ARTICLE IN PRESS

Engineering Science and Technology, an International Journal xxx (xxxx) xxx

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Contents lists available at ScienceDirect

Engineering Science and Technology, an International Journal

journal homepage: www.elsevier.com/locate/jestch

Full Length Article

Vortex search and Chu-Beasley genetic algorithms for optimal location and sizing of distributed generators in distribution networks: A novel hybrid approach

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ARTICLE INFO

Article history:
Received 2 June 2019
Revised 3 July 2020
Accepted 3 August 2020
Available online xxxx

Keywords:
Combinatorial optimization
Distributed generators
Genetic algorithm
Distribution networks
Power losses reduction
Vortex search optimization algorithm

ABSTRACT

In this study, we analyzed the optimal location and sizing of distributed generators (DGs) in radial distributed networks using a hybrid master-slave metaheuristic technique. The master stage corresponds to the selection of suitable points for the locations of the DGs, whereas the slave stage is the optimal dimensioning problem. The Chu-Beasley genetic algorithm (CBGA) is employed to solve the master stage, and the optimal power flow (OPF) method via the vortex search algorithm (VSA) is employed to solve the slave stage. The OPF solution from the VSA technique uses a successive approximation power flow to determine the voltage profiles and power losses by guaranteeing the energy balance in all the nodes of the network. The conventional and widely used 33- and 69-node test feeders are used to validate the hybrid CBGA-VSA for analyzing the optimal location and sizing of the DGs in the distribution networks using MATLAB software. The numerical results demonstrate the efficiency of the proposed optimization method in terms of power loss reduction as compared with the results available in the literature. An additional 24-h dimensioning analysis is included for demonstrating the efficiency and applicability of the proposed methodology for daily operations with renewable generation.

1. Introduction

1.1. General context

Electrical networks are shifting from a classical vertical connection, i.e., generation, transmission, distribution, and commercialization where generation power plants are typically located far from the consumers (power systems with hydro-dominated generation, as in the case of Colombia), to horizontal connections where the loads and generators are located closer to each other [1]. This paradigm shift is induced by the rapid advancement in power electronics by the integration of distributed energy resources at distribution levels [2,3]. The integration of these devices (renewable generation and energy storage technologies) has improved the technical operating conditions of distribution networks (medium-voltage AC networks), in terms of an improvement in voltage profile, a reduction of active and reactive power losses, and the possibility of attending to new users [4–8].

Classical distributed energy resources integrated into distribution networks include distributed generators (DGs) that provide active power for minimizing the total power loss. In some cases, DGs provide reactive power support, which increases their impact with regard to power losses [9]. In addition, capacitor banks are integrated into AC networks to support a part of the reactive power consumed by constant power loads, which help increase the voltage profiles and reduce power loss. In the case of energy storage devices [10], batteries (chemical storage devices) are used for supporting the power in grids with a high penetration of renewable generators to deal with the weather dependency of the latter devices [1,11].

Regarding the distributed generation, several challenges need to be addressed for installing plants at optimal locations and operating them in medium-voltage distribution systems. Some of the solutions to these challenges are as follows [2]:

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https://doi.org/10.1016/j.jestch.2020.08.002
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- Define the optimal location of these devices as a function of the grid topology and load conditions.
- Consider renewable energy resources with daily energy profiles for providing a suitable power dispatch.
- Solve the nonlinear, nonconvex mixed-integral programming model that represents the optimal location and distribution operation with adequate processing times.
- Consider the problems of metaheuristic optimization techniques that introduce uncertainties in planning due to its non-deterministic structure.

The aforementioned challenges are the classical problems related to the optimal location of DGs in AC distribution networks. These problems allow providing efficient optimization methodologies and robust solutions, considering free computational toolboxes with low computational efforts, and the possibility of achieving optimal solutions [4,12]. In the following section, the motivation of this study is presented.

1.2. Motivation

The DGs need to be placed at optimal locations in AC networks, to reduce fossil fuel consumption [13,14]. Pollution from fossil fuels has been a severe worldwide concern, which led to the signing of the Paris Agreement [1]. The Colombian government is no stranger to this issue; in 2014, the country's national congress approved Law 1715, which is related to the integration of renewable resources into the power system [15]. This law has promoted multiple renewable generation projects primarily based on photovoltaic (PV) and wind energy, ranging from tens of kilowatts to hundreds of megawatts, throughout the country. Currently, the Colombian electrical network generates 16800 MW of power from the power system, with the following distribution: 66% hydraulic, 28% thermal, and approximately 6% from smaller plants. However, it is expected that the Colombian electrical system will experience significant changes in its energy matrix by 2023, integrating at least 1.7 GW of wind plants and solar PV that currently have a connection concept from the mining-energetic planning unit UPME. According to UPME, these numbers are expected to increase significantly in the future, as requests for the connection of approximately 10 GW of solar and wind plants have been received [16].

1.3. Review of the state-of-the-art techniques

In the literature, the problem of the optimal location and placement of DGs in AC medium-voltage distribution systems has been widely studied using metaheuristic optimization techniques and nonlinear optimizers. These techniques are required because they correspond to a mixed-integer nonlinear programming (MINLP) problem, which is discrete due to the presence of binary variables in the location of the DGs [12]. In general, the location and sizing of DGs in distribution networks corresponds to an extension of the classical optimal power flow (OPF) problem with discrete variables [17]. The combination of continuous and discrete variables leads to the preference of metaheuristic techniques in addressing the MINLP problem [18]. These techniques allow the decoupling of the location problem to the sizing problem by proposing master–slave optimization methodologies [18,19], where the master stage locates the DGs, and the slave stage is responsible for their sizing.

In the master stage, multiple discrete optimizers have been proposed for selecting the candidate points suitable for the locations of the DGs. Classical methodologies include genetic algorithms [20,5], simulated annealing methods [21], krill herd algorithms [22–24], tabu search algorithms [25], population-based incremental learning [2], teaching-based learning optimizers [26], bat and firefly algorithms [27–30], harmonic search algorithms [31], imperialist competitive algorithms [18], symbiotic organism search algorithms [32], and bio-geography-based algorithms [33].

When the sizing problem is analyzed at the slave stage, the challenge is to determine the OPF dispatch for some previously located DGs, which is a classical problem known as the "OPF problem" [34]. To this end, the most classical slave algorithm is the particle swarm optimizer because it guarantees optimal solutions to continuous problems (comparable to convex and interior point approaches [5,35]). It is also easily implemented using any free programming language [2,36]. Nevertheless, multiple recently proposed OPF methods are also suitable for dimensioning the DGs, such as the black hole optimizer [6], sine–cosine optimization algorithm [37], and vortex-search algorithm [3].

Remark 1. The use of pure-algorithm approaches such as master–slave optimizers requires that the slave stage be embedded in a classical power flow approach for solving the power balance equations, as these approaches are nonlinear and require numerical methods for analysis. Typically used methods include the Newton–Raphson method [2,33,38], classical backward/forward power flow [39,40], and recent approaches such as successive approximations [37,3,9].

Additionally, some approaches are based on branch-and-bound methods that deal with the problem of the optimal location and sizing of DGs simultaneously, as reported in [12,41]. Nevertheless, due to the nonconvexities in the MINLP problem, these methods become stuck in local optimum solutions in some cases. Therefore, metaheuristics are preferred for the problem reported in this study.

However, some studies proposed a heuristic method for locating and sizing DGs in distribution grids. Some of these studies were based on loss-sensitivity factors [2,38], stability index [36], and search methods over tree graphs [9]. Although these methods are easy to implement and have lower computational times, they are usually stuck in local optimums.

In the following section, we present the proposed master–slave optimization algorithm and the main contributions of this study.

1.4. Contribution and scope

This study deals with the solution of the MINLP model for the problem of the optimal location and sizing of DGs in electrical AC distribution networks. We propose a master–slave optimization algorithm based on the classical Chu-Beasley genetic algorithm (CBGA) at the master stage and the vortex search algorithm (VSA) at the slave stage. To the best of our knowledge, this hybrid optimization approach has not been previously reported. The main advantages of the proposed approach are as follows:

- There is a possibility of achieving the optimal global solution of the OPF problem by implementing the VSA and successive approximation power flow method, which guarantees an adequate solution for each possible location of the DGs as demonstrated by [3,42,9].
- An integer codification of the CBGA facilitates a more straightforward implementation with a small matrix that corresponds to the population, which is efficient in comparison with classical binary formulations. This is because the infeasibilities caused by recombination and mutation operators are eliminated in the proposed codification.

Furthermore, this study focuses on the location and sizing of DGs in AC networks, considering a unity power factor. In contrast to previously used methods, we include the Colombian daily operative curves of load consumption and PV generators to determine the

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optimal sizing of DGs. This approach is different from the mutated salp swarm algorithm proposed by [10], which focused on the optimal location of PV generators without considering the daily load and generation variations. In addition, our approach is different from [43], which focused on the sizing of the DGs in electrical distribution networks without considering the optimal location problem.

Remark 2. The proposed master–slave CBGA-VSA approach is more efficient than the recently proposed approaches. The proposed approach achieves the best-reported optimal solution with negligible standard deviations and low computational times. Additionally, it is a pure-algorithmic approach easily implementable in any programming package without requiring a specialized optimization software.

1.5. Document organization

The remainder of this paper is organized as follows: Section 2 presents the compact mathematical formulation of the problem using complex variables. Section 3 shows the master–slave methodology by presenting the proposed integer CBGA in conjunction with the slave-stage approach based on the VSA and successive approximation power flow method. Section 4 presents the configuration of the test systems composed of the conventional 33- and 69-node test feeders. Section 5 presents the numerical results for the load peak condition and daily operative scenario considering PV generators. Section 6 highlights the concluding remarks and suggests possible future studies.

2. Mathematical modeling

The optimal location and dimensioning of DGs in radial distribution networks correspond to a mixed-integer nonlinear optimization problem [12], which is nondifferentiable and nonconvex with multiple local minimums and nondeterministic polynomial time [37]. The mathematical model for this problem is expressed as follows:

Objective function:

$$\min p_{\text{loss}} = \text{real}\{\mathbb{V}^T \mathbb{Y}_L^{\star} \mathbb{V}^{\star}\}, \tag{1}$$

where p_{loss} corresponds to the objective function value associated with the power loss in all the branches of the network, $\mathbb{V}$ is a vector that contains all the voltage profiles in complex form, $\mathbb{Y}_L$ is the complex component of the admittance matrix associated with the branches, and real$\{\cdot\}$ represents the real value of the complex number contained in its argument.

Set of constraints:

$$\mathbb{S}_{CG}^{\star} + \mathbb{S}_{DG}^{\star} - \mathbb{S}_{D}^{\star} = \textbf{diag}(\mathbb{V}^{\star})[\mathbb{Y}_L + \mathbb{Y}_N]\mathbb{V}, \tag{2}$$

$$V^{\min} \leqslant |\mathbb{V}| \leqslant V^{\max}, \tag{3}$$

$$S_{GC}^{\min} \leqslant |\mathbb{S}_{GC}| \leqslant S_{GC}^{\max}, \tag{4}$$

$$x_{DG} S_{DG}^{\min} \leqslant |\mathbb{S}_{DG}| \leqslant x_{DG} S_{DG}^{\max}, \tag{5}$$

$$x_{DG}^T \mathbb{S}_{DG} \leqslant \alpha \mathbf{1}^T \text{real}\{\mathbb{S}_D\}, \tag{6}$$

$$\textbf{imag}\{\mathbb{S}_{DG}\} = 0, \tag{7}$$

$$\mathbf{1}^T x_{DG} \leqslant N_{DG}, \tag{8}$$

where $\mathbb{S}_{CG}$ and $\mathbb{S}_{DG}$ are the complex vectors of power generation from conventional generators and DGs, respectively; $\mathbb{S}_D$ represents the complex vector of power consumptions; $\mathbb{Y}_N$ is the component of the admittance matrix related to the loads modeled as constant impedances (note that $\mathbb{Y} = \mathbb{Y}_L + \mathbb{Y}_N$); $V^{\min}$ and $V^{\max}$ are the minimum and maximum magnitudes, respectively, allowed for all the voltage profiles in the network; $S_{GC}^{\min}$ and $S_{GC}^{\max}$ represent the minimum and maximum power limits in the conventional generators, respectively; $S_{DG}^{\min}$ and $S_{DG}^{\max}$ correspond to the minimum and maximum power bounds for the DGs, respectively; x_{DG} is the vector containing all the binary variables related to the location of a DG in the grid; α defines the percentage of penetration of the distributed generation in the network; N_{DG} is the maximum number of DGs allowed for installation; **1** is a vector filled by ones with an appropriate dimension. In addition, **diag**$(\cdot)$ generates a diagonal matrix of the vector contained in its argument and **imag**$\{\cdot\}$ extracts the imaginary component of the complex number in its argument.

The interpretation of the mathematical model expressed by Eqs. (1)–(8) is as follows: the objective function (1) represents the minimization of the active power loss in all the resistive effects related to the branches of the network. Expression (2) presents the complex power balance in all the nodes of the grid, i.e., active and reactive power balance; the acceptable bounds of voltage regulation are defined in Eq. (3). The maximum apparent power capability in the slack node is defined in Eq. (4); in expression (5), the apparent power capability of each DG is bounded by considering x_{DG} as a binary decision variable that can activate its location in the network. Eq. (6) limits the penetration of the total distributed generation in the network by the percentage factor of penetration, denoted α. In Eq. (7), the reactive power capability of the distributed generation is nulled, which indicates that all the DGs that will be installed in the network operate with unity power factor. Finally, Eq. (8) bounds the maximum number of DGs allowed for allocation to the network.

Remark 3. The optimization model (1)–(8) is nonlinear and nonconvex for two main reasons [22]. First, the power balance equation is a non-affine constraint that involves products between voltage variables. Second, the presence of binary variables makes the problem unsolvable using classical nonlinear methods due to the impossibility of using derivatives in its analysis.

Based on the complexity of the optimization problem, metaheuristic optimization tools are required to address the nonlinear nonconvexities for the problems of the optimal location and sizing of DGs in distribution networks. In the following section, we present a new hybrid approach based on a classical CBGA with an integer formulation that operates together with an emerging continuous optimizer named VSA. This optimization approach produces the best optimal solution reported in the literature with an easy implementation structure (i.e., pure-algorithmic method), without approximating (linearizing) the power flow equations or using sensitive indicators for reducing the size of the solution space.

3. Proposed methodology

The solution of the optimal location and sizing of the DGs in the distribution networks based on metaheuristics is typically obtained using master–slave optimization strategies [2]. At the master stage, a discrete optimization algorithm (i.e., CBGA) defines the optimal location of all the DGs. At the slave stage, the OPF is solved by a continuous optimizer (i.e., VSA). When the OPF problem is solved using a continuous metaheuristic approach, a conventional power flow method is required for solving the power balance equations (see (2)) [3]. Here, we employ an emerging power flow method named the successive approximation method, as it is applicable to radial and mesh grid structures [37].

3.1. Chu-Beasley genetic algorithm

CBGA is a classical metaheuristic approach with the capability of solving continuous and discrete optimization problems by

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implementing three basic rules of evolution [5,44,45], namely, selection, recombination, and mutation. These rules are applied to individuals that conform to a population (potential solution in optimization problems) from an initial to the final state after multiple generations. To summarize it, "genetic algorithms represent the capability of survival of the strongest population (animal) over time.".

Here, we present the general concepts for implementing the genetic algorithm proposed by Chu-Beasley, as reported in [46], to select the candidate nodes for the optimal location of DGs.

3.1.1. Codification

Traditionally, CBGA is applied to binary populations. For the problem of the optimal location of DGs, an individual x has the dimensions $1 \times (n-1)$, where the distribution network has n nodes and each position x_i can be either zero or one. An example of a classical individual is presented in Fig. 1.

Such a higher occurrence indicates that the computational processing time increases owing to the additional calculations required in correcting the infeasibilities for each generational cycle. To simplify this, we propose a discrete combination where each individual takes the form presented in Fig. 2.

The proposed codification can contain each position number between 2 and n, i.e., it is possible to locate a DG at all the nodes except the slack node, which is typically located at node 1. In addition, an individual x_i does not repeat nodes to maintain this codification.

3.1.2. Generation of the initial population

Now, if each individual x_i is placed inside a vector, the population will take the form of a matrix with the dimensions $N_I \times N_{DG}$, where N_I is the number of individuals in the population. Observe that t is the iterative counter.

$$x^t = \begin{bmatrix} x_1^t \\ x_2^t \\ \vdots \\ x_{N_I}^t \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1N_{DG}} \\ x_{21} & x_{22} & \cdots & x_{2N_{DG}} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_I 1} & x_{N_I 2} & \cdots & x_{N_I N_{DG}} \end{bmatrix}. \tag{9}$$

Each component of the initial population x_{ij} is a random natural number between 0 and n, and the following constraint must be fulfilled to preserve the feasibility of the solution:

$$x_{ij} \neq x_{ik}, \ \forall k = 1, 2, \ldots, N_{DG} \tag{10}$$

with $k \neq j$.

3.1.3. Fitness function evaluation

As in any metaheuristic optimization procedure, the development through the solution space is guided by a fitness function that helps deal with infeasibilities. This function corresponds to an adaptation of the constraints as penalizations. To propose a fitness function in this study for CBGA, we shall make the following assumptions:

2	3	4	$\cdots$	$n-1$	n
0	0	1	$\cdots$	1	0

Fig. 1. Classical codification in genetic algorithms.

1	2	3	$\cdots$	$N_{DG}-1$	N_{DG}
2	$n-3$	5	$\cdots$	$n-10$	8

Fig. 2. Proposed codification.

Assumption 1. The power balance Eq. (2) is guaranteed through a power flow methodology, as it has strong nonlinearities in equality that can be solved numerically using power flow tools. This will be addressed later in the methodology.

Assumption 2. The capability of the slack generator is unlimited, which indicates that expression (4) is fulfilled, including the initial case, i.e., when the distribution networks do not include the DGs.

Assumption 3. The constraints associated with the power outputs at the distributed generation (see (5)–(7)) will be satisfied by the VSA in the slave stage, which will be discussed later in this paper.

Note that expression (8) is directly fulfilled by the way the initial population has been created. Moreover, if we consider the aforementioned assumptions, then the fitness function employed by our proposed CBGA takes the following form:

$$z_{f_1} = p_{loss} - \beta_1 \left(\min\left\{0, \mathbb{V} - \mathbb{V}^{\min}\right\} - \min\left\{0, \mathbb{V}^{\max} - \mathbb{V}\right\} \right), \tag{11}$$

where β_1 is a penalty factor that penalizes the voltage deviations in the grid for a particular generation condition.

Now, considering the structure of the fitness function (11), all the individuals in the population are evaluated as $z_{f_1}(x)$.

Remark 4. The evaluation of the objective function requires the voltage profiles, which indicates that the evaluation of z_{f_1} is indispensable in the slave stage. This evaluation is the VSA in conjunction with a new power flow approach named the successive approximation method.

3.1.4. Selection

The selection in the proposed CBGA is the initial step for obtaining a potential offspring individual that will be included in the population. Here, we select four arbitrary individuals contained in the population, which are submitted to a tournament according to a fitness function. Two of the chosen individuals have lower fitness values (best solution in terms of power loss).

3.1.5. Recombination

In this step, we generate two offspring individuals by recombining their genetic information. Suppose that the individuals x_i^t and x_j^t have been obtained after the selection process as follows:

$$\begin{aligned} x_i^t &= [x_{i1} \quad x_{i2} \quad \cdots \quad x_{ik} \quad \cdots \quad x_{iN_{DG}}], \\ x_j^t &= [x_{j1} \quad x_{j2} \quad \cdots \quad x_{jk} \quad \cdots \quad x_{jN_{DG}}], \end{aligned} \tag{12}$$

Now, assume that an arbitrary position of the vector is selected as the recombination point, i.e., $k-1$. Therefore, the individuals x_i^{t+1} and x_j^{t+1} can recombined as follows:

$$\begin{aligned} x_i^{t+1} &= [x_{i1} \quad x_{i2} \quad \cdots \quad x_{jk} \quad \cdots \quad x_{jN_{DG}}], \\ x_j^{t+1} &= [x_{j1} \quad x_{j2} \quad \cdots \quad x_{ik} \quad \cdots \quad x_{iN_{DG}}]. \end{aligned} \tag{13}$$

Remark 5. If repeated components in the offspring individual x_i^{t+1} and x_j^{t+1} appear during the recombination process, each of the offspring individuals must be corrected by eliminating the repeated components, until rule (10) is guaranteed.

3.1.6. Mutation

Once the two potential individuals are generated as given in expressions (13), an arbitrary position of each individual is mutated (changed) by replacing its value with an alternative value between 2 and n, always guaranteeing feasibility on each one.

3.1.7. Population replacement

To decide if one of the individuals in the current population x^t is replaced by x_i^{t+1} and x_j^{t+1} to generate a new population, i.e., x^{t+1}, it is necessary to evaluate the fitness function of each offspring individual. Then, x_i^{t+1} is selected instead of x_j^{t+1} only if $z_{f_1}\left(x_i^{t+1}\right) \leqslant z_{f_1}\left(x_j^{t+1}\right)$; otherwise, x_j^{t+1} is selected.

For the selected individual (e.g., x_i^{t+1}) to join the population, two conditions must be satisfied:

- The fitness function $z_{f_1}\left(x_i^{t+1}\right)$ is at least better than or equal to that of the worst individual contained in the population.
- Its genetic material (structure) is different from that of all the individuals contained in the population (aspiration criterion).

An example of a possible offspring population is presented as follows:

$$x = \begin{bmatrix} x_1^t & x_i^{t+1} & \cdots & x_{N_I}^t \end{bmatrix}^\top$$

Remark 6. The CBGA only replaces one individual in the current population at each time, which considerably improves its processing times compared with that of the classical genetic algorithm approach.

3.1.8. Stopping criteria

The search through the solution space in the proposed CBGA stops if one of the following two criteria is satisfied:

- The maximum number of iterations $t_{\max}$ is reached.
- The best individual in the population x^t has not been modified after $c_{\max}$ consecutive iterations.

Note that $c_{\max}$ is selected as a percentage of the total iterations; here, we select it as 25%, as recommended in [2].

3.1.9. Algorithmic implementation of the CBGA

Algorithm 1 presents the necessary steps for implementing a CBGA for the optimal location and sizing of DGs in radial distribution networks.

Algorithm 1 Proposed master-slave optimization approach for the optimal location and sizing of DGs in AC distribution networks

Inputs:
Read the data of the AC network;
Define the number of generators available, i.e., N_{DG};
Define the percentage of penetration of distributed generation i.e., α;
Generate the initial population x^0;
Evaluate the fitness function for each individual in the population (see the slave stage);
Make $t = 0$;
while $t \leqslant t_{\max}$ **do**
  Create the tournament for selecting four parents;
  Make recombinations for generating four offspring individuals;
  Apply the mutation operator to the offspring population;

Algorithm 1 (*continued*)

Algorithm 1 Proposed master-slave optimization approach for the optimal location and sizing of DGs in AC distribution networks

  Evaluate the fitness function of the offspring individuals, i.e., slave stage;
  Select the offspring with the minimum fitness function;
  Replace the worst individual in the population with the selected offspring if the offspring has the best objective function and fulfills the diversity criterion;
  if $c \geqslant c_{\max}$ **then**
    Select as the solution of the problem, the individual with the best objective function in the current population x^t;
    Return the optimal solution concerning the location and sizing of DGs (see the slave stage);
    break;
  end if
  $t = t + 1$;
end while
Output:
The best solution is found for the MINLP model;

3.2. Vortex search algorithm

The VSA is an optimization technique for solving continuous problems based on the behavior of the stirred fluids that generate vortex demeanors in pipes [3,47]. A significant advantage of this optimization method in OPF analysis is that it operates with a Gaussian distribution and variable radius that allow for the exploration and exploitation of the solution space [48]. The authors of [3,9] have proven that this method converges to the optimal solution in power flow problems with minimal standard deviations [47]. Subsequently, we present the main steps in solving the OPF problems in AC grids using the VSA technique combined with the successive approximation power flow method.

The VSA technique operates with nonconcentric hyperspheres, where the outer diameter represents the boundaries of the solution space and the center of the hypersphere corresponds to the current solution. Initially, the center of the hypersphere is defined as follows:

$$\mu_0 = \frac{y^{\max} + y^{\min}}{2}, \tag{14}$$

where $y^{\max}$ and $y^{\min}$ are $d \times 1$ vectors that define the upper and lower bounds, respectively, in an optimization problem inside a d-dimensional space. The number of solutions in the neighborhood is denoted $C_\tau(s)$, where τ is the iteration number. Initially, $\tau = 0$ and $C_0(s)$ is generated by a random process using a Gaussian distribution in the d-dimensional space. Here, $C_0(s) = \{s_1, s_2, \ldots, s_n\}$, where n represents the number of candidate solutions. Note that a general Gaussian distribution can be defined in a multivariable space as follows:

$$p(x|\mu,\Sigma) = \left((2\pi)^d|\Sigma|\right)^{-\frac{1}{2}} exp\left\{-\frac{1}{2}(y-\mu)^T\Sigma^{-1}(y-\mu)\right\} \tag{15}$$

where $y \in \mathbb{R}^{d\times 1}$ corresponds to a random vector of variables, $\mu \in \mathbb{R}^{d\times 1}$ represents a simple mean (center) vector, and $\Sigma \in \mathbb{R}^{d\times d}$ corresponds to the covariance matrix.

If the elements of the covariance matrix are equal and the off-diagonal elements of this matrix are considered as zero, then the

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resulting form of the Gaussian distribution will be generated as hyperspheres in a d-dimensional space [49].

A simple form to calculate Σ, considering zero covariances and equal variances is

$$\Sigma = \sigma^2 \mathscr{I}_{d\times d}, \tag{16}$$

where σ is the variance of the Gaussian distribution and $\mathscr{I}_{d\times d}$ is an identity matrix. Note that the standard deviation of the Gaussian distribution (i.e., σ_0) can be defined as follows:

$$\sigma_0 = \frac{\max\{y^{\max}\} - \min\{y^{\min}\}}{2}, \tag{17}$$

where σ_0 is considered the radius of the hypersphere in a d-dimensional space, i.e., r_0. To attain an adequate exploration in the solution space, σ_0 is the biggest possible hypersphere initially. During the search process, its radius decreases closer to the optimal solution. At the selection stage, the best solution $s^{0,*} \in C_0(s)$ is selected and memorized to modify the current center of the hypersphere μ_0. Each solution s_k^l needs to lie within its limits before the selection step. The following rule is employed for this purpose:

$$s_k^l = \begin{cases} (y_l^{\max} - y_l^{\min})r_l + y_l^{\min}, & s_k^l < y_l^{\min} \\ s_k^l, & y_l^{\min} \leqslant s_k^l \leqslant y_l^{\max} \\ (y_l^{\max} - y_l^{\min})r_l + y_l^{\min}, & s_k^l > y_l^{\min} \end{cases} \tag{18}$$

where $k = 1, 2, \ldots, n$, $l = 1, 2, \ldots, d$, and r_l represent a uniformly distributed random number between 0 and 1. Note that the best solution $s^{\tau,*} \in C_\tau(s)$ is updated if the current solution is better, which produces an updating of the center μ_τ and its radius r_τ.

One of the most critical procedures in the implementation of the VSA is the adaptive adjustment of the radius of the hypersphere using a variable-step approach [50]. To perform this task, we consider an exponential decrement of the radius, as recommended in [47]. This exponential form is expressed as

$$r_\tau = \sigma_0\left(1 - \frac{\tau}{\tau_{\max}}\right)e^{-a\frac{\tau}{\tau_{\max}}}, \tag{19}$$

where the parameter a is defined heuristically as 6, and $\tau_{\max}$ is the total number of iterations [47].

Remark 7. The VSA technique is used in this study to solve the resulting OPF problem (i.e., to determine the values of $\mathbb{S}_{DG}$) once the CBGA defines the location of the DGs, i.e., it determines the optimal sizing of these DGs by evaluating the fitness function (11).

The evaluation of the fitness function requires the solution of the resulting power flow problem. Here, we employ the successive approximation method reported in [3], which is formulated from Eq. (2). Let us split Eq. (2) as follows:

$$\mathbb{S}_{CG}^{\star} = \mathbf{diag}(\mathbb{V}_s^{\star})[\mathbb{Y}_{ss}\mathbb{V}_s + \mathbb{Y}_{sd}\mathbb{V}_d], \tag{20}$$

$$\mathbb{S}_{DG}^{\star} - \mathbb{S}_D^{\star} = \mathbf{diag}(\mathbb{V}_d^{\star})[\mathbb{Y}_{ds}\mathbb{V}_s + \mathbb{Y}_{dd}\mathbb{V}_d], \tag{21}$$

where the subscripts s and d refer to the slack (conventional generators) nodes and demands, respectively. Notably, $\mathbb{V}_s$ is perfectly known, and the interest lies in the solution for $\mathbb{V}_d$. In addition, Eq. (20) is linear, as $\mathbb{S}_{CG}^{\star}$ are free variables that absorb any change in the demand. This indicates that, if $\mathbb{S}_{DG}^{\star}$ is known (provided by the VSA), then $\mathbb{V}_d$ can be obtained by iteratively solving Eq. (21) as presented below:

$$\mathbb{V}_d^{m+1} = \mathbb{Y}_{dd}^{-1}\left[\mathbf{diag}(\mathbb{V}_d^{m,\star})\left[\mathbb{S}_{DG}^{\star} - \mathbb{S}_D^{\star} - \mathbb{Y}_{ds}\mathbb{V}_s\right]\right], \tag{22}$$

where m is the iterative counter. In addition, this iterative procedure is repeated until the voltage tolerance error between two consecutive iterations is satisfied.

Finally, Algorithm 2 shows the steps to implement the VSA technique used herein.

Algorithm 2 Proposed slave optimization based on the VSA for the optimal sizing of DGs

Inputs:
The initial center μ_0 is calculated from (14);
The initial radius r_0 (or the standard deviation σ_0) is calculated in (19);
Set the initial best fitness function as $p_{loss}(s_{best}) = \infty$ (minimization problem);
Make $\tau = 0$;
while $\tau \leqslant \tau_{\max}$ **do**
  Generate the candidate solutions using a Gaussian distribution around the center μ_τ with a standard deviation (radius) r_τ as defined in (15) to obtain $C_\tau(s)$ with d-dimension rows and b columns (b represents the number of DGs considered in the OPF problem);
  If $C_\tau(s)$ crosses any upper or lower bound, place it within its bounds using (18);
  Evaluate the successive approximation power flow problem (see (22)) for each s_k in $C_\tau(s)$ and calculate its corresponding fitness function as (11);
  Select the best solution s^* as the argument that produces the minimum z_f contained in $C_\tau(s)$;
  if $z_f(s^*) < z_f(s_{best})$ **then**
    $s_{best} = s^*$;
    $z_f(s_{best}) = z_f(s^*)$;
  else
    Retain the best solution attained so far s_{best};
  end if
    Make the center $\mu_{\tau+1}$ equal to the best solution s_{best};
    Update the current radius $r_{\tau+1}$ as given by (19);
    $\tau = \tau + 1$;
end while
Output:
The best solution is found for s_{best} and its fitness function $z_f(s_{best})$;

Remark 8. Note that the roles of the CBGA and the VSA in the optimal location and sizing of DGs in electrical distribution networks are the following:

- The CBGA is a discrete optimization algorithm that deals with the problem of the optimal location of DGs, as it defines the subset of nodes, which will be located as shown in the codification illustrated in Fig. 2.
- The VSA is a continuous optimization approach that addresses the problem of the optimal sizing of DGs (i.e., OPF);

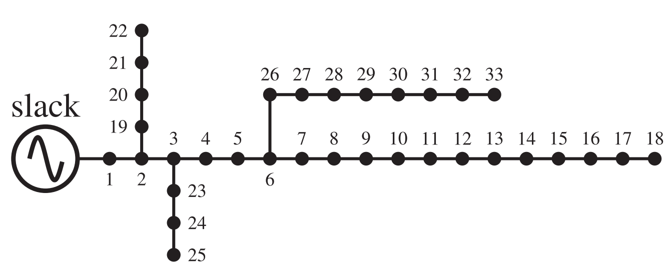

Fig. 3. Electrical configuration of the 33-node test system.

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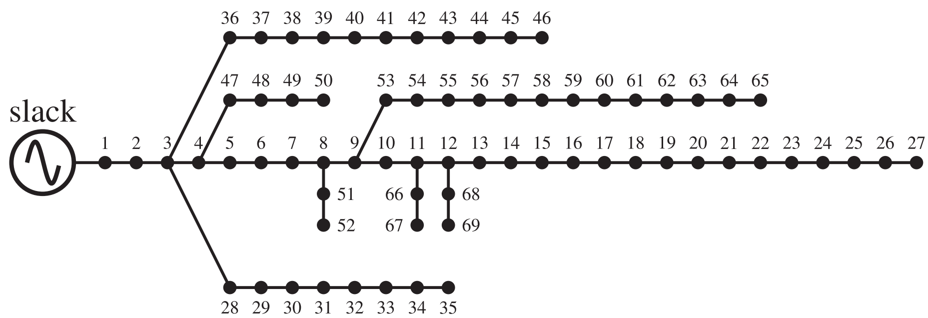

Fig. 4. Electrical configuration of the 69-node test system.

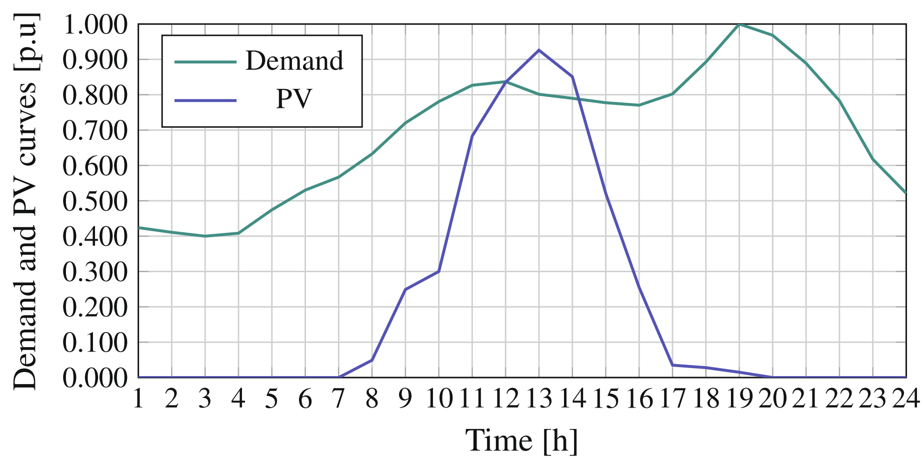

Fig. 5. Percentage of power consumption and availability on a typical sunny day in the Caribbean region of Colombia.

this algorithm takes the location of the DGs and defines their sizing via continuous searching. The main advantage of the VSA is that it guarantees the convergence to the global optimum for nonlinear optimization problems as reported in [3,48].

4. Test systems and simulation cases

This section presents the electrical configuration and test system information of the radial distribution systems employed in this study for validating the MINLP formulation and its solution in the GAMS package [8]. Two test systems were used: a 33-node test system and a 69-node test feeder. The complete details of these test systems are presented below.

4.1. 33-node test feeder

This test system was composed of 33 nodes and 32 branches with an operating voltage of 12.66 kV. The slack node is located at node 1, and its configuration is presented in Fig. 3. This feeder had the total active and reactive power demands of 3715 kW and 2300 kvar, respectively. The initial active power losses of this system were 210.9876 kW. For this test system, the possibility of installing three DGs was considered because it was the most commonly reported solution in the literature [2]. The power of each DG was limited from 300 kW to 1200 kW[1]. In addition, we considered the voltage and power base values of 12.66 kV and 1000 kW, respectively.

[1] Note that these bounds were selected to guarantee equal conditions for comparison with the techniques reported in the literature.

The information of all the branches and the load consumption of the 33-node test feeder are listed in Table 6.

4.2. 69-node test feeder

This test system consisted of 69 nodes and 68 branches with an operating voltage of 12.66 kV. The slack node is located at node 1, and its configuration is depicted in Fig. 4. This feeder had the total active and reactive power demands of 3890.7 kW and 2693.6 kvar, respectively. The initial active power losses of this system were 224.9520 kW. We considered the installation of three DGs, with the power of each DG ranging from 0 to 2000 kW. We also considered 12.66 kV and 1000 kW as the voltage and power base values, respectively.

The information of all the branches and the load consumption of the 69-node test feeder are presented in Table 7.

4.3. Simulation cases

To validate the proposed master–slave optimization algorithm, we considered two simulation cases as follows:

- **Case 1:** This simulation case evaluated the possibility of locating three DGs for each test system, considering that the load was under peak consumption condition, as reported in Tables 6 and 7. This simulation case was the typical simulating condition reported in the literature for locating DGs in AC distribution networks. For simplicity, we considered a unity power factor for operating all the DGs.
- **Case 2:** In this case, we considered the possibility of integrating three DGs with PV technology, considering the typical curves of

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a Colombian utility [51]. These curves were the percentages of the load variation and the availability of renewable generation of the PV plants considering daily operations. Fig. 5 displays these curves in terms of per-unit representation [52].

5. Computational validation

To solve the general MINLP model for the problem of the optimal location and sizing of DGs in radial distribution systems, we employed the MATLAB software version 2017*b* on a desktop computer with an INTEL(R) Core(TM) $i5-3550$ 3.5-GHz processor and 8 GB of RAM running on a 64-bit version of Microsoft Windows 7 Professional.

To implement the proposed master–slave optimization approach to deal with the optimal location and sizing of DGs in distribution networks, we consider the information reported in Table 1. Note that these parameters correspond to the CBGA, VSA, and successive approximation power flow method, respectively.

5.1. Case 1

For a comparison on the 33-node test feeder, we considered the following published papers in the literature: krill herd algorithm (KHA) [22], loss sensitivity factor simulated annealing (LSFSA) [22], combined genetic algorithm (GA) and particle swarm optimization (PSO) (GA-PSO) [5], teaching–learning based optimization (TLBO) [4], quasi-oppositional teaching–learning-based optimization (QOTLBO) [4], harmony search algorithm with PSO embedded artificial bee colony (HSA-PABC) [53], hybrid intelligent water drops and GA (GA-IWD) [54], heuristic approach (AHA) [55], mutated salp swarm algorithm (MSSA) [10], MINLP model [12], and constructive heuristic vortex search algorithm (CHVSA) [9].

The results of all the previous methods and the proposed master–slave approach are listed in Table 2 for a comparison. Note that we evaluated each size reported in each corresponding literature to obtain the same number of decimals in the power loss value, for a fair comparison.

From Table 2, it can be observed that the proposed approach reaches the global minimum of the problem (72.7853 kW), which can be considered a technical tie with the results of the MSSA and MINLP models (72.7854 and 72.7862 kW, respectively). Furthermore, these techniques identified the same group of nodes for locating all the DGs, i.e., nodes 13, 24, and 30, with similar power dispatches (dimension of the DGs). Thus, the slave stage is crucial in obtaining the optimal solution. Furthermore, the proposed CBGA in discrete form is adequate for solving the MINLP model for the optimal location and sizing problems of DGs in AC grids with lower standard deviations and speed convergence when hybridized using the VSA method.

Table 1
Parameters for implementing the proposed hybrid CBGA-VSA algorithm.

Parameter	Value (Method)
Chu-Beasley genetic algorithm	
Population size	10
Number of iterations	100
Population generation	Gaussian Distribution
Non-improvements	$c_{max} = 25$
Tournament individuals	4 (random selected)
Recombination	100 %
Mutation	50 %
Vortex search algorithm	
Population size	4
Number of iterations	100
Population generation	Gaussian Distribution
Successive approximation power flow	
Number of iterations	1000
Tolerance	1×10^{-10}
Experimental tests per system	
Number of evaluations	100

Table 2
Comparative results for the optimal sizing and dimensioning of DGs for the 33-node test feeder under simulation case 1.

Method	**Loss** [kW]	**Location** (Node)	**Size** [MW]
KHA [22]	75.4116	{13,25,30}	{0.8107,0.8368,0.8410}
LSFSA [22]	82.0525	{6,18,30}	{1.1124,0.4874,0.8679}
GA-PSO [5]	103.3600	{11,16,32}	{0.9250,0.8630,1.2000}
TLBO [4]	75.5400	{10,24,31}	{0.8246,1.0311,0.8862}
QOTLBO [4]	74.1008	{12,24,29}	{0.8808,1.0592,1.0714}
HSA-PABC [53]	72.8129	{14,24,30}	{0.7550,1.0730,1.0680}
GA-IWD [54]	110.5100	{11,16,32}	{1.2214,0.6833,1.2135}
AHA [55]	72.8340	{13,24,30}	{0.7920,1.0680,1.0270}
MSSA [10]	72.7854	{13,24,30}	{0.8010,1.0910,1.0530}
MINLP [12]	72.7862	{13,24,30}	{0.8000,1.0900,1.0500}
CHVSA [9]	78.4534	{6,14,31}	{1.1846,0.6468,0.6881}
CBGA-VSA	72.7853	{13,24,30}	{0.8018,1.0913,1.0536}

Both previous hybrid approaches that use GA present worse performances, as shown in Table 2 (see the GA-PSO and GA-IWD methods), compared with the results presented in this study. The difference may be attributed to the parameterization of the GA and the use of binary codification in the previous methods, whereas this study presents an integer codification that helps obtain the best numerical results.

In Fig. 6, the power loss reductions achieved by each method are presented. The losses confirm that the best approach for the 33-node test feeder is the proposed hybrid method between the CBGA and VSA, which is conclusively tied with the MSSA and MINLP methods.

For the voltage profile in the base case (i.e., without distributed generation), the worst voltage profile occurred at node 18 with 0.9038 p.u. Nevertheless, after applying the proposed CBGA-VSA, the worst voltage profile occurred at node 33 with 0.9687 p.u. This indicates a general improvement of approximately 6.49%, which exceeds the typical value of 0.95 p.u. required by regulatory policies in the distribution system operation [12].

The proposed hybrid CBGA-VSA approach was evaluated 100 consecutive times, demonstrating an average processing time of approximately 20.2001 s for reaching the optimal solution, with a minimal power loss of approximately 72.7853 kW, a maximum of 74.5616 kW, a mean of 72.9895 kW, and a standard deviation of 0.3592 kW. These results confirm that the proposed approach has a lower processing time, and it is at least 88 times faster in comparison with [2]. Moreover, for the dispersion of the solutions when compared with the KHA reported in [22] (a mean of 75.4940 kW and a maximum of 75.6380 kW), our approach produces better results inclusive of the worst case, as our approach showed an improvement by approximately 0.8500 kW with respect to the best solution reported by the KHA.

For the 69-node test system based on the literature reports, we removed the HSA-PABC and included the hybrid teaching–learning-based optimization-grey wolf optimizer (HTLBOGWO) reported by [56] for a comparison. HSA-PABC was removed because it did not report any information for the 69-node test feeder.

Table 3 reports the numerical validation of the proposed approach and the previous methods. First, four approaches, i.e., MSSA, MINLP, CHVSA, and CBGA-VSA showed a tie as the difference among them was lower than 1.30 W, which is negligible for

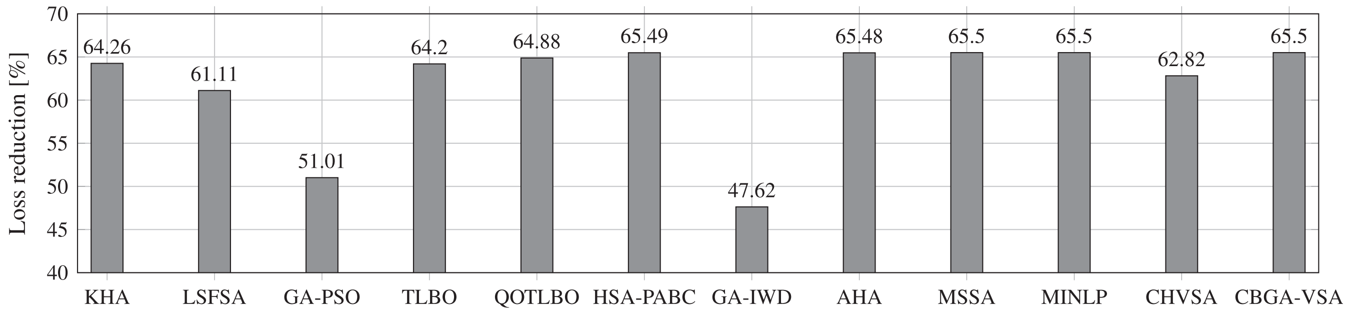

Fig. 6. Total reduction of the power loss in the 33-node test feeder achieved by the different methods.

Table 3
Comparative results for the optimal sizing and dimensioning of DGs for the 69-node test feeder under simulation case 1.

Method	**Loss** [kW]	**Location** (Node)	**Size** [MW]
KHA [22]	69.5730	{12,22,61}	{0.4962,0.3113,1.7354}
LSFSA [22]	72.1120	{18,60,65}	{0.4204,1.3311,0.4298}
GA-PSO [5]	84.5909	{21,61,63}	{0.9105,1.1926,0.8849}
TLBO [4]	72.4157	{15,61,63}	{0.5919,0.8188,0.9003}
QOTLBO [4]	71.6345	{18,61,63}	{0.5334,1.1986,0.5672}
HTLBOGWO [56]	71.7281	{18,61,62}	{0.5330,1.0000,0.7730}
GA-IWD [54]	80.9100	{20,61,64}	{0.9115,1.3926,0.8059}
AHA [55]	69.6669	{12,21,61}	{0.4710,0.3120,1.6890}
MSSA [10]	69.4077	{11,18,61}	{0.5260,0.3800,1.7180}
MINLP [12]	69.4090	{11,17,61}	{0.5300,0.3800,1.7200}
CHVSA [9]	69.4088	{11,17,61}	{0.5284,0.3794,1.7186}
CBGA-VSA	69.4077	{11,18,61}	{0.5268,0.3801,1.7190}

practical purposes. Nevertheless, the CBGA-VSA and MSSA achieved the same numerical solution, i.e., power losses of 69.4077 kW, and were hence the most efficient algorithms in comparison with the other approaches. The KHA and AHA methods were adequate algorithms for the 69-node test feeder because both reduced the power loss up to 67.5730 kW and 69.6669 kW, respectively, i.e., lower than 70 kW, contrary to the other algorithms.

From the literature approaches (including our proposal) reported in Table 3, we identify nodes 11, 17, 18, 61, and 63 as the most sensitive nodes for the optimal location of DGs. Additionally, the best results were obtained for nodes 11, 18, and 61 as for the MSSA and CBGA-VSA.

Fig. 7 reports the percentage of power loss reduction for the base case. This plot confirms that six techniques exceeded a power loss reduction of 69%, with the MSSA and the proposed approach being the best models (comparing the power losses in Table 3). The approaches that use GA (e.g., GA-PSO and GA-IWD) were implemented using binary codifications that were difficult to develop through the solution space; consequently, both approaches were stuck in local optimums for both test feeders. However, our proposed integer CBGA demonstrates the capability of reaching the global optimum when combined with efficient OPF approaches, as reported in [3,9] for the VSA.

After 100 consecutive iterations, we observed a maximum, minimum, mean, and standard deviation of approximately 70.7219 kW, 69.4077 kW, 69.5409 kW, and 0.3400 kW, respectively. The worst result from our approach (70.7219 kW) was better than those of the KHA, LSFSA, GA-PSO, TLBO, QOTLBO, HTLBOGWO, and GA-IWD approaches (see the second column of Table 3. Additionally, a lower standard deviation was obtained in comparison with the results reported in [22]. For the computational time, the average time taken by the CBGA-VSA for the 69-node test feeder was 62.2219 s, which was at least 32 times faster than the method reported in [2].

Regarding the voltage profile improvement, the worst voltage value for the base case corresponded to node 65 with 0.9092 p.u. After applying the proposed CBGA-VSA, a minimum voltage of approximately 0.9790 p.u was obtained in the same node, which indicated an improvement of approximately 6.98% in the voltage profile of the network after sizing and locating all the DGs.

5.2. Case 2

In this simulation case, we explored the possibility of including three DGs at each test feeder, considering that they are all PVs and follow the typical Colombian power output curve reported in Fig. 5. Moreover, we consider that all the demands vary with the percentage of consumption per hour reported in the same plot.

Table 4 reports the best solution for the proposed hybrid master–slave optimizer after 100 consecutive evaluations. The location of the DGs in this scenario coincides with the nodes reported in the first case for the 33-node test system (i.e., nodes 13, 24, and 30 in Table 2). For the 69-node test feeder (Table 3), nodes 11 and 61 are equal, and node 18 is moved to node 17, which is a neighbor as plotted in Table 7.

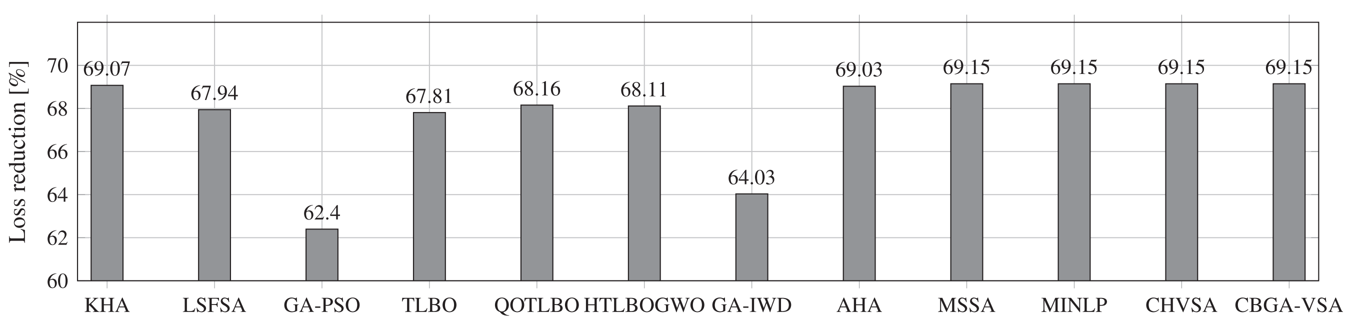

Fig. 7. Total reduction of the power loss in the 69-node test feeder achieved by the different methods.

Table 4
Optimal sizing and dimensioning of PV sources in AC distribution networks using the proposed approach.

System	Loss [kWh/day]	Location (Node)	Size [MW]
33-nodes	1915.6694	{13,24,30}	{0.9676,1.9999,1.2000}
69-nodes	1999.6510	{11,17,61}	{0.6538,0.4446,2.0000}

After the dimensions achieved by the proposed approach in the 33-node test feeder were observed, two PV nodes having the maximum capacity of 1200 kW were selected, whereas only the DG located at node 61 was sized as 2000 kW in the 69-node test feeder. Nonetheless, these increases in the DG dimensions occurred because the PV generators can only support power for 12 h during an operative day in Colombia. This duration was different from the first case where the DGs continuously generated the same power irrespective of the time of the day.

The initial daily losses for the 33- and 66-node test feeders were 2508.6343 kWh/day and 2664.7952 kWh/day, respectively, which indicate that the CBGA-VSA hybrid approach achieved daily reductions of approximately 23.64 % and 24.96 %, respectively (Table 4). This reduction corresponds to the third part of the first simulation case, which is caused by the time dependency of the second simulation case, as previously explained.

5.3. Complementary analysis

We perform 100 consecutive evaluations for the 33- and 69-node test feeders to observe the effectiveness of the proposed master–slave CBGA-VSA for dealing with the problem of the optimal location and sizing of DGs in radial distribution networks. These repetitions allow observing the maximum and minimum values of the objective functions achieved by our proposed method. Table 5 reports the best and worst solutions for the presented case **Case 1**. In addition, Fig. 8 shows the processing time behaviors for each test feeder.

From Table 5, we can observe that the difference between the best and worst solutions in the 33-node test feeder is 1.7762kW, which indicates that our approach is better than the following literature approaches: KHA [22], LSFSA [22], GA-PSO [5], TLBO [4], QOTLBO [4], GA-IWD [54], and CHVSA [9]. Note that the same behavior can be observed for the 69-node test feeder (the difference between the best and worst results is 1.3142 kW) when compared with Table 3 and the worst solution for this system in Table 5. These comparisons confirm that the proposed hybrid optimization method shows the best performance inclusive of the worst case when compared with previous literature results.

From Table 5, it can also be observed that, in the case of the 33-node test feeder, the best and worst solutions share two nodes, i.e., nodes 13 and 24, and differ in the third one; nevertheless, node 30 is electrically near node 32 as presented in Fig. 3. Therefore, these solutions are near in terms of power loss reduction. In the case of the 69-node test feeder, the difference in the locations of the DGs between the worst solutions indicates that they are electrically near the best one. Thus, there exist alternative solutions near the optimal one, which can also be considered for utilities in case the optimal solution is restricted and cannot be implemented owing to environmental, geographical, or social factors.

Table 5
Best and worst solutions for the 33- and 69-node test feeder at the peak load condition.

Solution type	Loss [kW]	Location (Node)	Size [MW]
		33-node test feeder	
Best	72.7853	{13,24,30}	{0.8018,1.0913,1.0536}
Worst	74.5615	{13,24,32}	{0.8406,1.1298,0.8941}
		69-node test feeder	
Best	69.4077	{11,18,61}	{0.5268,0.3801,1.7109}
Worst	70.7219	{10,17,60}	{0.5446,0.3957,1.6903}

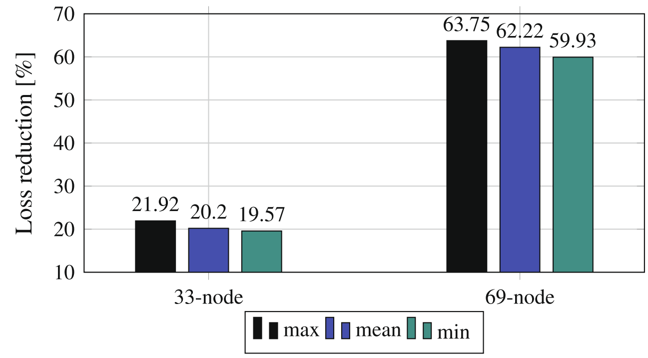

Fig. 8. Processing time behaviors for the proposed hybrid approach in the 33- and 69-node test feeders.

In terms of computational performance, Fig. 8 shows the minimum, maximum, and mean processing times required for locating and sizing DGs in AC distribution networks with the proposed master–slave CBGA-VSA approach. Note that the approach takes approximately 20 s for the 33-node test feeder, and approximately 60 s for the 69-node one. This computational performance allows evaluating multiple scenarios for planning and operation in distribution systems with minimum requirements. This can be attractive for distribution companies as the tools presented in this study can be simulated in multiple operative scenarios for taking economic decisions regarding investment, operation, and maintenance.

Notably, we decided not to compare the processing times with previous results from the literature, as they depend on the characteristics of the computers used during the simulations. However, the processing times of our approach are in the range of one minute, which can be considered highly efficient according to metaheuristic standards. This is an additional contribution of our proposed hybrid approach.

Fig. 9 illustrates the probability of the proposed method finding the best-presented solution. The CBGA-VSA approach presents a likelihood of finding the best solution of 94.5 % and 93.3 % for the 33- and 69-node test feeders, respectively. These results indicate that the proposed approach is well calibrated, and it will find the best solution in most cases.

To demonstrate the effectiveness of the proposed hybrid optimization method for solving the problem of the optimal location and sizing of DGs in AC distribution networks independent of the stochastic nature of the metaheuristic optimization approach, the nonparametric statistical test known as the Wilcoxon test is applied for both test feeders. This test attempts to identify if two (or more) independent samples have the same median, which entails that they are statistically comparable [57]. In the case of the proposed hybrid CBGA-VSA, we consider the first simulation with 100 evaluations for both the test feeders as the control sample. In addition, 10 new simulations are performed with 100 evaluations. These data are evaluated through the Wilcoxon test, which provides the mean p-values of approximately 0.8098 for the 33-node test feeder and 0.1159 for the 69-node test feeder. These results indicate that the different simulations performed in both test feeders have the same medians, i.e., each simulation is statistically comparable to the other ones. Thus, the proposed CBGA-VSA optimization method has the ability to reach the global optimum each time when it is evaluated at least 100 times. Note that this result is especially important for evaluating the effectiveness of a

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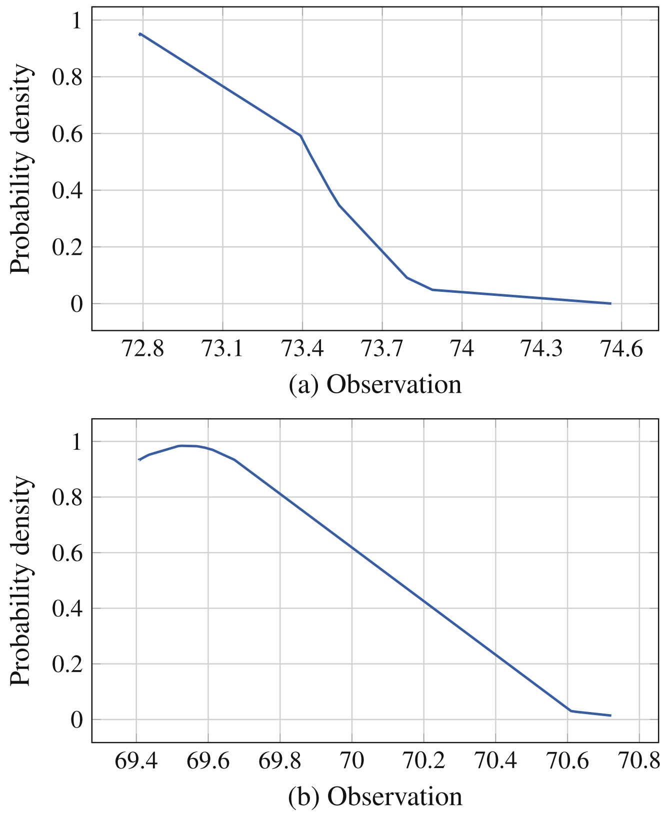

Fig. 9. Normal distribution for the proposed method: (a) 33-node test feeder and (b) 69-node test feeder.

metaheuristic optimization technique regarding the stochasticity present in its evolution performance inside the solution space, i.e., its ability to reach the global optimum of the studied problem.

6. Conclusions and future work

A novel hybrid master–slave optimization approach for solving the problem of the optimal location and sizing of DGs in AC distribution networks based on the CBGA and VSA was presented. At the master stage, the CBGA was employed to solve the location problem using an integer representation, and at the slave stage, the VSA was employed for defining the optimal dimension of the DGs (i.e., OPF solution). The numerical results confirmed that this hybrid optimizer achieved the optimal solutions reported in the literature for 33- and 69-node test feeders. The findings also demonstrated that an integer codification for CBGA allows the optimal global solution to be reached in contrast to the classical GA-PSO and GA-IWD methods. In addition, after 100 consecutive iterations, the results of the proposed approach were better than those of some approaches reported in the literature. Even in the worst case, the solutions of the hybrid CBGA-VSA were better than those of TLBO, QOTLBO, LSFSA, and HTLBOGWO.—.

Regarding the computational requirements, the proposed hybrid approach takes a few seconds for solving the complete MINLP model, which is an advantage in planning projects where multiple evaluations are required before taking a decision regarding inversion by the utilities. The processing time is approximately 20 s for the 33-node test feeder and approximately 60 s for the 69-node test system. In addition, the proposed approach is a pure-algorithmic approach and does not require any specialized software; hence, it can be regarded as free software design.

For the optimal location and dimensioning of PV plants, considering a daily operative environment, the proposed method demonstrated a daily energy reduction of approximately 23% for both the test systems. This energy minimization is adequate in distribution networks, as PV sources are limited to a maximum of 12 h of operation in the Colombian power system scenario, which conditions the effective power injection to a reduced number of hours per day due to the Gaussian form of the PV generation curve.

The Wilcoxon test allowed demonstrating that, independent of the stochastic nature, the proposed hybrid CBGA-VSA optimization algorithm allows reaching the optimal solution of the problem when multiple evaluations are performed, as for each run (100 consecutive evaluations), the median of the data is the same as that of the control set in statistical terms. This indicates that the solution strategy finds, on average, the same set of solutions each time it is employed to solve the studied optimization problem.

In future studies, the proposed approach can be embedded into a distribution system planning project for the optimal selection and sizing of renewable energy resources in AC power systems. Additionally, the proposed method can be adapted for the optimal operation of battery energy storage systems in distribution networks, considering the penetration of renewables in daily operating conditions and the typical demand and generation curves.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Electrical parameters

Tables 6 and 7 present the electrical parameters of the 33- and 69-node test feeders, respectively.

Table 6
Electrical parameters of the 33-node test feeder.

Node i	Node j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.0922	0.0477	100	60
2	3	0.4930	0.2511	90	40
3	4	0.3660	0.1864	120	80
4	5	0.3811	0.1941	60	30
5	6	0.8190	0.7070	60	20
6	7	0.1872	0.6188	200	100
7	8	1.7114	1.2351	200	100
8	9	1.0300	0.7400	60	20
9	10	1.0400	0.7400	60	20
10	11	0.1966	0.0650	45	30
11	12	0.3744	0.1238	60	35
12	13	1.4680	1.1550	60	35
13	14	0.5416	0.7129	120	80
14	15	0.5910	0.5260	60	10
15	16	0.7463	0.5450	60	20
16	17	1.2890	1.7210	60	20
17	18	0.7320	0.5740	90	40
2	19	0.1640	0.1565	90	40
19	20	1.5042	1.3554	90	40
20	21	0.4095	0.4784	90	40
21	22	0.7089	0.9373	90	40
3	23	0.4512	0.3083	90	50
23	24	0.8980	0.7091	420	200
24	25	0.8960	0.7011	420	200
6	26	0.2030	0.1034	60	25
26	27	0.2842	0.1447	60	25
27	28	1.0590	0.9337	60	20
28	29	0.8042	0.7006	120	70
29	30	0.5075	0.2585	200	600
30	31	0.9744	0.9630	150	70
31	32	0.3105	0.3619	210	100
32	33	0.3410	0.5302	60	40

Please cite this article as: O. D. Montoya, W. Gil-González and C. Orozco-Henao, Vortex search and Chu-Beasley genetic algorithms for optimal location and sizing of distributed generators in distribution networks: A novel hybrid approach, Engineering Science and Technology, an International Journal, https://doi.org/10.1016/j.jestch.2020.08.002

Table 7
Electrical parameters of the 69-node test feeder.

Node i	Node j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.0005	0.0012	0	0
2	3	0.0005	0.0012	0	0
3	4	0.0015	0.0036	0	0
4	5	0.0251	0.0294	0	0
5	6	0.3660	0.1864	2.6	2.2
6	7	0.3810	0.1941	40.4	30
7	8	0.0922	0.0470	75	54
8	9	0.0493	0.0251	30	22
9	10	0.8190	0.2707	28	19
10	11	0.1872	0.0619	145	104
11	12	0.7114	0.2351	145	104
12	13	1.0300	0.3400	8	5
13	14	1.0440	0.3450	8	5.5
14	15	1.0580	0.3496	0	0
15	16	0.1966	0.0650	45.5	30
16	17	0.3744	0.1238	60	35
17	18	0.0047	0.0016	60	35
18	19	0.3276	0.1083	0	0
19	20	0.2106	0.0690	1	0.6
20	21	0.3416	0.1129	114	81
21	22	0.0140	0.0046	5	3.5
22	23	0.1591	0.0526	0	0
23	24	0.3460	0.1145	28	20
24	25	0.7488	0.2475	0	0
25	26	0.3089	0.1021	14	10
26	27	0.1732	0.0572	14	10
3	28	0.0044	0.0108	26	18.6
28	29	0.0640	0.1565	26	18.6
29	30	0.3978	0.1315	0	0
30	31	0.0702	0.0232	0	0
31	32	0.3510	0.1160	0	0
32	33	0.8390	0.2816	14	10
33	34	1.7080	0.5646	19.5	14
34	35	1.4740	0.4873	6	4
3	36	0.0044	0.0108	26	18.55
36	37	0.0640	0.1565	26	18.55
37	38	0.1053	0.1230	0	0
38	39	0.0304	0.0355	24	17
39	40	0.0018	0.0021	24	17
40	41	0.7283	0.8509	1.2	1
41	42	0.3100	0.3623	0	0
42	43	0.0410	0.0475	6	4.3
43	44	0.0092	0.0116	0	0
44	45	0.1089	0.1373	39.22	26.3
45	46	0.0009	0.0012	39.22	26.3
4	47	0.0034	0.0084	0	0
47	48	0.0851	0.2083	79	56.4
48	49	0.2898	0.7091	384.7	274.5
49	50	0.0822	0.2011	384.7	274.5
8	51	0.0928	0.0473	40.5	28.3
51	52	0.3319	0.1114	3.6	2.7
9	53	0.1740	0.0886	4.35	3.5
53	54	0.2030	0.1034	26.4	19
54	55	0.2842	0.1447	24	17.2
55	56	0.2813	0.1433	0	0
56	57	1.5900	0.5337	0	0
57	58	0.7837	0.2630	0	0
58	59	0.3042	0.1006	100	72
59	60	0.3861	0.1172	0	0
60	61	0.5075	0.2585	1244	888
61	62	0.0974	0.0496	32	23
62	63	0.1450	0.0738	0	0
63	64	0.7105	0.3619	227	162
64	65	1.0410	0.5302	59	42
11	66	0.2012	0.0611	18	13
66	67	0.0047	0.0014	18	13
12	68	0.7394	0.2444	28	20
68	69	0.0047	0.0016	28	20

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