2) are given in Plücker coordinates and detailed in [14]. Likewise, considering that, $\$}_i$, is a line in Plücker coordinates pointed from the points, A_i , to the points, B_i , as shown in Figure 1. Therefore, the Klein form [16] was applied systematically for each line $Li(i = 1, 2, 3)$ on both sides of the Equation (2), representing the reciprocal screw or wrenches, ${}^1\$}_i^2$ and ${}^2\$}_i^3$, for each kinematic chain, as shown in Equation (3).

$$\{\$}_i; {}^0V^3\} = \dot{q}\{{}^0\$}_i^1; \$}_i\}. \quad (3)$$

Finally, casting Equation (3), into matrix-vector form, we find that the input-output equation of velocity of the 3-RRR planar parallel robot, as observed in Equation (4).

$$J^t \Delta^0 V^3 = \dot{q}, \quad (4)$$

where, $J=[\$}_1, \$}_2, \$}_3]$, is the 3x3 reduced screw-coordinate Jacobian matrix of the robot, Δ , is an operator of polarity defined by, $\Delta = [0 \ 1 \ 0; \ 0 \ 0 \ 1; \ 1 \ 0 \ 0]$, while, $\dot{q}=[\dot{q}_1 \ \{{}^0\$}_1^1; \$}_1\} \ \dot{q}_2 \ \{{}^0\$}_2^1; \$}_2\} \ \dot{q}_3 \ \{{}^0\$}_3^1; \$}_3\}]^T$, is the first-order driver matrix of 3-RRR planar parallel robot.

3.3. Acceleration analysis

Acceleration analysis consists of a formulation of equations involving the linear and angular acceleration of each element of the robot. For the development of this analysis, the intention was to determine the acceleration of the active joints from the acceleration measured in the end-effector using screw theory [14], as show in Equation (5).

$${}^0A_p^3 = \begin{bmatrix} {}^0\alpha^3 \\ a_p - {}^0\omega^3 \times V_p \end{bmatrix}. \quad (5)$$

On the other hand, the 6D vector, Equation (5), of acceleration for each kinematic chain may be written in screw form, as observed in Equation (6).

$${}^0A_p^3 = {}_0\omega_1 {}^0\mathcal{S}_i^1 + {}_1\omega_2 {}^1\mathcal{S}_i^2 + {}_2\omega_3 {}^2\mathcal{S}_i^3 + \mathcal{A}_{lie}, \quad i = 1, 2, 3, \quad (6)$$

where the last term is known as Lie screw of acceleration is given by Equation (7).

$$\mathcal{A}_{lie} = [{}_0\omega_1 {}^0\mathcal{S}_i^1 + {}_1\omega_2 {}^1\mathcal{S}_i^2 + {}_2\omega_3 {}^2\mathcal{S}_i^3] + [{}_1\omega_2 {}^1\mathcal{S}_i^2 + {}_2\omega_3 {}^2\mathcal{S}_i^3]. \quad (7)$$

Note that the time rate of the passive joints A_i , B_i may be obtained by the dot product on both sides of Equation (7), by a vector which is perpendicular to the screw in the other two joints. Therefore, ${}^1\dot{\omega}_i^2$, denotes the time rate of the passive joints for the robot, as shown by Equation (8).

$${}^1\dot{\omega}_i^2 = \frac{({}^0\mathcal{S}_i^1 \times {}^2\mathcal{S}_i^3)^T V_O}{({}^0\mathcal{S}_i^1 \times {}^2\mathcal{S}_i^3)^T ({}^1\mathcal{S}_i^2)}. \quad (8)$$

Finally, the input–output equation of acceleration of 3-RRR planar parallel robot results in Equation (9).

$$J^T \Delta^0 A_p^3 = G\ddot{q} + \mathcal{A}_{lie}, \quad (9)$$

where, $\ddot{q} = [\ddot{q}_1 \ \ddot{q}_2 \ \ddot{q}_3]^T$; meanwhile, $J = [L_1 \ L_2 \ L_3]$ is the 3x3 reduced active screw-coordinate Jacobian matrix of the manipulator where the last terms in the rows were removed from the 6D vectors $Li(i = 1, 2, 3)$, because it is a planar mechanism. Furthermore, based on the theory of helicoidal vector fields [2], the reduced acceleration state of body 3 as measured from body 0 taking point 3 as the reference pole, and simulate the 6D vector, ${}^0A_p^3$, (section 4) [14].

3.4. Jerk analysis

Jerk analysis consists of a formulation of equations involving the time rate of acceleration linear and angular for bodies in relative motion, relevant for the mechanism performance in complex trajectories [17]. For the development of this analysis for the 3-RRR, the representation of the reduced jerk state of body 3 as measured from body 0 taking point, P , as the reference pole in the end-effector, and considering the conditions of helicoidal vector fields is represent with a 6D vector, as shown by Equation (10). Where, ρ , is the angular jerk in the end-effector.

$${}^0\mathcal{J}_p^3 = \begin{bmatrix} {}^0\rho^3 \\ {}^0\rho_p^3 - 2{}^0\alpha^3 \times {}^0V_p^3 - {}^0\omega^3 \times {}^3a_p^0 \end{bmatrix}. \quad (10)$$

The representation of Equation (11), in screw form may be expressed as follows:

$${}^0\mathcal{J}_p^3 = [{}_0\rho_1 {}^0\mathcal{S}_i^1 + {}_1\rho_2 {}^1\mathcal{S}_i^2 + {}_2\rho_3 {}^2\mathcal{S}_i^3 + \mathcal{J}_{lie}], \quad i = 1, 2, 3, \quad (11)$$

where, \mathcal{J}_{lie} , represent the Lie screw of jerk and is given by Equation (12).

$$\begin{aligned} \mathcal{J}_{lie} = & 2[{}_0\omega_1 {}^0\mathcal{S}_i^1 \alpha_2 {}^1\mathcal{S}_i^2 \alpha_3 {}^2\mathcal{S}_i^3] + [{}_1\omega_2 {}^1\mathcal{S}_i^2 \omega_3 {}^2\mathcal{S}_i^3] + 2[{}_1\omega_2 {}^1\mathcal{S}_i^2 \alpha_3 {}^2\mathcal{S}_i^3] + [{}_0\alpha_1 {}^0\mathcal{S}_i^1 \omega_2 {}^1\mathcal{S}_i^2 \omega_3 {}^2\mathcal{S}_i^3] \\ & + [{}_1\alpha_2 {}^1\mathcal{S}_i^2 \omega_3 {}^2\mathcal{S}_i^3] + [{}_0\omega_1 {}^0\mathcal{S}_i^1 [{}_0\omega_1 {}^0\mathcal{S}_i^1 \omega_2 {}^1\mathcal{S}_i^2 \omega_3 {}^2\mathcal{S}_i^3]] + [{}_1\omega_2 {}^1\mathcal{S}_i^2 [{}_1\omega_2 {}^1\mathcal{S}_i^2 \omega_3 {}^2\mathcal{S}_i^3]]. \end{aligned} \quad (12)$$

In this case, the joint-acceleration rates are computed according to Equation (13) as follows:

$$\dot{\Omega}_i = J_i^{-1}({}^0A_p^3 + \mathcal{A}_{lie}), \quad (13)$$

where, $\dot{\Omega}_i = [{}^0\dot{\omega}_1^i \quad {}^1\dot{\omega}_2^i \quad {}^2\dot{\omega}_3^i]$, is a matrix containing the the i th limb, as show in Equation (14).

$$B^0 \mathcal{J}_p^3 = C \dot{q} + \mathcal{J}_{lie}. \quad (14)$$

Finally, the input–output equation of jerk of 3-RRR planar parallel robot results in Equation (14); where, $\ddot{q} = [\ddot{q}_1 \quad \ddot{q}_2 \quad \ddot{q}_3]^T$, is the third-order driver matrix of the parallel robot.

3.5. Hyper-jerk analysis

The hyper-jerk analysis consists of a formulation of equations involving the time rate of linear and angular jerk for bodies in relative motion [2]. The authors considered is relevant performing this type of analysis because the 3-RRR planar parallel robot has singular configurations that could be architecturally mobile according to hyper-jerk analysis [18]. The reduced hyper-jerk state of end-effector with respect to active joint, represented in 6D vectors, ${}^0H_p^3$, is given by the Equation (15).

$${}^0H_p^3 = \begin{bmatrix} {}^0\sigma^3 \\ {}^0\sigma_p^3 - 3{}^0\rho^3 \times {}^0v_p^3 - 3{}^0\alpha^3 \times {}^0a_p^3 - {}^0\omega^3 \times {}^3\rho_p^0 \end{bmatrix}. \quad (15)$$

Equation (15) may be written in screw form through the limbs of the 3-RRR, as follows in Equation (16).

$${}^0H_p^3 = {}^0\sigma_1^i {}^0\mathcal{S}_i^1 + {}^1\sigma_2^i {}^1\mathcal{S}_i^2 + {}^3\sigma_3^i {}^3\mathcal{S}_i^3 + \mathcal{H}_{lie}, \quad i = 1, 2, 3, \quad (16)$$

where, \mathcal{H}_{lie} , is the Lie screws of hyper-jerk for each kinematic chain of the robot. \mathcal{H}_{lie} is computed as follow in Equation (17).

$$\begin{aligned} \mathcal{H}_{lie} = & 3[{}^0\omega_1^0 \mathcal{S}_i^1 \rho_2^1 \mathcal{S}_i^2 + {}^2\rho_3^2 \mathcal{S}_i^3] + 3[{}^1\omega_2^1 \mathcal{S}_i^2 \rho_3^2 \mathcal{S}_i^3] + 3[{}^0\alpha_1^0 \mathcal{S}_i^1 \alpha_2^1 \mathcal{S}_i^2 + {}^2\alpha_3^2 \mathcal{S}_i^3] \\ & + 3[{}^1\alpha_2^1 \mathcal{S}_i^2 \alpha_3^2 \mathcal{S}_i^3] + 3[{}^0\omega_1^0 \mathcal{S}_i^1 [{}^0\omega_1^0 \mathcal{S}_i^1 \alpha_2^1 \mathcal{S}_i^2 \alpha_3^2 \mathcal{S}_i^3]] + 3[{}^1\omega_2^1 \mathcal{S}_i^2 [{}^1\omega_2^1 \mathcal{S}_i^2 \alpha_3^2 \mathcal{S}_i^3]] \\ & + 2[{}^0\alpha_1^0 \mathcal{S}_i^1 [{}^0\omega_1^0 \mathcal{S}_i^1 \omega_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3]] + 2[{}^1\alpha_2^1 \mathcal{S}_i^2 [{}^1\omega_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3]] + [{}^0\rho_1^0 \mathcal{S}_i^1 \omega_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3] \\ & + [{}^1\rho_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3] + [{}^0\omega_1^0 \mathcal{S}_i^1 [{}^0\alpha_1^0 \mathcal{S}_i^1 \omega_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3]] + [{}^1\omega_2^1 \mathcal{S}_i^2 [{}^1\alpha_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3]] \\ & + [{}^0\omega_1^0 \mathcal{S}_i^1 [{}^0\omega_1^0 \mathcal{S}_i^1 [{}^0\omega_1^0 \mathcal{S}_i^1 \omega_2^1 \mathcal{S}_i^2 \omega_3^2 \mathcal{S}_i^3]]]. \end{aligned} \quad (17)$$

Finally, the input–output equation of hyper-jerk results in Equation (18).

$$J^T \Delta^0 H_p^3 = [\ddot{q}_1 \{{}^0\mathcal{S}_1^1; \mathcal{S}_1\} \ddot{q}_2 \{{}^0\mathcal{S}_2^1; \mathcal{S}_2\} \ddot{q}_3 \{{}^0\mathcal{S}_3^1; \mathcal{S}_3\}]^T + \{\mathcal{S}_1 \mathcal{H}_{lie1}\} \{\mathcal{S}_2 \mathcal{H}_{lie2}\} \{\mathcal{S}_3 \mathcal{H}_{lie3}\}]^T, \quad (18)$$

where, for each limb the Lie screw of hyper-jerk, \mathcal{H}_{lie} , is calculated according to Equation (17); meanwhile, $\ddot{q} = [\ddot{q}_1 \quad \ddot{q}_2 \quad \ddot{q}_3]^T$, is the fourth-order driver matrix of the parallel robot.

4. Simulation and results

To verify the theoretical developments and show the application of higher-order kinematic analysis described above, a series of numerical simulations were developed, as detailed in this section using MATLAB environment. Freeth's Nephroid was the trajectory selected, as shown in Figure 2, which is a strophoid (tritrochoid) of a circle with respect to two points, with a characteristic polar equation $r = a[1 + 2 \sin(\frac{1}{2}\theta)]$, where the time selected for the simulation is in the interval, $0 \leq t \leq 2\pi$. The details of the geometric parameters of 3-RRR planar parallel robot are: $x_{o1} = y_{o1} = 0$, $x_{o2} = d = 0.35$ m, $y_{o2} = 0$, $x_{o3} = d/2 = 0.175$ m, $y_{o3} = d/\sqrt{3} = 0.202$ m, $l_1 = l_2 = 0.25$ m, $l_3 = 0.07$ m.

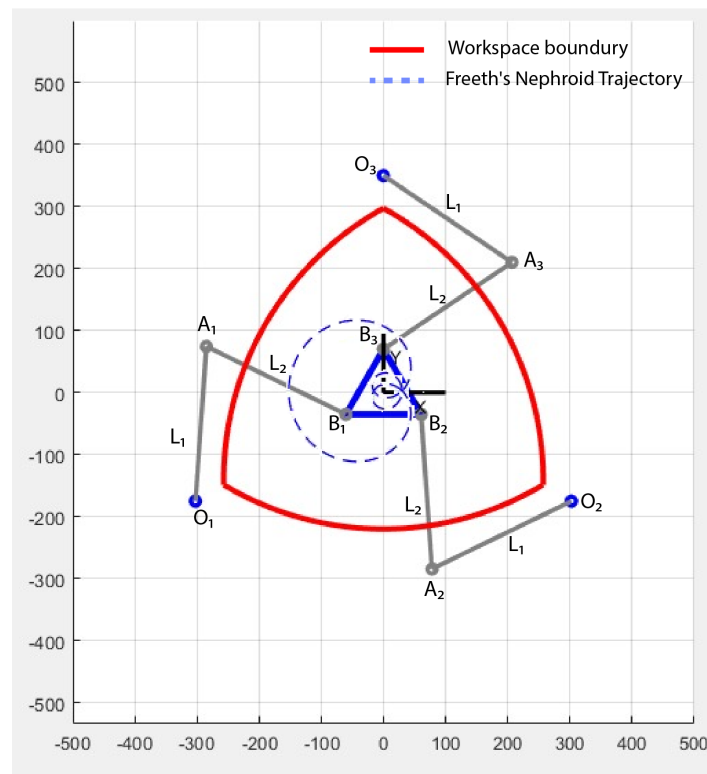


Figure 2. Tracking freeth's nephroid trajectory using the 3-RRR planar parallel robot.

The relevant results are presented in Figure 3, where time history for angular velocity, acceleration, jerk and hyper-jerk of the center of the end-effector are provided. Figure 3(a) shows the behavior of angular velocity for the desired path was smooth and oscillates with a maximum of 0.4 rad/s, consistent with the screw-coordinate Jacobian matrix. In addition, Figure 3(b), presents the acceleration state where the effect of Lie's accelerator is observed, constraining it to a maximum of 1 rad/s², avoiding abrupt changes in the end-effector. Moreover, Figure 3(c) represents the jerk analysis that allows to check the probability of resonance or chatter vibrations in the end-effector, showing acceptable behavior, not exceeding 0.5 rad/s³. Finally, Figure 3(d) it is observed that if we increase the order of jerk to Hyper-jerk, it is possible to identify the singular areas and the mechanism's performance trying to avoid them. Therefore, it is evident for the kinematics developed the end-effector tracks the desired path with high precision and smooth movements inside the workspace, considering areas of instability, as result of the singular configurations present in the robot. Likewise, it is noted that the kinematics performance has a good performance, that justifies this type of high-order analysis.

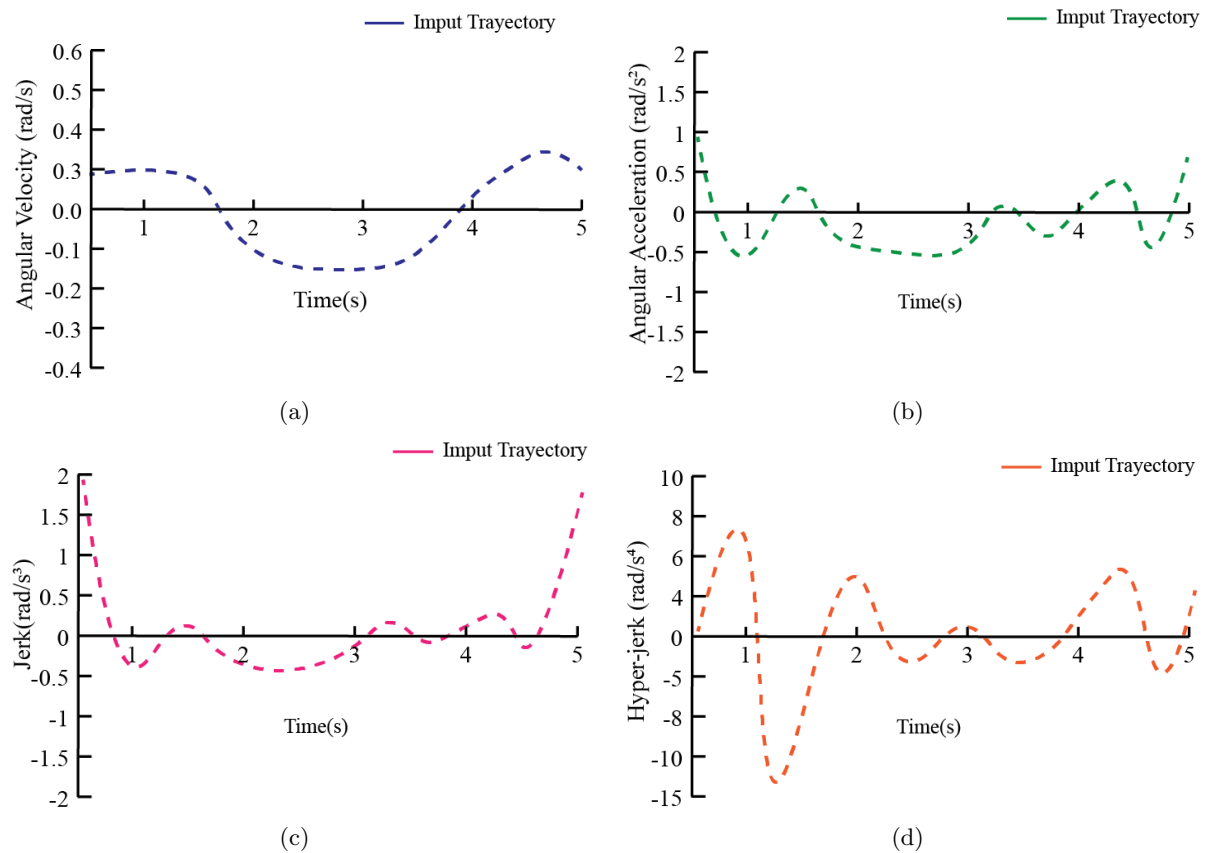


Figure 3. (a) Time history for angular velocity, (b) acceleration, (c) jerk, (d) and Hyper-jerk.

5. Conclusion

In this paper a 6D compact higher-order kinematics was developed, applying screw theory and Lie algebra $SE(3)$ for a 3-RRR planar parallel robot, proved has greater advantages with high precision, reliability and smaller processing time than traditional 3D classical formulations. The validation was achieved by tracking the end-effector of mechanism under a complex path-planning, with results showing a smooth and good performance, considering the singularities of this mechanism. This methodology could be extended when the complexity and the number of links in the robot increases which will be studied in future research.

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