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An exact MINLP model for optimal location and sizing of DGs in distribution networks: A general algebraic modeling system approach

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ABSTRACT

This paper addresses the classical problem of optimal location and sizing of distributed generators (DGs) in radial distribution networks by presenting a mixed-integer nonlinear programming (MINLP) model. To solve such model, we employ the General Algebraic Modeling System (GAMS) in conjunction with the BONMIN solver, presenting its characteristics in a tutorial style. To operate all the DGs, we assume they are dispatched with a unity power factor. Test systems with 33 and 69 buses are employed to validate the proposed solution methodology by comparing its results with multiple approaches previously reported in the specialized literature. A 27-node test system is also used for locating photovoltaic (PV) sources considering the power capacity of the Caribbean region in Colombia during a typical sunny day. Numerical results confirm the efficiency and accuracy of the MINLP model and its solution is validated through the GAMS package.

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1. Introduction

1.1. General context

Nowadays, around the world, electricity is mainly produced by large-scale plants that operate using conventional sources of energy, such as hydraulic and thermal technologies. Electric plants are usually located far from final consumers and, therefore, energy losses associated with transmission lines increase [1,2]. Additionally, the voltage profile can exceed its lower and upper bounds [3,4]. For that reason, distributed generators (DGs) have become a local solution for medium- and low-voltage power systems [5–7]. DGs enable the injection of active and reactive power closer to consumers, which can produce benefits in terms of quality of service [8,1,9]. Integrating DGs into the electric system has both positive and negative effects because they modify the behavior of the state variables of the grid, which absolutely depend on their location and sizing in the power system [10]. Advances in solid-state electronics and software have boosted the high penetration of renewable energy into electrical networks, mainly at distribution levels. Hence, strategies or methods that allow the correct integration of these emerging technologies are necessary [11,9].

In the last decade, different models, methods, and optimization techniques for sizing and locating DGs in electric distribution networks have been proposed. They have allowed the integration of renewable energy sources (e.g., wind and photovoltaic (PV) generation), small-scale hydraulic generation, and biomass generation, among others, in an appropriated way [12–14]. DGs enable an improvement of different technical aspects, such as voltage profiles, the power capacity of the lines, and the reliability and quality of service, as well as a reduction of active and reactive power losses [15]. Said generation technologies also allow utility companies to diversify their energy matrix and transform electric power grids into autonomous and smart systems [16].

1.2. Motivation

Advances in power electronics transform the possibility of having electrical networks with a high penetration of distributed generation at distribution levels into a reality, mainly with the

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integration of multiple renewable energy sources. Therefore, mathematical models and new solution methodologies should be continuously developed to address the problem of optimal location and sizing of such sources in power distribution networks. For that reason, the motivation behind this study was providing the specialized literature with a powerful tool (a GAMS optimization package) for solving large-scale nonlinear discrete problems via mathematical interpretation. Such tool focuses on the mathematical modeling itself by concentrating the attention of the researches in the correct mathematical modeling by using a compact and structured architecture. For that purpose, this paper presents a simple implementation of the problem under study in order to explain all the basic concepts for GAMS usage.

1.3. Brief state-of-the-art

The literature about the optimal location and sizing of DGs in distribution networks is extensive and rich. This topic has a strong background in terms of mathematical formulations and solution techniques. Regarding its mathematical formulation, this problem corresponds to a nonlinear non-convex optimization model with discrete and continuous variables [17]; its mathematical structure is an extension of the classical distribution system power flow problem with discrete variables [18]. In terms of solution methodologies, the approaches most commonly adopted are metaheuristic optimization techniques [19]. Such optimization approaches allow the separation of the location problem from the sizing problem by adopting a master-slave methodologies [19,20].

In the case of master-slave approaches, multiple discrete optimization methods have been proposed: genetic algorithms [21], ant lion optimizers [22], tabu search algorithms [23], simulated annealing methods [24], krill herd algorithms [25–27], population-based incremental learning [28], teaching-based learning optimizers [29], bat and firefly algorithms [30–33], symbiotic organism search algorithms [34], harmonic search algorithms [35], and imperialist competitive algorithms [19].

Regarding the methodology for solving the sizing problem, the most common approach is particle swarm optimization [6,28], since it is easy to implement in any computational language and its results are comparable with interior-point and convex optimization methods [36].

The specialized literature has also proposed exact models for addressing the problem studied in this paper. In [17], a MINLP model for the problem of optimal location and sizing of DGs in distribution systems was proposed by implementing a master-slave approach in the decoupled form. That model combines sequential quadratic programming methods with a branch and bound approach, which implies that, so far, a compact formulation has not been used as proposed in this paper. In [37], a MINLP model was proposed to address the same problem, and the GAMS software was used for its solution. Nevertheless, its implementation has not yet been extended to daily operation with photovoltaic (PV) sources, as proposed by us.

1.4. Contribution and scope

Based on the review of the state-of-the-art above, this paper presents a solution to the problem of optimal location and sizing of DGs in distribution networks in a tutorial style by taking advantage of the compact modeling available in the GAMS software and its nonlinear optimization packages. Note that the main contribution of our research is the possibility of implementing the exact MINLP model of the problem using compact sets in GAMS without adopting decoupling methods (e.g., master-slave algorithms), which allows us to focus on the mathematical formulation itself. In addition, the scope of our study is mainly defined by electrical distribution networks and power losses minimization via the integration of DGs. This work presents, in a numerical simulation, the possibility of extending our proposed MINLP model for the optimal integration of renewable energy resources in a typical electrical distribution network in Colombia, which is not typically addressed in metaheuristic or conventional MINLP models. Furthermore, this paper contains a simple example with the implementation of the MINLP model, which will help researchers and students to use the GAMS package for evaluating future studies in this area and as a powerful comparative approach when emerging optimization models are tested and validated.

1.5. Document structure

The rest of this document is organized as follows. Section 2 presents the complete mathematical formulation of the problem by describing and discussing all the equations along with their variables and meanings. In Section 3, we provide all the necessary elements for using GAMS as an optimization package; in addition, such section reports the complete mathematical implementation of the MINLP model analyzed in this work as an opportunity to identify all the concepts that compose the GAMS package. Section 4 presents all the information related to the 33 and 69-node test feeders. Section 5 details all the numerical results of the proposed GAMS approach compared with approaches reported in the literature; in addition, we present the extension of the model for the daily operation of distribution networks with PV integration in the context of a Colombian electrical system located in the Caribbean region. Section 6 draws the main conclusions derived from this work as well as some possible future works, followed by the acknowledgments and the references.

2. Problem description

2.1. Mathematical formulation

The mathematical model of the optimal location and sizing of DGs in RDN corresponds to a MINLP problem [17]. Here, integer (binary) variables represent the decision variables associated with the location or not of a DG in the grid, while continuous variables are associated to the classical power flow formulation, which is represented by magnitudes and angles of the voltage per node. The following is the detailed mathematical model proposed in this paper [17].

Objective function

$$\min\ z = \sum_{i \in \Omega_N} V_i \left(\sum_{j \in \Omega_N} V_j Y_{ij} \cos\left(\theta_i - \theta_j - \phi_{ij}\right) \right) \tag{1}$$

where z is the value of the objective function, which corresponds to the power losses in all the branches of the network under a load peak scenario of demand; Ω_N, the set associated with the nodes of the network; V_i and V_j, the voltages' magnitudes at nodes i and j, respectively; θ_i and θ_j, the voltages' angles at nodes i and j, respectively; Y_{ij}, the magnitude of the admittance associated with the line connected between i and j nodes; and ϕ_{ij}, its angle.

Constraints

$$\begin{aligned} P_i^{CG} + P_i^{DG} &= V_i \sum_{j \in \Omega_N} V_j Y_{ij} \cos\left(\theta_i - \theta_j - \phi_{ij}\right) \\ &+ P_i^{D}, \quad \{\forall i \in \Omega_N\} \end{aligned} \tag{2}$$

where P_i^{GC} represents the active power generated at node i by a conventional generator; P_i^{DG}, the active power generated by a DG located at node i; and P_i^{D}, the total active power demanded at node i.

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Eq. (2) represents the active power balance at each node in the network.

$$Q_i^{CG} - Q_i^D = V_i \sum_{j \in \Omega_N} V_j Y_{ij} \sin\left(\theta_i - \theta_j - \phi_{ij}\right) \quad \{\forall i \in \Omega_N\} \tag{3}$$

where Q_i^{GC} denotes the reactive power generated at node i by a conventional generator; Q_i^{DG}, the reactive power generated by a DG located at node i; and Q_i^D, the total reactive power demanded at node i.

Eq. (3) represents the reactive power balance at each node in the network.

$$V_i^{\min} \leqslant V_i \leqslant V_i^{\max} \quad \{\forall i \in \Omega_N\} \tag{4}$$

where $V_i^{\min}$ and $V_i^{\max}$ represent the minimum and maximum allowed voltage values at each node. Note that (4) corresponds to the voltage regulation constraint.

$$0 \leqslant P_i^{DG} \leqslant x_i P_i^{DG,\max} \quad \{\forall i \in \Omega_N\} \tag{5}$$

where $P_i^{DG,\max}$ is the maximum allowed active power injection at node i by a DG and x_i represents the decision variable, which takes a value of 1 if the DG is located at node i and 0 otherwise. Eq. (5) shows the possibility of locating and sizing a DG at any node in the RDN. We considered only active power injection in the DGs, which means that $Q_i^{DG} = 0$ in this paper.

$$\sum_{i \in \Omega_N} x_i \leqslant N_{ava}^{DG} \tag{6}$$

where N_{ava}^{DG} is the available number of DGs, which implies that (6) limits the number of location possibilities for the distributed generation in the RDN.

$$x_i \in \{0, 1\} \quad \{\forall i \in \Omega_N\} \tag{7}$$

Finally, (7) expresses the binary nature of the decision variable.

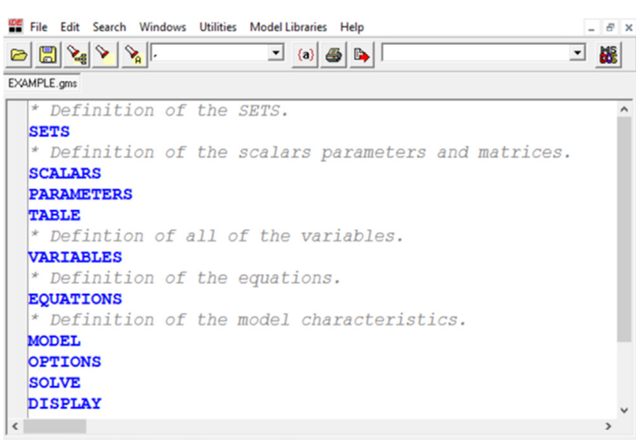

Fig. 1. GAMS software environment.

2.2. General comments

The MINLP model described from (1) to (7) represents problem of optimal location and sizing of DGs in a RDN [17]. Such model only focuses on the technical aspects related to active power losses in the branches of the network, respecting classical constraints of the power flow problem [1]. Note that this model corresponds to an adaptation of the optimal power flow problem reported by [38], in order to allow the location and sizing of DGs as a function of the total active power consumption.

An adaptation for obtaining a power flow time-varying formulation can be easily extracted for the model, as mentioned earlier, by adding some sub-indexes and sums [39]. Here, we used the demand peak hour to define the optimal location and sizing of each distributed generator because it represents the worst operating point in the RDN, with the highest power losses and voltage deviations. In addition, we also extended this model to the daily operation of an electrical network in order to evaluate the possibility of sizing PV generators.

This mathematical formulation can be directly implemented in the GAMS platform [40], which allowed us to obtain an adequate solution with a low computational effort. Such solution can be local or global, depending on the characteristics of the problem under analysis.

The next section presents a possible GAMS implementation for a small radial distribution network. Such implementation uses sets and a compact formulation [41].

3. General algebraic modeling system: GAMS

The GAMS software is a powerful optimization package developed for interpreting and solving nonlinear large-scale optimization problems based on a compact formulation [40,42]. Said software works with a simple plain text structure, where the optimization model is written using five essential components [43]:

i. The sets where the variables make sense, e.g., set of nodes: $i \in \Omega_N$.
ii. All the scalars, parameters, and matrices involved in the model, i.e., number of generators, matrices, and vectors.
iii. All the variables in the model, e.g., voltages, powers, angles, etc.
iv. The equations' names and their mathematical structures, e.g., expressions (2) and (3) associated with the power balance constraints.
v. The nature of the model (i.e., MINLP) and displaying options.

Fig. 1 presents the GAMS interface and the words reserved for implementing an optimization model.

Note that, at the bottom of Fig. 1, each reserved word is needed to define all the particular components of the model under study. In that sense, we present a simple example that can illustrate the complete structure of an optimization model implemented in the GAMS software [41]. Such example aims at guiding readers on the easy utilization of this optimization toolbox for addressing optimization problems in engineering. For that purpose, let us consider the grid depicted in Fig. 2, an electrical network composed of 7 nodes and 6 lines operated at 23 kV as voltage output at the sub-

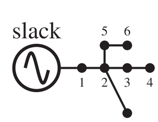

Fig. 2. Electrical configuration of the 7-node test system used in the GAMS implementation example.

station (slack node). Its line parameters, as well as power consumption, are reported in Table 1.

The 7-node test system was implemented as an example in GAMS considering 23 kV and 1 MVA as voltage and base power, respectively. We also considered the possibility of installing one distributed generator with unlimited capability.

Algorithm 1. GAMS implementation of the model in (1)–(7) for the 7-node example

```
1  SETS
2  G Index associated with slack nodes /G1/
3  N Index associated with nodes /N1*N7/
4  MAP(G,N) Relates generators and nodes /G1.N1/;
5  ALIAS(N,NP);
6  SCALARS
7  PGmax Maximum power output per DG /10/
8  NGmax DGs available /1/;
9  TABLE LINE(N,N,*) YBUS matrix: YBUS = Y<PHI
10             Y                  PHI
11 N1.N1    901.921127450169   -0.541881487533056
12 N2.N1    901.921127450169    2.599711166056740
13 N1.N2    901.921127450169    2.599711166056740
14 N2.N2    4225.31203745061   -0.627679749517968
15 N3.N2    1116.21013633344    2.583440665436250
16 N5.N2    1226.36440161427    2.668614055686530
17 N7.N2    1058.84470205320    2.183227654872620
18 N2.N3    1116.21013633344    2.583440665436250
19 N3.N3    2401.88600124091   -0.511488986218966
20 N4.N3    1287.94374735150    2.670540998703630
21 N3.N4    1287.94374735150    2.670540998703630
22 N4.N4    1287.94374735150   -0.471051654886168
23 N2.N5    1226.36440161427    2.668614055686530
24 N5.N5    1706.01794624463   -0.540610678882462
25 N6.N5    489.524057395199    2.430858585727870
26 N5.N6    489.524057395199    2.430858585727870
27 N6.N6    489.524057395199   -0.710734067861924
28 N2.N7    1058.84470205320    2.183227654872620
29 N7.N7    1058.84470205320   -0.958364998717178;
30 TABLE BUS(N,*) Demand behavior
31 VMIN   VMAX   PL    QL
32 * (V)  (V)  (W)  (VAr)
33 N1     0.90   1.10  0.00  0.00
34 N2     0.90   1.10  1.00  0.60
35 N3     0.90   1.10  0.90  0.50
36 N4     0.90   1.10  2.50  1.20
37 N5     0.90   1.10  1.20  0.95
38 N6     0.90   1.10  1.05  0.78
39 N7     0.90   1.10  2.00  1.15;
40 VARIABLES
41 z Objective function
42 p(G) Active power output from the slack generator
43 q(G) Reactive power output from the slack
      generator
44 v(N) Voltage magnitude at node N
45 d(N) Voltage angle at node N
46 pdg(N) Active power output from the DGs at node N
      ;
47 BINARY VARIABLES
48 x(N) Variable for optimal location of the DG;
49 v.lo(N) = BUS(N,'VMIN'); v.up(N)=BUS(N,'VMAX');
50 d.fx('N1')=0;  v.fx('N1')=1.0;
51 EQUATIONS
52 OBJFUC  Objective function
53 PBAL(N)  Active power balance per node
54 QBAL(N)  Reactive power balance
55 TGEN     Number of DGs available
56 MAXGD(N) Maximum reactive power output from the
      DG
57 MINGD(N) Minimum reactive power output from the
      DG;
58 OBJFUC..   z =E= SUM(N,v(N)*SUM(NP,LINE(N,NP,'Y')
      *v(NP)*COS(d(N)-d(NP)-LINE(N,NP,'PHI'))));
59 PBAL(N)..  SUM(G$MAP(G,N),p(G))-BUS(N,'PL') + pdg
      (N)=e= v(N)*SUM(NP,LINE(N,NP,'Y')*v(NP)*
60 COS(d(N)-d(NP)-LINE(N,NP,'PHI')));
61 QBAL(N)..  SUM(G$MAP(G,N),q(G))-BUS(N,'QL')=e= v(
      N)*SUM(NP,LINE(N,NP,'Y')*v(NP)*
62 SIN(d(N)-d(NP)-LINE(N,NP,'PHI')));
63 TGEN..     SUM(N,x(N)) =L= NGmax;
64 MAXGD(N).. pdg(N) =G= 0;
65 MINGD(N).. pdg(N) =L= x(N)*PGmax;
66 MODEL SevenNodes /ALL/;
67 OPTIONS decimals = 8;
68 SOLVE SevenNodes using MINLP minimizing z;
69 DISPLAY z.l,x.l,pdg.l;
```

From Algorithm 1, it can be seen that all the components in the optimization model (1)–(7) were included. Hence, the following are the most important features of this implementation:

i. The command `ALIAS(N,NP)`, in line 5, allows the duplication of the set of nodes N in the set NP to evaluate the power balance equations and the objective function.
ii. All the parametric information of the model was defined between lines 6 and 39.
iii. The set of variables was classified into continuous variables (voltage, angles, and powers) and binary variables (optimal location of the DG), as can be seen from lines 40 to 48.
iv. Lines 49 and 50 define the voltage constraint (4) and the typical behavior of the slack node in a radial distribution network, i.e., plane voltage.
v. Lines 51 to 57 define the name of the equations, while lines 58, 59, and 61 are the objective function and the active and reactive power balance constraints (i.e., the compact representation of (1)–(3)).
v. Lines 63 to 65 represent the maximum number of DGs available as well as their minimum and maximum power outputs (i.e., constraints (5) and (6)).
v. Lines 66 to 69 define the characteristics of the model and its type (minimization), as well as its displaying features.

Note that, if we solve this model in GAMS by fixing the number of available DGs at zero, then the base case of the network is achieved. Figs. 3 and 4 present the GAMS outputs when zero and one distributed generator are considered.

Note that, in Fig. 3, the total power losses without distributed generation reach 128.058 kW, while in Fig. 4 the final losses decrease to 56.9563 kW when one DG is installed. To reduce such losses, GAMS determined that the distributed generator must be located at node 3 with a total capacity of 6.3610 MVA.

It is important to mention that the model implemented in GAMS is general for the problem analyzed in this paper, which implies that, based on the parametric information of the test system, it can find an optimal solution to the proposed MINLP model, as will be confirmed in the next section. For detailed information about GAMS and a complete description of its functionalities, refer to [40,43,41].

4. Test systems and simulation cases

This section presents the electrical configuration, as well as the test system information, of the radial distribution systems employed in this work for validating the MINLP formulation and its solution in the GAMS package. Two test system were used: a 33-node test system and a 69-node test feeder. The complete information of these test systems is presented below.

4.1. 33-node test feeder

This test system is composed of 33 nodes and 32 branches with 12.66 kV of operating voltage. The slack node is located at node 1, and its configuration is presented in Fig. 5. This feeder has 3715 kW and 2300 kVAr of total active and reactive power demand. The initial active power losses of this system equal 210.9876 kW. For this test system, the possibility of installing 3 distributed generators was considered since that is the most commonly reported solution in the specialized literature [28]. Each distributed generator is limited from 0 kW to 2500 kW.[1] In addition, we considered voltage and power base values of 12.66 kV and 1000 kW, respectively.

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Table 1
Electrical parameters of the 7-node test feeder used in the GAMS implementation example

Node i	Node j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.5025	0.3025	1000	600
2	3	0.4020	0.2510	900	500
3	4	0.3660	0.1864	2500	1200
2	5	0.3840	0.1965	1200	950
5	6	0.8190	0.7050	1050	780
2	7	0.2872	0.4088	2000	1150

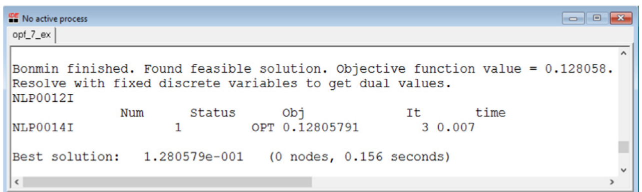

Fig. 3. GAMS output with zero DGs.

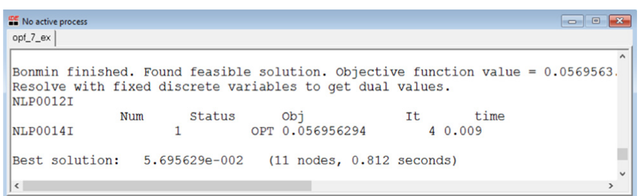

Fig. 4. GAMS output with a unique DG.

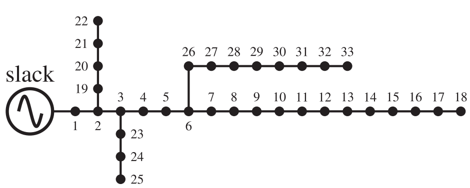

Fig. 5. Electrical configuration of the 33-node test system.

The information of all the branches, as well as the load consumption of the 33-node test feeder, is listed in Table 2.

4.2. 69-node test feeder

This test system consists of 69 nodes and 68 branches with 12.66 kV of operating voltage. The slack node is located at node 1, and its configuration is depicted in Fig. 6. This feeder has 3890.7 kW and 2693.6 kVAr of total active and reactive power demand. The initial active power losses of this system equal 225.0718 kW. For this test system, we also considered the possibility of installing 3 distributed generators, and each of them limited from 0 kW to 2000 kW. In addition, we also considered 12.66 kV and 1000 kW as voltage and power base values, respectively.

The information of all the branches, as well as the load consumption of the 69-node test feeder, is presented in Table 3.

5. Computational validation

To solve the general MINLP model that represents the problem of optimal location and sizing of DGs in radial distribution systems, we employed the GAMS optimization package with the solver BONMIM in a desktop computer with an INTEL(R) Core(TM) $i5-3550$ 3.5-GHz processor and 8 GB of RAM running a 64-bit version of Windows 7 Professional. The implemented mathematical model is the same as the one presented in Section 3, except that the information of each test feeder was modified.

To demonstrate the robustness and efficiency of the GAMS package for locating and sizing DGs in distribution networks, we compared our results with the solutions previously reported in [34,24]. In addition, we considered that all the DGs were operated with a unity power factor, as recommend in [28].

5.1. 33-node test feeder

Table 4 presents a list of solutions provided by [34] for the 33-node test feeder with the corresponding location, size, and power losses when 3 DGs are considered.

Note that the power losses results reported in Table 4 show that the GAMS optimization package in conjunction with the BONMIN solver finds the best solution with respect to the all comparative methods, i.e., 72.79 kW, followed by the REPSO method with 76.91 kW and the LSFSA approach with 82.03 kW, in the first three positions. It is also important to highlight that the MINLP model we proposed, solved through a GAMS implementation, finds an alternative set of nodes for locating all the distributed generators (e.g., nodes 6, 18, and 30) with a total power injection of 2.9336 MW, while the REPSO and LSFSA approaches reach 2.5212 MW and 2.4677 MW, respectively. Such values imply that the solutions provided by REPSO and LSFSA can be stuck in local optima, while our approach allows the improvement of those solutions by increasing the total power injection. In order to find the best solution.

Fig. 7 presents a comparison of the power losses reduction percentages of all the approaches reported in Table 4, along with the initial power losses. This figure confirms that the GAMS approach allows the highest power losses reduction, 65.50 %, followed by the REPSO and LSFSA approaches with 63.55 % and 61.12 %, respectively.

5.2. 69-node test feeder

Table 5 presents a list of solutions provided by [34] for the 69-node test feeder with the corresponding location, size, and resulting power losses, when 3 DGs were considered.

The behavior of the power losses presented in Table 5 proves that the GAMS approach effectively converges to the best solution, in contrast with the other methodologies. In this system, our MINLP model, solved through the BONMIN solver, reaches final power losses of 72.09 kW, followed by the LSFSA and TLBO methods with 77.10 kW and 81.00 kW, respectively.

Fig. 8 shows the power losses reduction achieved by the proposed GAMS approach as well as the other methods. Note that the GAMS approach achieves the highest reduction in power losses, with 67.97 %, which confirms the efficiency and accuracy of the proposed MINLP model and its solution by GAMS.

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Table 2
Electrical parameters of the 33-node test feeder.

Node i	Node j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.0922	0.0477	100	60
2	3	0.4930	0.2511	90	40
3	4	0.3660	0.1864	120	80
4	5	0.3811	0.1941	60	30
5	6	0.8190	0.7070	60	20
6	7	0.1872	0.6188	200	100
7	8	1.7114	1.2351	200	100
8	9	1.0300	0.7400	60	20
9	10	1.0400	0.7400	60	20
10	11	0.1966	0.0650	45	30
11	12	0.3744	0.1238	60	35
12	13	1.4680	1.1550	60	35
13	14	0.5416	0.7129	120	80
14	15	0.5910	0.5260	60	10
15	16	0.7463	0.5450	60	20
16	17	1.2890	1.7210	60	20
17	18	0.7320	0.5740	90	40
2	19	0.1640	0.1565	90	40
19	20	1.5042	1.3554	90	40
20	21	0.4095	0.4784	90	40
21	22	0.7089	0.9373	90	40
3	23	0.4512	0.3083	90	50
23	24	0.8980	0.7091	420	200
24	25	0.8960	0.7011	420	200
6	26	0.2030	0.1034	60	25
26	27	0.2842	0.1447	60	25
27	28	1.0590	0.9337	60	20
28	29	0.8042	0.7006	120	70
29	30	0.5075	0.2585	200	600
30	31	0.9744	0.9630	150	70
31	32	0.3105	0.3619	210	100
32	33	0.3410	0.5302	60	40

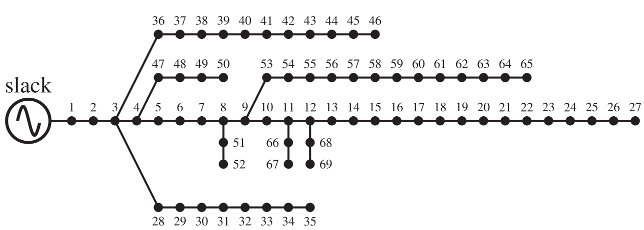

Fig. 6. Electrical configuration of the 69-node test system.

5.3. Optimal location of renewable generators in a daily operational environment

Here, we explore the possibility of using GAMS for locating renewable generators (PV systems) in radial distribution systems by considering the typical solar radiation performance in a Colombian system in the Caribbean region. For that purpose, we employ the 27-node test feeder reported in [42] with the branch and peak load information reported in Table 6. The grid configuration of this test feeder is illustrated in Fig. 9. To evaluate the daily operation of this system including PV systems, we employ the demand variation and the PV generation capacity in Fig. 10. In addition, we use 13.8 kV and 1000 kW as voltage and power bases, respectively; during GAMS implementation.

In this test system, we evaluate the possibility of installing from 1 to 3 PV generators with the curve of power generation reported in Fig. 10. Note that, in this test system, the total power losses per day are 2094.01 kWh/day when renewable power generation has not yet been installed, while such losses are lower when different numbers of PV generators are installed, as reported in Table 7. In addition, said Table shows the size of each PV generator, e.g., in the case of 2 PV generators, the GAMS package suggests locating them at nodes 10 and 16 with maximum capacities of 1.321 p.u and 1.008 p.u, respectively[2]

Note that the GAMS package solves the problem for all the different options; in the case of 1 PV generator, it achieves a reduction of 7.54 % in the total power losses per day, while with 2 and 3 generators, the reductions are 12.53 % and 17.32 %, respectively. These results imply that, as the number of PV generators increases, power losses decrease. Notwithstanding, these reductions tend to the saturation due to the impossibility of generating power at night, as depicted in Fig. 11, where the energy reduction in the system exhibits an exponential decreasing asymptotic behavior approaching 1600 kWh/day.

5.4. General comments

The numerical validation presented in the section above shows that:

- The MINLP model and its implementation in GAMS can produce excellent solutions in terms of power losses reduction for the 33-node test feeder and the 69-node test feeder.
- The 27-node test feeder, with a daily operation, revealed the possibility of using the MINLP model with time-varying variables for solving the problem of optimal location and sizing of renewable generators (e.g., PV systems) through its GAMS implementation.
- The integration of multiple PV modules for generating renewable power in distribution grids allows the reduction of their total daily power losses. However, a massive integration of such modules does not cause important reductions in said losses.

[2] To determine the total power output of each generator, the maximum capacities reported in Table 7 should be multiplied by the typical generation provided in Fig. 10.

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Table 3
Electrical parameters of the 69-node test feeder.

Node i	Node j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.0005	0.0012	0	0
2	3	0.0005	0.0012	0	0
3	4	0.0015	0.0036	0	0
4	5	0.0251	0.0294	0	0
5	6	0.3660	0.1864	2.6	2.2
6	7	0.3811	0.1941	40.4	30
7	8	0.0922	0.0470	75	54
8	9	0.0493	0.0251	30	22
9	10	0.8190	0.2707	28	19
10	11	0.1872	0.0619	145	104
11	12	0.7114	0.2351	145	104
12	13	1.0300	0.3400	8	5
13	14	1.0440	0.3450	8	5
14	15	1.0580	0.3496	0	0
15	16	0.1966	0.0650	45	30
16	17	0.3744	0.1238	60	35
17	18	0.0047	0.0016	60	35
18	19	0.3276	0.1083	0	0
19	20	0.2106	0.0690	1	0.6
20	21	0.3416	0.1129	114	81
21	22	0.0140	0.0046	5	3.5
22	23	0.1591	0.0526	0	0
23	24	0.3463	0.1145	28	20
24	25	0.7488	0.2475	0	0
25	26	0.3089	0.1021	14	10
26	27	0.1732	0.0572	14	10
3	28	0.0044	0.0108	26	18.6
28	29	0.0640	0.1565	26	18.6
29	30	0.3978	0.1315	0	0
30	31	0.0702	0.0232	0	0
31	32	0.3510	0.1160	0	0
32	33	0.8390	0.2816	10	10
33	34	1.7080	0.5646	14	14
34	35	1.4740	0.4873	4	4
3	36	0.0044	0.0108	26	18.55
36	37	0.0640	0.1565	26	18.55
37	38	0.1053	0.1230	0	0
38	39	0.0304	0.0355	24	17
39	40	0.0018	0.0021	24	17
40	41	0.7283	0.8509	102	1
41	42	0.3100	0.3623	0	0
42	43	0.0410	0.0478	6	4.3
43	44	0.0092	0.0116	0	0
44	45	0.1089	0.1373	39.22	26.3
45	46	0.0009	0.0012	39.22	26.3
4	47	0.0034	0.0084	0	0
47	48	0.0851	0.2083	79	56.4
48	49	0.2898	0.7091	384.7	274.5
49	50	0.0822	0.2011	384.7	274.5
8	51	0.0928	0.0473	40.5	28.3
51	52	0.3319	0.1140	3.6	2.7
9	53	0.1740	0.0886	4.35	3.5
53	54	0.2030	0.1034	26.4	19
54	55	0.2842	0.1447	24	17.2
55	56	0.2813	0.1433	0	0
56	57	1.5900	0.5337	0	0
57	58	0.7837	0.2630	0	0
58	59	0.3042	0.1006	100	72
59	60	0.3861	0.1172	0	0
60	61	0.5075	0.2585	1244	888
61	62	0.0974	0.0496	32	23
62	63	0.1450	0.0738	0	0
63	64	0.7105	0.3619	227	162
64	65	1.0410	0.5302	59	42
11	66	0.2012	0.0611	18	13
66	67	0.0047	0.0014	18	13
12	68	0.7394	0.2444	28	20
68	69	0.0047	0.0016	28	20

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Table 4
Location and dispatch of the generators in the 33-node test feeder.

Method	Power generation [p.u] (Node)			Losses [kW]
GA [21]	1.5000 (11)	0.4228 (29)	1.0714 (30)	106.30
PSO [21]	1.1768 (8)	0.9816 (13)	0.9297 (32)	105.35
TLBO [29]	0.8847 (9)	0.8953 (18)	1.1958 (31)	104.00
REPSO [44]	1.2274 (6)	0.6068 (14)	0.6870 (31)	76.91
HSA [45]	0.5927 (16)	0.2133 (17)	0.1913 (18)	135.69
SOS [34]	2.2066 (6)	0.2000 (28)	0.7167 (29)	104.19
LSFSA [24]	1.1124(6)	0.4874 (18)	0.8679 (30)	82.03
GAMS	0.7709 (14)	1.0969 (24)	1.0658 (30)	**72.79**

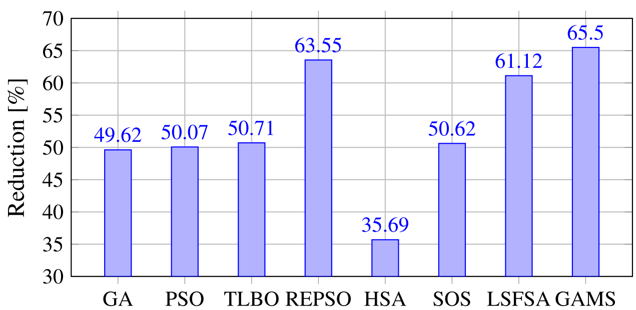

Fig. 7. Power losses reduction of different methods in the 33-node test feeder.

Table 5
Location and dispatch of generators in the 69-node test feeder.

Method	Power generation [p.u] (Node)			Losses [kW]
GA [21]	0.9297 (21)	1.0752 (62)	0.9925 (64)	89.00
PSO [21]	0.9925 (17)	1.1998 (61)	0.7956 (63)	83.20
TLBO [29]	0.7574 (25)	1.0188 (60)	1.1784 (63)	81.00
HSA [45]	1.6283 (63)	0.1416 (64)	0.0149 (65)	86.66
SOS [34]	0.2588 (57)	0.2000 (58)	1.5247 (61)	82.08
LSFSA [24]	0.4962 (18)	0.3113 (60)	1.7354 (65)	77.10
GAMS	0.8131 (12)	1.4447 (61)	0.2896 (64)	**72.09**

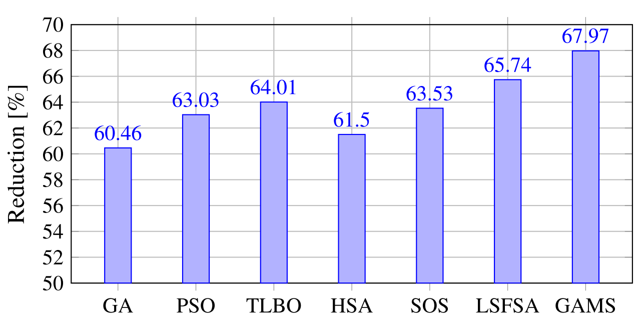

Fig. 8. Power losses reduction of different methods in the 69-node test feeder.

This situation occurs because PV generators only inject power during sunny hours, which makes them unusable during the night period.

6. Conclusions

An exact mathematical model to represent the optimal location and sizing of DGs in radial distribution networks, using a MINLP representation, was presented in this paper. Such mathematical model was solved by the GAMS optimization package, via compact formulation through the BONMIM nonlinear large-scale discrete

Table 6
Electrical parameters of the 27-node test feeder.

Node i	Node j	R_{ij} [Ω]	X_{ij} [Ω]	P_j [kW]	Q_j [kW]
1	2	0.15208	0.19855	0	0
2	3	0.65805	0.59745	0	0
3	4	0.19742	0.17924	297.5	184.4
4	5	0.43848	0.26038	0	0
5	6	0.48720	0.28931	255	158
6	7	0.48197	0.22732	0	0
7	8	0.87630	0.41330	212.5	131.7
8	9	1.09540	0.51663	0	0
9	10	0.87630	0.41330	266.1	164.9
2	11	0.87630	0.41330	85	52.7
11	12	1.07780	0.50836	340	210.7
12	13	0.65722	0.30998	297.5	184.4
13	14	0.49073	0.23145	191.3	118.5
14	15	0.87630	0.41330	106.3	65.8
15	16	0.87630	0.41330	255	158
3	17	0.87630	0.41330	255	158
17	18	0.52578	0.24798	127.5	79
18	19	0.78867	0.37197	297.5	184.4
19	20	0.83248	0.39263	340	210.7
20	21	0.87630	0.41330	85	52.7
4	22	0.87630	0.41330	106.3	65.8
5	23	0.87630	0.41330	55.3	34.2
6	24	0.35052	0.16532	69.7	43.2
8	25	0.52578	0.24798	255	158
8	26	0.52578	0.24798	63.8	39.5
26	27	0.70104	0.33064	170	105.4

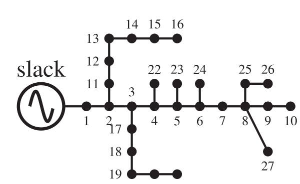

Fig. 9. Electrical configuration of the 27-node test system.

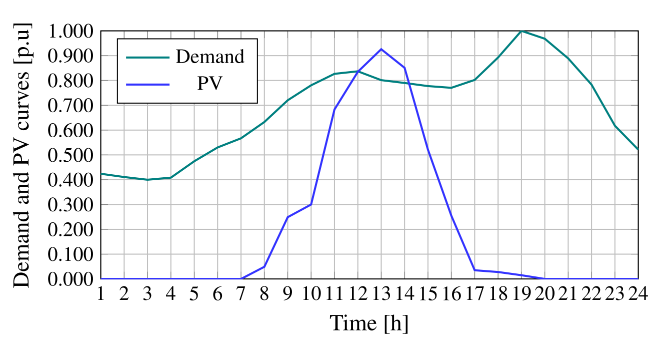

Fig. 10. Percentage of power consumption and availability on a typical sunny day in the Caribbean region of Colombia.

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Table 7
Power losses per day in the 27-node test feeder when different numbers of PV generators are installed.

Number of PV DGs	Losses [kWh/day]	Location
1	1936.20	1.520 (20)
2	1831.58	1.321 (10) - 1.008 (16)
3	1731.30	1.128 (10) - 0.975 (16) - 1.234 (20)

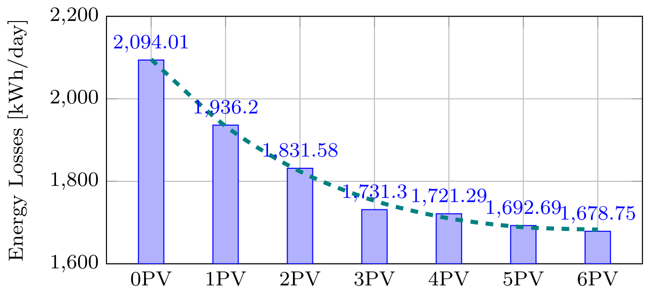

Fig. 11. Analysis of power losses reduction by increasing the number of the PV systems.

solver, and compared with multiple methodologies reported in the specialized literature, which confirmed its efficiency and accuracy in terms of power losses reduction. Additionally, an illustrative example of the GAMS implementation in a small test feeder was presented to show its ease of use for solving nonlinear optimization models.

Furthermore, an extension of the static MINLP model for optimal location and sizing of distributed generators in radial distributed networks was simulated to assess the possibility of addressing, through GAMS implementations, daily operation problems with renewable energy resources, as in the case of PV generators. This extension allowed an evaluation of the impact of PV location and sizing on total energy losses during a typical sunny day in an electrical system in the Caribbean region in Colombia.

In the future, the proposed MINLP model and its solution in GAMS can be used to size renewable generators (e.g., PV and wind generators) with variable power factor capacities in radial test feeders. In addition, such model will be modified to include capacitors and battery energy storage systems for islanded microgrid applications.

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