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Linear power flow formulation for low-voltage DC power grids

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ABSTRACT

This paper presents a reformulation of the power flow problem in low-voltage dc (LVDC) power grids via Taylor's series expansion. The solution of the original nonlinear quadratic model is achieved with this proposed formulation with minimal error when the dc network has a well defined operative conditions. The proposed approach provides an explicit solution of the power flow equations system, which avoids the use of iterative methods. Such a characteristic enables to provide accurate results with very short processing times when real operating scenarios of dc power grids are analyzed. Simulation results verify the precision and speed of the proposed method in comparison to classical numerical methods for both radial and mesh configurations. Those simulations were performed using C++ and MATLAB, which are programming environments commonly adopted to solve power flows.

1. Introduction

1.1. General context

Low-voltage dc (LVDC) power grids are a promising distribution energy technology specially under the concepts of smart grids and microgrids [1]. An LVDC is essentially a microgrid operating with direct current, which can interconnect multiple distributed energy resources such as: energy storage devices, electric vehicles, distributed generators and consumers [2,3]. In LVDC, the concept of electrical substation is replaced by an ac–dc power converter as depicted in Fig. 1. In such an example the microgrid supplies three different types of load, where Load 1 corresponds to a constant impedance load, Load 2 models a constant power consumption and Load 3 represents a dynamic dc load. Moreover, that microgrid also includes two renewable energy resources and an energy storage device [4].

To study the steady state behavior of any power grid, it is necessary to perform a power flow analysis [5], which solves iteratively the mathematical model of the network for an arbitrary operating point [6]. In the case of dc power grids, the set of equations representing the power flow problem corresponds to a quadratic and non-convex set of equations with multiple possible solutions [4]. To the best of the authors knowledge, currently there is not reported in the literature an explicit solution for such a set of equations describing the dc power flow. Taking into account the accelerated grow of dc microgrids discussed afterwards [4,7,8], that gap in the dc power grids analysis must be addressed.

1.2. Motivation

The LVDC grid is a suitable solution to manage clean energy resources [9], which is a topic with growing interest in the research community due to the positive environmental impact of renewable energy, but also due to the possibility of providing electrical service in off-the-grid zones. For planning and management of LVDC systems, it is required to perform a power flow analysis, which enables to calculate different technical and operative aspects of the grid, such as: voltage regulation, energy losses and conductor's chargeability, among others [10]. However, the process to solve the set of quadratic equations modeling the dc power flow formulation is not a trivial task, which is mainly due to the non-convex characteristic of the solution space [6]. Therefore, to determine the steady state solution of the LVDC, iterative methods have been used. This is the case of the gradient method adopted in [11], the Newton–Raphson method used in [12,13] and the

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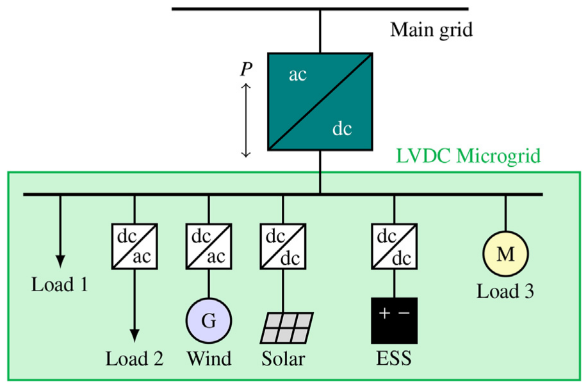

Fig. 1. Typical interconnection of distributed energy resources in a LVDC microgrid.

successive approximation algorithm reported in [4], among others.

The main drawback of iterative methods is the long processing times required to solve the power flow formulation [14]; this taking into account that such equations system must be solved several times in both planning and operation processes [15,16]. Therefore, this paper proposes a non-iterative method to solve the power flow equations in LVDC microgrids under the presence of constant-power loads. This method is designed to be applied to dc grids with radial or mesh configurations of any size and load condition, offering accurate enough results with low-computational effort, hence with shorter processing times in comparison with the iterative methods commonly adopted for this problem.

1.3. Brief state-of-the-art

In the specialized literature there is an increasing interest in LVDC under the microgrid concept, since this electrification technology allows to provide electrical service to remote areas using renewable energy resources [17]; moreover LVDC enable to improve the efficiency of classical urban distribution networks via hybrid ac–dc interconnections [18]. The works reported in [1,19] review the state-of-the-art about the importance of dc microgrids, analyzing the challenges and opportunities in terms of optimization and control. Those works identify the power flow analysis as an essential tool to perform control and optimization processes. Following this line, the authors of [12,20] present adaptations of classical methods such as Newton–Raphson or Gauss–-Seidel to this problem.

In the case of convex and linear formulations, there exist multiple references mainly devoted to ac grids [5,21]. Concerning dc grids, the work reported in [22] presents a convex reformulation of the power flow in high-voltage dc grids via semidefinite programming. One of the most analyzed problems in dc grids is the presence of constant power loads; for this reason the work reported in [4] defines the necessary conditions to guarantee the uniqueness of the solution of dc power flow equations independent of the solution method. Moreover, the works reported in [6,23] present the sufficient conditions to guarantee the existence of the solutions of power flow problems in ac and dc grids with constant power loads. Those works were carry out using linear matrix inequalities. Multiple linear techniques have been developed for solving the power flow problem in ac power grids [24–26]. This is the case of the work reported in [4], in which the Lauren series has been adopted for linearizing the power flow equations by using their complex representation. In [27] is proposed a linear method based on the Jacobian approach for radial distribution network with lateral derivations and distributed energy resources. Similarly, in [28] it is presented a linear power flow representation, which is based on a curve-fitting technique to derive a voltage-dependent load model to split the loads as a combination of impedances and current sources; moreover, numerical approximations on the imaginary part of the nodal voltages are also considered.

However, to the best of the authors knowledge, there is not reported in literature a widely accepted linear approximation to solve the power flow equations in dc power grids; which is the gap this paper is intended to fulfill.

1.4. Contribution and scope

This paper proposes a linear approximation method to solve explicitly the power flow problem in dc power grids. This method does not require any iterative process to determine the power flow solution, which provides a reduced computational effort when compared with iterative approaches. Moreover, the result is mainly oriented to LVDC grids which encompasses dc distribution, dc microgrids and nanogrids, independently of the topology and load conditions. Therefore, both radial and mesh grids can be analyzed with the proposed method.

A general expression is developed to obtain the solution of the power flow equations in LVDC grids, which requires only the admittance nodal matrix of the grid to be properly defined. Moreover, computational comparison of the proposed linear approximation with the classical non-linear methods and convex relaxation is made by using both C+ + and MATLAB implementations to validate the robustness and advantages of the proposed solution.

1.5. Document organization

The remain of paper is organized as follows: Section 2 presents both the classical nonlinear dc power flow formulation and the proposed linear approximation based on a Taylor series expansion. Then, Section 3 presents the general characteristics of the test system and the comparative methods. Afterwards, the simulation considerations, results and analyses are provided in Section 4. Finally, the conclusions given in Section 5 close the paper.

2. Power flow formulations

This section presents the derivation of the classical power flow equations and the proposed linear reformulation for dc power grids. Those models are based on the realistic assumption that the dc grid has well defined operative conditions in terms of quality service and electrical connections [4].

2.1. Network structure and considerations

Considering a dc power grid formed by a set of $\mathcal{N} = \{1, 2, ..., N\}$ nodes, it can be divided in three subsets depending on node nature: constant voltage nodes or generator nodes $\mathcal{V}$, constant impedance nodes $\mathcal{R}$ and constant power nodes $\mathcal{P}$, hence $\mathcal{N} = \{\mathcal{V} \cup \mathcal{R} \cup \mathcal{P}\}$. For practical purposes, the sets $\mathcal{R}$ and $\mathcal{P}$ can be compacted in a unique set named demands, which is defined as $\mathcal{D}$. The set of branches is represented by $\mathcal{E} = \mathcal{N} \times \mathcal{N}$.

Moreover, the following practical considerations hold [22]:

Assumption 1. The dc power grid contains a least one constant voltage node.

Assumption 2. The graph is connected, this is, there are not isolated nodes on the dc power grid.

Assumption 3. The voltage profile solutions are contained within the interval $V_{\min} \leq V \leq V_{\max}$, where $V_{\min} > 0$.

Assumption 4. The dc power grid is operating under steady state conditions, this is, there are not external perturbations.

2.2. Mathematical relations and nonlinear formulation

The classical formulation of the power flow equations for dc grids is based on the admittance nodal matrix, which relates the nodal voltages and the nodal injected currents as follows [22]:

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_k \\ \vdots \\ I_n \end{pmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1k} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2k} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ G_{k1} & G_{k2} & \cdots & G_{kk} & \cdots & G_{kn} \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nk} & \cdots & G_{nn} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_n \end{pmatrix} \tag{1}$$

In (1) I_k corresponds to the injected current at node k, while V_k represents its nodal voltage, G_{ki} is the conductance value associated to the physical interconnection between nodes k and i, and G_{kk} corresponds to the algebraic sum of all conductances connected at node k, respectively. In this model G_{kk} contains all the information of the line resistive effects related to the kth node, including the possible constant resistive loads, which reduces the power flow analysis to constant power nodes or step-nodes.

The instantaneous power in electrical networks corresponds to the product between the voltage and current on a specific point of the network, in case of the k-th node the electrical power can be defined as:

$$P_k^G - P_k^D = P_k = V_k I_k, \quad \forall\, k \in \mathcal{N}, \tag{2}$$

where P_k^D and P_k^G represent the active power demanded and generated at node k, respectively, while P_k corresponds to the total active power injected at node k. If the kth row of (1) is replaced in (2), the power flow equations take the form presented in (3).

$$P_k = V_k \sum_{j \in \mathcal{N}} G_{kj} V_j, \quad \forall\, k \in \mathcal{N} \tag{3}$$

2.3. Linear reformulation of the power flow

The linear approximation of the power flow equations is based in the Taylor's series expansion [29], for this reason the following consideration need to be fulfilled for each generator node modeled as constant voltage node:

Assumption 5. The node has the capability of generating or consuming the amount of active power needed to support the active power balance of the grid.

Under the light of the previous assumptions, expression (1) is re-written as follows [5]:

$$\begin{pmatrix} I_G \\ I_D \end{pmatrix} = \begin{bmatrix} \mathbb{G}_{GG} & \mathbb{G}_{GD} \\ \mathbb{G}_{DG} & \mathbb{G}_{DD} \end{bmatrix} \begin{pmatrix} V_G \\ V_D \end{pmatrix} \tag{4}$$

Eq. (4) I_G represents the currents provided by the generators, I_D represents the currents demanded by the loads (constant impedance and constant power nodes), V_G corresponds to the voltage values at the generators terminals, which are known, V_D corresponds to the voltage profile in all demand nodes of the network, $\mathbb{G}_{GG}$ represents the component of the conductance matrix associated to the generator interconnections, $\mathbb{G}_{DD}$ is the component of the conductance matrix associated to the load interconnections, and $\mathbb{G}_{GD} = \mathbb{G}_{DG}{}^T$ corresponds to the component of the conductance matrix relating the generators and loads.

Assumption 5 means that the generators provide the current needed to keep constant the terminals voltage, hence those are considered ideal devices. The generators voltages are calculated using V_G/V_{base}, where V_{base} is the base voltage assigned to the microgrid; in the case that a generator voltage is equal to V_{base}, that generator voltage value is 1 p.u. Moreover, the voltage profiles of the ideal generators are known, which implies that the first row of Eq. (4) is not required in the linear formulation. Therefore, Eq. (4) is simplified as follows:

$$I_D = \mathbb{G}_{DG} V_G + \mathbb{G}_{DD} V_D \tag{5}$$

2.3.1. Taylor's series expansion

To obtain an approximated linear model for the power flow equations system, it is necessary to obtain an equivalent linear form for expression (2). For linearization purposes, the inverse value of the voltage profile at the kth node was considered as a nonlinear term, this taking into account the alternative form of (2) given in (6).

$$\frac{P_k}{V_k} = I_k \tag{6}$$

The nonlinear term $\frac{1}{V_k}$ in (6) is approximated using a first-order (linear) Taylor's series expansion around the operating point V_k^0. Such a term is selected to be linearized because V_k does not take zero values; this is not the case of the current variable, which can take zero values in all step-nodes. Moreover, V_k is limited as it imposed in Assumption 3. The linearization of the term $\frac{1}{V_k}$ is based on the general form of the Taylor's formula for a continuous nonlinear function $f(x)$ around x_0 [29] presented below:

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{d^n}{dx^n} f(x_0) \right) (x - x_0)^n \tag{7}$$

Applying the definition given in (7) for a first-order expansion, i.e. $n = \{0, 1\}$, the nonlinear term $\frac{1}{V_k}$ around V_k^0 is approximated as follows:

$$\frac{1}{V_k} \approx \frac{1}{V_k^0} - \left(\frac{1}{V_k^0} \right)^2 (V_k - V_k^0) \tag{8}$$

Finally, the linear approximation of the active power balance given in (9) is obtained by substituting (8) into (6).

$$\left(\frac{2}{V_k^0} - \left(\frac{1}{V_k^0} \right)^2 V_k \right) P_k = I_k \tag{9}$$

2.3.2. Explicit calculation of the voltage profile

It must be noted that the Taylor's linearization is applied only to the nodes with unknown voltage, therefore the nodes with generators exhibiting voltage control capability are not included in the linearization process because their voltages are known [5]. Under the light of the previous analysis, expression (2) is reduced to $P_k = -P_k^D$, hence the linear approximation of the active power balance (9) is rewritten as it is given in (10), in which P_D is a vector including all constant power loads ordered in the same form previously defined in (4).

$$-2\mathrm{diag}(V_D^0)^{-1} P_D + (\mathrm{diag}(V_D^0)^{-1})^2 \mathrm{diag}(V_D) P_D = I_D \tag{10}$$

The compact form (10) uses the diagonal operator presented below:

$$\mathrm{diag} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & x_n \end{bmatrix} \tag{11}$$

Taking advantage of the diagonal operator (11) property, expression (12) is obtained:

$$-2\mathrm{diag}(V_D^0)^{-1} P_D + (\mathrm{diag}(V_D^0)^{-1})^2 \mathrm{diag}(P_D) V_D = I_D \tag{12}$$

Finally, replacing the I_D definition given in (5) into (12) leads to the linear approximation of the V_D solution given in (13).

$$V_D = ((\mathrm{diag}(V_D^0)^{-1})^2 \mathrm{diag}(P_D) - \mathbb{G}_{DD})^{-1} (\mathbb{G}_{DG} V_G + 2\mathrm{diag}(V_D^0)^{-1} P_D) \tag{13}$$

Expression (13) enables the explicit calculation of the voltage profile in the demand nodes of a dc power grid, hence iterative processes are not required. This characteristic enables to reduce, significantly, the computational time required to find the voltage profile in comparison with other solutions based on recursive algorithms. Moreover, this linear approach can be applied to both radial and mesh dc power grids.

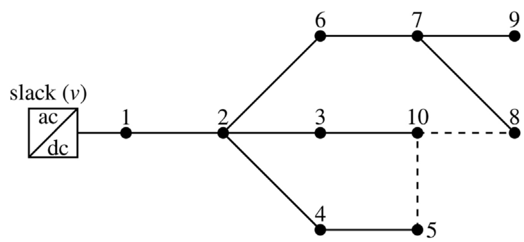

Fig. 2. Electrical configuration for the 10-bus test feeder.

Table 1
Electrical parameters of the 10 bus test system.

From	To	*R* (pu)	Type of node	*P* (pu)–*R* (pu)
1 (slack)	2	0.0050	Step-node	–
2	3	0.0015	*P*	−0.8
2	4	0.0020	*P*	−1.3
4	5	0.0018	*P*	0.5
2	6	0.0023	*R*	2.0
6	7	0.0017	Step-node	–
7	8	0.0021	*P*	0.3
7	9	0.0013	*P*	−0.7
3	10	0.0015	*R*	1.25

The following section presents the evaluation of the accuracy and speed of the proposed linear method using simulations.

3. Test systems and comparison methods

3.1. First test system

The first test system corresponds to the 10-node LVDC microgrid proposed in [4]. The electrical configuration and the network parameters of this test system are reproduced in Fig. 2 and Table 1, respectively.

The fourth column of Table 1 corresponds to the type of the receiving node (second column), while the fifth column corresponds to the value of the resistive or constant power load located at that node.

This test system was selected to validate the capability of applying the proposed linear analysis to both radial and mesh configurations: the radial topology is evaluated by preserving the configuration described in [4]; the mesh topology is evaluated by adding two lines to the test system following the connections and parameters given in Table 2.

3.2. Second test system

This test system is formed by 21 nodes with multiple constant power loads and small distributed generators. This system was originally presented in [13] for analyzing the convergence of the Newton's method in the power flow problem of DC power grids. Fig. 3 presents the test system configuration and Table 3 shows the network parameters and power consumption.

3.3. Comparison methods

To validate the effectiveness and robustness of the proposed linear method, three additional solution methods for the power flow equations in dc power grids, which are widely adopted in literature, are implemented for comparison purposes. To provide a fair comparison, techniques able to solve the problem of power flow for both radial and mesh topologies were considered. This enable to test the proposed linear approximation for any possible configuration of the DC power grid.

Table 2
Proposed connections for testing the mesh grid case.

From	To	*R* (pu)	From	To	*R* (pu)
5	10	0.0035	8	10	0.0025

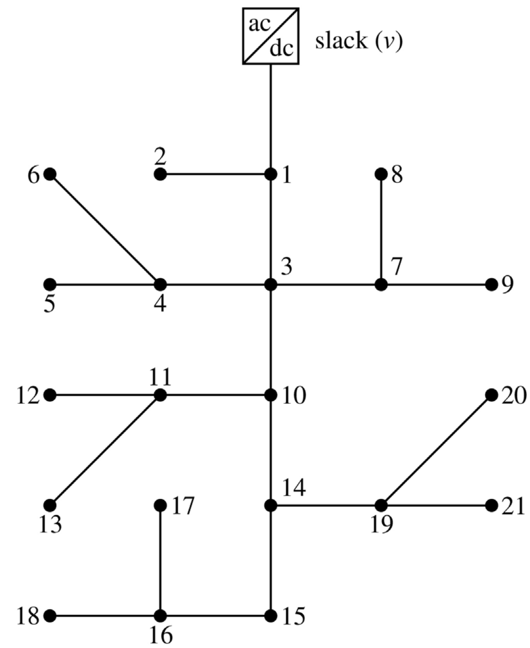

Fig. 3. Electrical configuration for the 21 bus test system.

Table 3
Electrical parameters of the 21 bus test system.

From	To	*R* (pu)	*P* (pu)	From	To	*R* (pu)	*P* (pu)
1 (slack)	2	0.0053	−0.70	11	12	0.0079	−0.68
1	3	0.0054	0.00	11	13	0.0078	0.10
3	4	0.0054	−0.36	10	14	0.0083	0.00
4	5	0.0063	−0.04	14	15	0.0065	0.22
4	6	0.0051	0.36	15	16	0.0064	−0.23
3	7	0.0037	0.00	16	17	0.0074	0.43
7	8	0.0079	−0.32	16	18	0.0081	−0.34
7	9	0.0072	0.80	14	19	0.0078	0.09
3	10	0.0053	0.00	19	20	0.0084	0.21
10	11	0.0038	−0.45	19	21	0.0082	0.21

The first method corresponds to the classical Newton–Raphson method [30], which solves the power flow equations given by (3) using successive approximations through the inverse of the Jacobian matrix. This numerical method finds a voltage profile using a recursive process as it is reported in [12], in which the Jacobian matrix of the dc power flow formulation is formed by the partial derivatives of the voltage variables only.

The second comparative method corresponds to the convex reformulation of the power flow equations given by (3) as a semidefinite programming problem [22]. This approach transforms the classical nonlinear set of dc power flow equations into a linear approximation that guarantees a unique solution [31]. The quality of the convex approximation can be measured using eigenvalue analysis as it is proposed in [22]. It must be noted that the convex reformulation, labeled in this paper as "Convex approximation", also uses a recursive process as reported in [32].

The third comparative method is the Gauss–Seidel numerical solution. This was the first numerical method applied in specialized literature for solving power flow problems [33]. Moreover, this method requires a simple implementation since it does not require matrix inversions to determine the voltage profile, but it guarantees numerical convergence for any power flow case under normal operating conditions (steady state operation), because the conductance matrix is diagonal dominant [34].

4. Computational validation

The simulations were carried out in a desk-computer with an INTEL (R) Core(TM) *i*5-3550 processor at 3.50 GHz, 8 GB RAM, running a 64-bits Windows 7 Professional operating system. The numerical validation of the proposed linear approximation was made using two different programming environments. The radial configurations were evaluated using C++, which is a classical programming language commonly used to solve power flows [35–37]. In this programming language were implemented the Gauss–Seidel, Newton–Raphson and the proposed linear method. The mesh configurations were evaluated using the MATLAB environment, which makes possible to solve convex models, via semidefinite programming, using the CVX toolbox [32]. Moreover, the proposed linear approximation, the Gauss–Seidel and Newton–Raphson methods were also implemented in MATLAB to provide a fair comparison in terms of accuracy and computational environment.

4.1. Analysis of the dc power grid with radial configurations

This section illustrates the application of the proposed technique to solve power flow in radial systems, for this case both 10-bus and 21-bus systems are considered. The performance of the proposed linear approximation is compared with the Newton–Raphson [13] and Gauss-–Seidel [4] numerical methods.

4.1.1. 10-bus test system

This simulation scenario considers the dashed lines in Fig. 2 disconnected from the main grid, which generates a radial configuration in the test system. The voltage profiles calculated with the proposed linear approach, and using the comparison methods, are reported in Table 4.

Both Newton–Raphson and Gauss–Seidel methods report almost the same values per node in Table 4, in which the error of the Gauss–Seidel method with respect to the Newton–Rahpson method is around $2.6872 \times 10^{-9}\%$. Similarly, the proposed linear method exhibits a maximum error equal to $4.9849 \times 10^{-6}\%$, which is observed in the less-significant digits of Table 4. For practical purposes, the accuracy of the linear approximation is satisfactory enough taking into account the precision of the experimental data commonly used to parametrize the model [5]. Moreover, the explicit representation of the proposed linear method avoids the use of iterative solvers, hence the processing times are significantly lower in comparison with the classical methods: after 1000 consecutive evaluations of the dc power flow, the mean value of the processing time required by the Newton–Raphson was 51.02 μs with a standard deviation of 2.70 μs; the Gauss–Seidel required 167.80 μs with a standard deviation of 13.00 μs; while the proposed linear solution required 9.34 μs with a standard deviation of 0.50 μs. Therefore, the proposed method only requires 5.56% and 18.29% of the time required by the Gauss–Seidel and Newton–Raphson methods, respectively.

Table 4
Voltage profiles for all nodes of the 10-bus test system for the radial topology (p.u.).

Node	Newton–Raphson	Gauss–Seidel	Linear approx.
1	1	1	1
2	0.983429492	0.983429494	0.983433344
3	0.981030463	0.981030465	0.981034755
4	0.981798881	0.981798883	0.981803314
5	0.982714712	0.982714714	0.982718867
6	0.981360772	0.981360775	0.981365053
7	0.980665875	0.980665878	0.980670477
8	0.981307876	0.981307878	0.981312250
9	0.979737055	0.979737057	0.979742042
10	0.979854637	0.979854640	0.979858924
8	0.981307876	0.981307878	0.981312250
9	0.979737055	0.979737057	0.979742042
10	0.979854637	0.979854640	0.979858924

Table 5
Voltage profile in p.u. at node 9 for different increments of the capacity of generation and consumption.

Load increasing	Newton–Raphson	Gauss–Seidel	Linear approx.
1	0.979737055	0.979737000	0.979742042
2	0.966725704	0.966726000	0.966753509
3	0.953351095	0.953351000	0.953435495
4	0.939581319	0.939581000	0.939775508
5	0.925379521	0.925380000	0.925760413
6	0.910702762	0.910703000	0.911376398
7	0.895500509	0.895501000	0.896608924
8	0.879712621	0.879713000	0.881442683
9	0.863266578	0.863267000	0.865861541
10	0.846073605	0.846074000	0.849848481

In order to evaluate the robustness of the proposed solution under critical load scenarios, Table 5 reports the behavior of the voltage profile in the node 9 (node with the worse voltage profile) when the load and distributed generation increase from 1 to 10 times their nominal values. The results show that the Gauss–Seidel follows the solution provided by the Newton–Raphson with a negligible error. Similarly, the proposed linear approximation accurately follows the solution generated by both classical methods, with a negligible error, when the load and generation capacity grows below 6 times their nominal values. For variations in the load and generation capacity equal or higher than 6 times their nominal values, the proposed linear method exhibits a slightly higher error; for example, a maximum error equal to 0.44% is obtained when the generation capacity and load consumption have been incremented 10 times. Such results put into evidence the robustness and efficiency of the proposed explicit method.

4.1.2. 21-bus test system

This simulation scenario considers the 21-bus system depicted in Fig. 3, which presents a radial configuration. Table 6 presents the voltage profiles obtained with the Newton–Raphson, Gauss–Seidel and proposed linear methods.

Similarly to the previous case, Table 6 reports a negligible difference (around $2.8070 \times 10^{-9}\%$) between the Gauss–Seidel and Newton–Raphson methods. The proposed linear method provides an

Table 6
Voltage profiles in all nodes of the 21-bus test system in (p.u.).

Node	Newton–Raphson	Gauss–Seidel	Linear approx.
1	1	1	1
2	0.996276133	0.996276133	0.996276185
3	0.999870879	0.999870878	0.999871048
4	0.999651190	0.999651189	0.999651354
5	0.999399038	0.999399037	0.999399202
6	1.001484468	1.001484467	1.001484628
7	1.001624231	1.001624230	1.001624243
8	0.999093938	0.999093937	0.999093952
9	1.007342248	1.007342247	1.007341953
10	0.997448213	0.997448212	0.997448775
11	0.993493968	0.993493967	0.993494969
12	0.988057035	0.988057034	0.988058821
13	0.994278457	0.994278455	0.994279431
14	1.002291131	1.002291129	1.002291349
15	1.002793986	1.002793983	1.002794128
16	1.001885027	1.001885024	1.001885107
17	1.005051035	1.005051032	1.005051034
18	0.999128625	0.999128622	0.999128707
19	1.006238881	1.006238879	1.006238866
20	1.007988901	1.007988898	1.007988774
21	1.007947304	1.007947302	1.007947181

Table 7
Voltage profile in p.u. at node 12 for different increments of the capacity of generation and consumption.

Load increasing	Newton–Raphson	Gauss–Seidel	Linear approx.
1	0.988057035	0.988057034	0.988058821
2	0.975439273	0.975439271	0.975455788
3	0.962101939	0.962101937	0.962165827
4	0.947988703	0.947988702	0.948161292
5	0.933028964	0.933028966	0.933411781
6	0.917133940	0.917133942	0.917883920
7	0.900190966	0.900190968	0.901541126
8	0.882054954	0.882054956	0.884343328
9	0.862535157	0.862535159	0.866246656
10	0.841373749	0.841373752	0.847203083

Table 8
Behavior of the active power losses for the mesh topology (p.u.).

Load increa.	Newton–Raphson	Gauss–Seidel	Convex approx.	Linear approx.
1	0.063602382	0.063602346	0.063602383	0.063573146
2	0.169358104	0.169358044	0.169358109	0.169091897
3	0.331695822	0.331695737	0.331695830	0.330567065
4	0.555455948	0.555455837	0.555455951	0.552103012
5	0.846210578	0.846210440	0.846210579	0.838113867
6	1.210428348	1.210428179	1.210428354	1.193349943
7	1.655690346	1.655690144	1.655690358	1.622926718
8	2.190977864	2.190977625	2.190977865	2.132356681
9	2.827063556	2.827063278	2.827063559	2.727584340
10	3.577055397	3.577055076	3.577055401	3.415024777

accurate calculation of the voltage profiles with a maximum error equal to 1.7058×10^{-6}%, which is enough for practical analyses. In this second test, the proposed linear approximation required 11.49% and 16.12% of the calculation time needed to process the Gauss–Seidel and Newton–Raphson methods, respectively. Therefore, the proposed solution provides a satisfactory trade-off between accuracy and processing speed in comparison with the classical Gauss–Seidel and Newton–Raphson techniques.

To evaluate the robustness of the proposed solution under critical load scenarios, Table 7 presents the behavior of the worse nodal voltage profile (node 12), when the loads and distributed generators increase from 1 to 10 times their nominal values. The results show that both Gauss–Seidel and Newton–Raphson methods provide almost identical results. Similarly to the previous test, the proposed linear approximation accurately follows the solution generated by both classical methods, with a negligible error, when the load and generation capacity grows below 6 times their nominal values. For variations in the load and generation capacity equal or higher than 6 times their nominal values, the proposed linear method exhibits a slightly higher error; in this case, a maximum error equal to 0.69% is obtained at node 10 when the generation capacity and load consumption have been incremented 10 times.

Finally, this second test put into evidence that the proposed method can be used to analyze large systems with different power demands and high penetration of distributed generation.

4.2. Analysis of the dc power grid with mesh topology

For this simulation case, the MATLAB software was used to implement the Newton–Raphson and Gauss–Seidel methods, the proposed linear approach and the convex approximation of the power flow problem, which is widely used in the specialized literature as it is reported in [22]. This case was implemented in MATLAB in order to use the CVX solver to process the convex approximation following the method reported in [32].

The adopted mesh configuration corresponds to the test system presented in Fig. 2 including the connection of the dashed lines reported in Table 2. For this system, it is illustrated the use of the proposed model in the evaluation of the active power losses when the load demand and generator injection increase from 1 to 10 times. Table 8 presents the active power losses calculated with the Newton–Raphson, Gauss–Seidel, Convex approximation and the proposed linear approaches, respectively. The active power losses were calculated from the voltage profile and conductance matrix using the following general expression:

$$p_{\text{loss}} = \sum_{i \in N} \left[\left(\sum_{j \in N} G_{ij} v_i v_j \right) - G_{i0} {v_i}^2 \right] \tag{14}$$

The first component of (14) corresponds to the power losses associated to the line conductance effects; if the system includes constant resistive loads, the associated power demand must be subtracted from the conductance matrix using the second component of (14). This last process is needed since all resistive elements interconnected to the ith node are lumped into the diagonal elements G_{ii} of the conductance matrix, including the constant resistive loads G_{i0}, which does not produce power losses.

The simulation results, reported in Table 8, put into evidence that the Gauss–Seidel and Convex approximation methods present estimation errors lower than 5.711×10^{-5}% and 2.943×10^{-6}% with respect to the Newton–Raphson method, respectively. The proposed linear method provides very accurate results, with errors lower than 0.957%, when the generation and consumption values increment from 1 to 5 times their nominal values, which is not common in real operating scenarios for dc power grids. For increments in the load demand and generation profiles higher than 5 times the errors are higher: 1.411% for 6 times, 1.979% for 7 times, 2.676% for 8 times, 3.519% for 9 times and 4.530% for 10 times.

On the other hand, the proposed linear method exhibits the shortest calculation time: the proposed method requires only the 0.67% of the average processing time required by the Newton–Raphson method. Similarly, the processing of the proposed linear method only takes the 0.21% and 0.02% of the average processing times required by the Gauss–Seidel and Convex solutions, respectively.

Therefore, the proposed solution exhibits the best performance when the generation and consumption values increment from 1 to 5 times their nominal values. For higher increments, the designer must analyze if the slightly higher error is acceptable. In any case, power flow analyzes with generation and consumption increments higher than 1.5 times of the nominal values are not frequent, since the over-current protection are usually calibrated to disconnect the system when the currents overpass 1.5 times the load current [38]. Hence, it is important to remark that increments in the power consumption up to 10 times of the nominal load were considered, in this section, just to validate the robustness of the proposed linear method, even when the power consumption increases drastically. Finally, the processing times required by the proposed linear approximation are much shorted in comparison with the other methods, hence a satisfactory trade-off between accuracy and speed is provided.

5. Conclusions and future works

The power flow analysis of dc power grids was explored in this paper. A linear equivalent model was proposed based on Taylor's series expansion, which leads to an explicit solution of the problem. This characteristic makes possible to solve the power flow equations without an iterative process, which it reduces the computational time significantly. In order to evaluate the performance of the proposed solution, three widely adopted methods were implemented and tested, namely the classical Newton–Raphson (nonlinear model), Gauss–Seidel and the Convex formulation methods.

The simulation results demonstrate the robustness and effectiveness of the proposed method, which exhibits a high grade of fidelity, when compared to the nonlinear model, for load and generation increments up to 500% of the nominal values. Therefore, the proposed linear model is suitable to be used in real operating scenarios for dc power grids, in which the levels of power variation are frequently close to 150%.

Then, the satisfactory accuracy and short processing times provided by the proposed linear approximation can be very effective for reducing the computational times of planning problems, and to implement tertiary control systems for dc microgrids, for example, to optimize the energy flows under different demand and generation scenarios. Similarly, the proposed solution could be useful for performing planning studies of dc power grids embedded into mixed-integer nonlinear programming models. This will minimize the computational efforts of those analysis by eliminating of iterative process required by classical power flow solution methods.

The proposed power flow analysis technique can be also used for power losses minimization, economic power dispatch or any optimization problem via master-slave optimization structures. In this way, the master algorithm defines the power generation or consumption in all non-controlled voltage nodes with distributed energy resources, using the proposed linear power flow approximation as the slave strategy. Moreover, the linear model can be combined with ac counterparts to solve power flow problems derived from hybrid ac-dc power grids. In terms of real-time applications, the proposed linear model can be integrated into control strategies for operating dc microgrids using real-time computation devices.

It is also possible develop a method, based on the proposed linear approximation, to analyze LVDC power microgrids with isolated zones and distributed generation capabilities. That process will require a distributed control approach to solve the voltage regulation problem: an arbitrary distributed energy resource must be selected as a virtual slack node; and the proposed linear model must be used, recursively, for solving the power flow equations until all distributed generation constraints are satisfied.

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