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A convex OPF approximation for selecting the best candidate nodes for optimal location of power sources on DC resistive networks

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ABSTRACT

This paper proposes a convex approximation approach for solving the optimal power flow (OPF) problem in direct current (DC) networks with constant power loads by using a sequential quadratic programming approach. A linearization method based on the Taylor series is used for the convexification of the power balance equations. For selecting the best candidate nodes for optimal location of distributed generators (DGs) on a DC network, a relaxation of the binary variables that represent the DGs location is proposed. This relaxation allows identifying the most important nodes for reducing power losses as well as the unimportant nodes. The optimal solution obtained by the proposed convex model is the best possible solution and serves for adjusting combinatorial optimization techniques for recovering the binary characteristics of the decision variables. The solution of the non-convex OPF model is achieved via GAMS software in conjunction with the CONOPT solver; in addition the sequential quadratic programming model is solved via quadprog from MATLAB for reducing the estimation errors in terms of calculation of the power losses. To compare the results of the proposed convex model, three metaheuristic approaches were employed using genetic algorithms, particle swarm optimization, continuous genetic algorithms, and black hole optimizers.

1. Introduction

1.1. General context

All over the world, electrical networks are the motor of the economy [3,33,37,42]. These grids are essential for providing other primary services to the population, such as: telecommunications, transportation, water, and wireless connectivity [13,19,20,23,35,43], among others. Nevertheless, the design of these networks is not an easy task for utility companies, since their planning, construction, operation and management require careful studies for making them economically profitable in the long term [15,17,21,22]. Electrical networks can be designed in alternating current (AC), direct current (DC) or hybrid configurations [34], in order to provide reliable, secure and quality service to all end-users [18]; one of the main challenge for utility companies is employing/proposing efficient mathematical models to analyze their electrical networks for making investments in them [4]. This paper provides a new mathematical tool to help utilities in their planning and operation, paying special attention to the DC paradigm as a promising approach for designing modern electrical networks.

1.2. Motivation

The optimal design of DC grids, from high-voltage to low-voltage applications, has become an important topic in the specialized literature [10,32], since these grids allow integrating multiple distributed energy resources directly to the DC network by avoiding additional power electronic inverters [11,24], which clearly permits a reduction of costs in terms of installation, operation and management [2,18]. The biggest advantage of DC networks in comparison to their AC counterparts is the elimination of the concepts of reactive power and frequency [31], which makes their control and operation easier [2]. Nevertheless, for both types of electrical networks, power flow analysis is the most important tool for knowing the steady state behavior of the grid when a determinate set of power injections, consumptions and grid topology is given [30]. This has given rise to the motivation of this paper, which focuses on providing an efficient method for solving the optimal power flow problem for a DC grid in the presence of multiple distributed generators, so as to identify their optimal locations.

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1.3. Literature survey

For DC networks there have recently been presented exact proofs of the convergence of the well-known Newton–Raphson [8] and Gauss–Seidel (successive approximation) methods [7] for power flow analysis; furthermore, a linear approximation has also been proposed based on Taylor's series [26,27,31] with results comparable to those of the conventional iterative methods. Nevertheless, when the main interest is to determine the best set of power injections to reduce power losses, then the optimal power flow (OPF) analysis appears to be the main tool [1]. Note that the OPF model for a DC network is a nonlinear non-convex problem, harder to solve efficiently [36]; for this reason, there have been recently proposed convex approximations based on semidefinite programming methods [11,9,25], second-order cone programming approaches [16], as well as approaches based on Taylor expansions [28] and sequential quadratic approximations based on the linearizations of Taylor and Newton–Raphson [30].

When we concentrate on identifying recent developments in DC grid planning, a few works related can be found in the specialized literature: [2] presented a general design for a hybrid AC-DC network for minimizing the investment and operating costs during the planning period. Moreover, [18] compared AC and DC planning models, illustrating their most important aspects from the economical and technical points of view. In [32], a planning approach for the design of DC microgrids with photovoltaic generation was presented. Nevertheless, the location of renewable generation and the DC grid topology correspond to well-known inputs to the planning problem, which reduces its mathematical complexity. Note that the optimal location and sizing of DGs in a DC grid has not been well-studied in the specialized literature, which thus constitutes the contribution of the research presented below.

1.4. Contribution

After this review of the state of the art, we see that no results about the optimal location of the power sources in a DC network have been reported in the specialized literature, except [32], where DC grids with photovoltaic (PV) generators are studied. Based on this research gap, the present paper proposes a reformulation of the OPF problem in conjunction with the relaxation of the binary characteristic of the variables associated to the optimal location of power sources, so as to obtain a convex formulation that allows identifying the best candidate nodes for the optimal location of the power sources in a DC network. The main difference of the proposed quadratic convex reformulation in comparison with the aforementioned convex OPF methods (see [30]) lies in the possibility of detecting the best set of candidate nodes for the allocation of the power sources in conjunction with the option of determining their optimal sizes by using any discretization method for treating all the binary variables that represents this problem.

1.5. Organization of the present paper

The remainder of this paper is organized as follows: Section 2 explores the classical formulation of the optimal power flow problem as well as the proposed convex reformulation using a method based on the Taylor series expansion. Section 3 presents the conventional mixed-integer nonlinear formulation of the problem of the optimal location of the power sources in a DC network. In addition, the proposed convex formulation is presented. Section 4 presents the main characteristics of two distribution test feeders, one composed of 21 and the other of 69 nodes, and multiple constant power loads. Section 5 provides all the details related to the computational implementation and results. Lastly, Section 6 presents the main conclusions derived from this research as well as possible avenues for future research.

2. Optimal power flow modeling

The mathematical modeling of the OPF problem is a nonlinear non-convex minimization problem [16], which tries to find the best combination of voltage variables and power generation to reduce the total power losses on the grid's conductors [28]. This section presents the conventional OPF model and the proposed convex approximation.

2.1. Nonlinear OPF modeling

The complete formulation of the nonlinear non-convex OPF problem is presented below [30].

Objective function:

$$\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij} v_i v_j, \tag{1}$$

where G_{ij} is the ij^{th} component of the conductance matrix, v_i and v_j represent the voltage values at nodes i and j, respectively, and z is the value of the objective function associated to the total power losses of the network [31]. Note that n is the total number of nodes.

Set of constraints:

$$p_i^g - p_i^d = v_i \sum_{j=1}^{n} G_{ij} v_j \quad \forall i \in \mathcal{N}, (2)$$

$$v_i^{\min} \leqslant v_i \leqslant v_i^{\max} \quad \forall i \in \mathcal{N}, (3)$$

$$p_i^{g,\min} \leqslant p_i^g \leqslant p_i^{g,\max} \quad \forall i \in \mathcal{N}, (4)$$

where p_i^g and p_i^d are the power generation and consumptions connected at node i, $p_i^{g,\min}$ and $p_i^{g,\max}$ are the minimum and maximum power generation capabilities at node i; while $v_i^{\min}$ and $v_i^{\max}$ are the lower and upper bounds of the voltage profile at each node. Note that $\mathcal{N}$ is the set of nodes of the DC network.

In the mathematical model given by (1)–(4), the expression (1) gives the objective function associated to the minimization of the power loss, (2) is the power balance equation which corresponds to Kirchhoff's laws in power form (Tellegen's first theorem), while (3) and (4) are the voltage regulation and power generation capability constraints, respectively.

It is important to highlight that there is only one constraint that makes the OFP model (1)–(4) a nonlinear non-convex formulation: the power balance equation (see Eq. (2)) [36]. Nevertheless, not all the constraints contained in (2) are nonlinear. Some of them are associated to the slack nodes (voltage controlled nodes) and are linear [6]. For this reason, the set of nodes $\mathcal{N}$ can be divided as $\mathcal{S} + \mathcal{D}$, where $\mathcal{S}$ represents the set of slack nodes and $\mathcal{D}$ the set of remaining nodes (demand nodes, i.e., $\mathcal{D} = \mathcal{N} - \mathcal{S}$). Based on these considerations, the power balance equation can be split as follows [30].

$$p_i^g - p_i^d = v_i \sum_{j=1}^{n} G_{ij} v_j \quad \forall i \in \mathcal{S}, \tag{5a}$$

$$p_k^g - p_k^d = v_k \sum_{m=1}^{n} G_{km} v_m \quad \forall k \in \mathcal{D}, \tag{5b}$$

Here, v_i in (5a) is the voltage profile at the voltage controlled nodes, which is constant and well-defined [7]. In addition, the presence of p_k^g on the demand nodes implies that there is the possibility of interconnecting power sources over these nodes without the ability to control their voltage profile, i.e., small-distributed generation [11].

The main challenge to the convexification of the OPF model is obtaining a linear equivalent representation of the nonlinear constraint given in (5b)[5], which is one of the main contributions of the present paper, and will be presented in the following section.

2.2. Convex approximation for the OPF problem

For transforming the OPF model into a convex approximation, let us to consider a simple product of continuous variables as follows [31].

$$f(x,y) = xy, \tag{6}$$

where the main interest is to find a linear representation of $f(x,y)$ around the operating point (x_0, y_0). To do this, we use the Taylor's series expansion around this point, as proposed in [31], which yields

$$f(x,y) = xy_0 + x_0y - x_0y_0 + \mathcal{O}(x,y), \tag{7}$$

where $\mathcal{O}(x,y)$ corresponds to the higher-order terms of the Taylor's series expansion [30]; nevertheless, for the purposes of power flow analysis, those terms can be omitted due to the fact that their contribution is small in comparison to the linear component [28].

Now, note that if we change the product xy to the product $v_k v_m$ around (v_{k0}, v_{m0}), then the expression (5b) can be transformed by (7) as presented below.

$$p_k^g - p_k^d = \sum_{m=1}^{n} G_{km}(v_{k0}v_m + v_{m0}v_k - v_{k0}v_{m0}); \forall k \in \mathcal{D}, \tag{8}$$

This is clearly a linear set of constraints, which can turn the OPF model into a convex model. For completeness, the full mathematical model with the proposed convex approximation is presented below.

Model 1 (*Convex OPF model for DC networks*).

$$\begin{aligned}
&\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij} v_i v_j, \\
&p_i^g - p_i^d = v_i \sum_{j=1}^{n} G_{ij} v_j \;\forall i \in \mathcal{S}, \\
&p_k^g - p_k^d = \sum_{m=1}^{n} G_{km}(v_{k0}v_m + v_{m0}v_k - v_{k0}v_{m0}); \forall k \in \mathcal{D}, \\
&v_i^{\min} \leqslant v_i \leqslant v_i^{\max} \;\forall i \in \mathcal{N}, \\
&p_i^{g,\min} \leqslant p_i^g \leqslant p_i^{g,\max} \;\forall i \in \mathcal{N},
\end{aligned} \tag{9}$$

Remark 1. The mathematical model (9) is a convex approximation of the optimal power flow problem that has a quadratic positive definite function, two affine hyperplanes, and two linear inequalities.

Remark 2. The linear hyperplanes (5a) and (8) may also be used for solving the classical power flow problem by employing a linear alternative form in comparison to the method based on the Taylor series used in [31] or the conventional Newton–Raphson form presented in [8].

Remark 3. The main advantage of the proposed convex reformulation of the OPF model in comparison to the classical semidefinite [25] or second-order cone approximations [16] lies in the fact that the model (9) does not create n^2 variables associated to the voltage profiles.

Finally, note that the proposed model given by (9) is different from previous work reported by [28,30], since the way the Taylor's series is employed here uses the product of linear variables for the linearization, whereas [28] uses a transformation of the hyperbola $\frac{1}{x}$ around x_0; besides, [30] proposed a convex model based on Newton–Raphson method as well as by using the voltage–current representation of the OPF model instead of the conventional power balance formulation, which make both models different from the approach proposed in the present paper.

3. Optimal locations and sizes of the power sources

Determining the optimal locations and sizes of the power sources in a DC network is a non-convex mixed-integer nonlinear programming (MINLP) model [12], where continuous variables are associated to the OPF model, i.e., expressions (1)–(4), and integer (binary) variables are associated to the possible locations of distributed generators in the network [38]. In this section, we present the exact MINLP model as well as the proposed convex relaxation for determining the set of best nodes for possible locations of power sources in a DC grid.

3.1. The exact MINLP model

To obtain an exact model that represents the optimal locations and sizes of the power sources in a DC network it is only necessary to add binary variables to the expression (4) when the power balance equation is split as presented in (5a) and (5b). The complete mathematical model of this problem is presented below.

Model 2 (*Exact MINLP model*).

$$\begin{aligned}
&\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij} v_i v_j, \\
&p_i^g - p_i^d = v_i \sum_{j=1}^{n} G_{ij} v_j \;\forall i \in \mathcal{S}, \\
&p_k^g - p_k^d = v_k \sum_{m=1}^{n} G_{km} v_m \;\forall i \in \mathcal{D}, \\
&v_i^{\min} \leqslant v_i \leqslant v_i^{\max} \;\forall i \in \mathcal{N}, \\
&p_i^{g,\min} \leqslant p_i^g \leqslant p_i^{g,\max} \;\forall i \in \mathcal{S}, \\
&x_k p_k^{g,\min} \leqslant p_k^g \leqslant p_k^{g,\max} \;\forall k \in \mathcal{D}, \\
&\sum_{k=1}^{|\mathcal{D}|} x_k \leqslant N_{ps}^{\max}, \\
&\sum_{k=1}^{|\mathcal{D}|} p_k^g \leqslant \alpha \sum_{k=1}^{|\mathcal{D}|} p_k^d, \\
&x_k \in \{0,1\}; \forall k \in \mathcal{D},
\end{aligned} \tag{10}$$

Here, $N_{ps}^{\max}$ is a scalar, the number of power sources available for installation, α is the percentage factor of power generation allowed for the power sources into the DC grid, i.e., $\alpha = 0.6$ implies that a maximum of 60% of the power consumption can be provided by distributed generation. Note that x_k is a binary variable associated to the possibility of location ($x_k = 1$) of a distributed generation at node k, or not ($x_k = 0$).

Remark 4. The MINLP model (10) is non-convex due to the presence of the power balance constraint associated to the demand nodes as well as the binary variables related to the locations of the power sources.

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For relaxing the model (10) with nonlinear sensitivity factor for the optimal selection of the best nodes for locating distributed generators in the DC network, we propose that the binary nature of x_k must be relaxed as follows.

$$0 \leqslant x_k \leqslant 1; \forall k \in \mathscr{D}. \tag{11}$$

The relaxation employed in (11) allows allocating multiple generators in the network in order to identify the most important nodes for the optimal location of distributed generators in the grid. With this consideration, we propose the convex relaxation for doing this task in the following section.

3.2. Proposed convex relaxation

The proposed convex relaxation corresponds to the inclusion in the convex OPF model defined by (9) of the set of constraints associated to the optimal location of power sources as presented in (11) by considering that the nature of the binary variable is relaxed as in (11). Based on these considerations, the proposed convex model takes the following form.

Model 3 (*Relaxed convex model*).

$$\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij} v_i v_j,$$

$$p_k^g - p_k^d = \sum_{m=1}^{n} G_{km}(v_{k0} v_m + v_{m0} v_k - v_{k0} v_{m0}); \forall k \in \mathscr{D},$$

$$p_i^g - p_i^d = v_i \sum_{j=1}^{n} G_{ij} v_j \ \forall i \in \mathscr{D}, \tag{12}$$

$$\begin{aligned} v_i^{\min} &\leqslant v_i \leqslant v_i^{\max} \ \forall i \in \mathscr{N}, \\ p_i^{g,\min} &\leqslant p_i^g \leqslant p_i^{g,\max} \ \forall i \in \mathscr{S}, \\ x_k p_k^{g,\min} &\leqslant p_k^g \leqslant p_k^{g,\max} \ \forall k \in \mathscr{D}, \\ \sum_{k=1}^{|\mathscr{D}|} x_k &\leqslant N_{ps}^{\max}, \\ \sum_{k=1}^{|\mathscr{D}|} p_k^g &\leqslant \alpha \sum_{k=1}^{|\mathscr{D}|} p_k^d, \\ 0 \leqslant x_k &\leqslant 1; \forall k \in \mathscr{D}, \end{aligned}$$

Remark 5. The main difference between model (11) and the proposed convex relaxation (12) is that the latter model guarantees a unique solution, while with the current optimization techniques this is not possible for nonlinear non-convex models as (10).

4. Test systems

For validating the proposed convex approximation for selecting the best candidate nodes for the optimal location of power sources in a DC network, we use two distribution DC feeders, one with 21 and the other with 69 nodes.

4.1. The 21-node test feeder

Fig. 1 depicts the test system, which is an adaptation of the 21-node test system proposed in [8,31]. The voltage value at the slack node is set to 1.0 p.u. All the parameters of this test system can be found in [30,28].

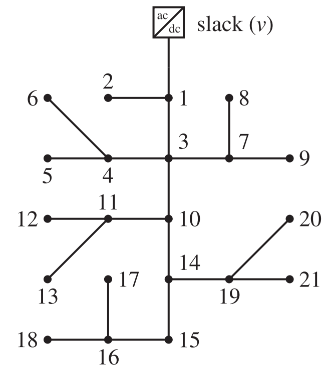

Fig. 1. Electrical configuration for the 21-node test system.

4.2. The 69-node test feeder

This test system is an adaptation of the classical AC 69-node test system employed for power loss reduction via distributed generation integration in AC networks [12,14,39]. To transform this system to a DC network, we use 12.66 kV and 100 kW as voltage and power bases. In addition, the reactance component in all branches as well as the reactive power consumption in all nodes is neglected. Fig. 2 presents the configuration of the 69-node test feeder. Its parameters can be found in [12].

5. Computational implementation and results

The solution of the non-convex nonlinear model (10) and the proposed convex approximation (12) has been carried out by the general algebraic modeling system (GAMS) in conjunction with the CONOPT solver. The simulations were carried out in a desk computer with an INTEL(R) Core(TM) $i5-3550$ processor at 3.50 GHz, 8 GB RAM, running a 64-bit Windows 7 Professional operating system.

For both test feeders, the possibility of locating three generators is evaluated. For the 21-node test feeder, generators with maximum capabilities of 1.5 p.u. are considered, while for the 69-node test feeder, this bound is extended to 12 p.u. Note that these power bounds for the DGs in both test systems were selected based on the simulation test developed in [30,28] for the 21-node test feeder, and in [12,39] for the 69-node test feeder.

5.1. The 21-node test feeder

For simulation purposes, in this test system the percentage of penetration of the power sources is fixed at 60% of the total power consumption.

Fig. 3 shows the nodes where the relaxation of the binary variable is activated, which implies that these nodes are the best candidates for the optimal location of power sources in the 21-node test system. Note that in this graphical solution 12 nodes appear as the most important nodes, which implies that for three possible locations of distributed generators with 12 nodes there are 286 combinations out of the 1140 when all nodes are considered as candidates (20 nodes without counting the slack node)[1], which constitutes an 80.70% reduction in the size of the solution space.

From Fig. 3 it is possible to observe that the exact MINLP model as well as the proposed convex relaxation identifies the same subset of nodes with the best performance for the optimal location of power sources, which implies that these models are equivalent.

[1] These values are calculated by using the formula for combinations without repetitions

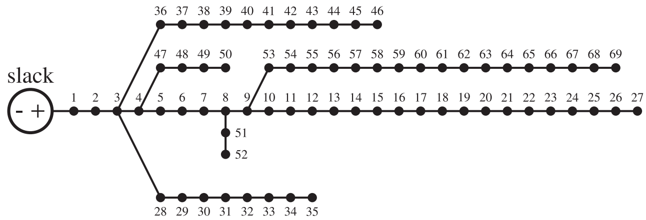

Fig. 2. Electrical configuration for the 69-node test system.

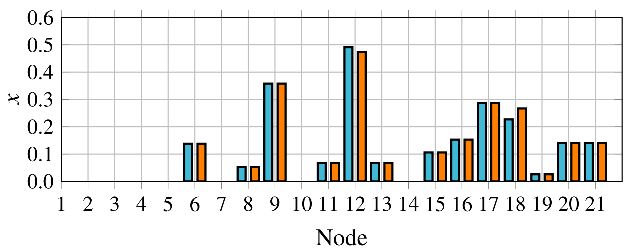

Fig. 3. Relaxed solution of the convex model for x for the 21-node test feeder.

Table 1
Behavior of the power losses in the 21-node test feeder.

Method	Initial power losses [p.u.]	Final power losses [kW]
Model 2	0.27603411	0.02085889
Model 3	0.27603411 (4)	0.02086787 (7)
Error [%]	0	0.04305

Those results are confirmed in Table 1, where the exact model and the convex proposed model attain the same value of the objective function when no power sources are installed. In addition, the minimal error, 0.04%, shows that after solving these models considering the relaxation of the binary variables, both models achieve quite similar solutions in terms of the objective function.

Note that for improving the solution of the proposed convex model (see **Model 3**), we employed the sequential quadratic programming approach recently proposed in [30]. This approach allows improving the estimation of the Taylor linearization by updating the initial point v_0 at each iteration. For this reason, the numbers four and seven appear in Table 1 in parentheses. These values indicate the number of iterations required by the sequential quadratic programming approach for solving the convex proposed model in each case when considering a convergence error lower than 1×10^{-10}. These results were obtained via MATLAB in conjunction with the quadprog toolbox.

5.2. The 69-node test feeder

In this test system we assume that the percentage of distributed generation penetration is fixed at 40% of the total power consumption.

Fig. 4 depicts the most important subset of nodes that are candidates for the optimal location of the power sources in the 69-node test feeder. Note that these nodes are concentrated per area, i.e., the nodes between 21 to 27 excluding nodes 23 and 25 (which are located at the end of the main feeder as can be seen in Fig. 2), as well as the nodes from 61 to 69 except node 63. Note that in this solution (see Fig. 4) there are 13 important nodes that combine three possibilities of allocating the power sources, which generates 286 combinations, while the original solution space has 50,116 options, which implies that the proposed relaxed models allow a reduction of 99.43% in the size of the solution space. It is important to mention that the MINLP as well as the proposed convex approximation yield the same subset of best candidate nodes for the optimal location of power sources, which is confirmed in the results presented in Table 2, which shows that both models are equivalent in terms of numerical performance by exhibiting results with errors lower than 0.04% in each case.

Note that just as happened in the 21-node test feeder, for the 69-node test feeder the sequential quadratic programming model uses seven iterations for solving the power flow model and four iterations for solving the relaxed proposed model.

5.3. Additional results

Note that the relaxed solution of the convex model for the 21- and 61-node test feeders shows that the maximum power loss reductions are 92.44% and 89.91%, respectively, which clearly are higher values taking into account the fact that for the 21-node test feeder the maximum penetration of power sources allowed is 60% of the total demand, while for the 69-node test feeder, this penetration does not exceed 40%.

Besides, for demonstrating that the solution space reduction proposed in this paper corresponds to the best possible selection of candidate nodes for the optimal location of power sources, we evaluated all 286 possibilities for both test feeders, which produced the results provided in Table 3.

From Table 3 it is possible to observe that the optimal solution (discrete solution) of the 21-node test feeder correspond to the higher bars plotted in Fig. 3. In addition, the same behavior is evidenced for the 69-node test feeder, with bars associated to nodes 21, 61 and 64. In terms of the reduction of power losses, note that the discrete solutions attain 88.31% and 89.78% for the 21- and 69-node test feeders, respectively, which are pretty close to the relaxed solutions reported previously in this subsection. Finally, these solutions confirm that the proposed method for selecting the best candidate nodes for the optimal location of power sources

Table 2
Behavior of the power losses in the 69-node test feeder.

Method	Initial power losses [p.u.]	Final power losses [kW]
Model 2	1.53847553	0.15523231
Model 3	1.53847556 (4)	0.15528870 (7)
Error [%]	1.9500×10^{-6}	0.0363

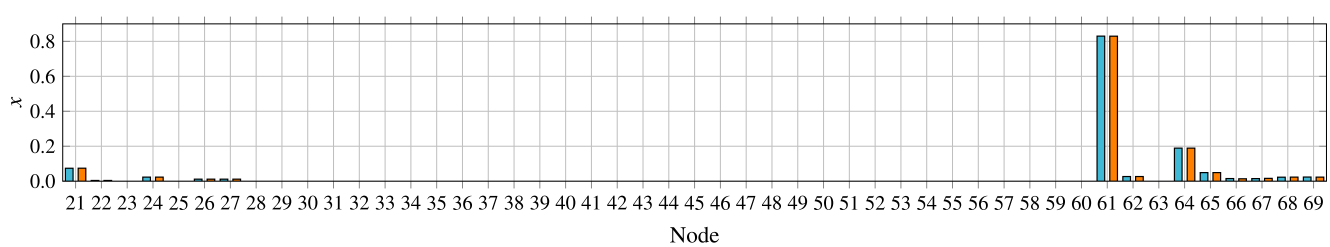

Fig. 4. Relaxed solution of the convex model for x for the 69-node test feeder.

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Table 3
Location and sizes of power sources for both test feeders.

Test feeder	Node	Size [p.u]	Node	Size [p.u]	Node	Size [p.u]	Losses [p.u]
21-node system	9	0.8350	12	1.0258	16	1.4632	0.0306
69-node system	21	1.4140	61	10.2627	64	3.8803	0.1573

in a DC grid has an excellent numerical performance for addressing the problem under analysis in this research, in terms of the quality of the final candidate nodes for possible allocation of power sources.

5.4. Comparison with combinatorial methods

To demonstrate that the proposed method effectively reaches the best possible solution when the reduced solution space is evaluated, in this section we present a comparison with a classical metaheuristic approach based on a genetic algorithm (GA) in conjunction with three optimal power flow (OPF) methods also based on metaheuristics. These methods are particle swarm optimization (PSO), reported in [41], black hole optimization (BHO) [40], and a continuous genetic algorithm (CGA) [29]. Note that the GA is a Chu & Beasley approach that is entrusted with the problem of optimal location, while the OPF methods allow solving the dimensioning problem.

To make a fair comparison between these metaheuristics, we employed 100 consecutive evaluations in order to determine the standard deviation σ, the mean value μ, the minimum value min of the power loss, as well as the average time t_{ave} that each method takes to solve the problem. In addition, the GA is parametrized with ten individuals in the population and 100 generational cycles, while the OPF methods work with ten individuals in the population and 200 iterations.

Table 4 shows the solution provided by each metaheuristic (i.e., GA-BHO, GA-CGA and GA-PSO) and contrasted with the proposed approach when all individuals in the reduced solution space are evaluated.

From the results in Table 4 we can affirm the following facts:

- The proposed convex approximation is faster than all the comparative methods presented. For the 21-node test feeder, if we add all the times in the column 6, then the proposed approach takes 4.23% of the computational time, while the GA-BHO takes 52.75% being the worst method in terms of processing time. In the case of the 69-node test feeder, our approach uses 1.81% of the processing time, while the GA-BHO approach consumes 53.49% of the computational time.
- The standard deviation in both test systems evidences values lower than 1×10^{-16}, which implies that for each consecutive evaluation the proposed approach reaches the same solution. This means that there is one and only one solution for the relaxed OPF model, due to its convexity.
- For both test feeders it is possible to observe that the optimal location of the distributed generators is identified by at least three methods: GA-CGA, GA-PSO and the proposed approach (see column 2 in Table 4. Nonetheless, each one of them evidences different minima (column 5), which implies that only the proposed approach permits attaining the global optimum for both simulation cases, namely, 0.0306 p.u. for the 21-node test feeder and 0.1573 p.u. for the 69-node test feeder, as previously reported in Table 3.
- The average values of the solution presented in column 4 of Table 4 confirm that when metaheuristic approaches are used for optimizing continuous problems, then multiple explorations are required, since such methods do not guarantee reaching a global solution. In addition, the combinatorial approaches can show high dispersion in the final solutions, as confirmed by the standard deviation in column 3.

6. Conclusion and future work

A convex approximation for the optimal power flow problem was addressed in this paper to provide an optimal subset of the best candidate nodes for the optimal location of power sources in a DC network. For doing so, a linearization method based on the Taylor series expansion was used for decomposing the product of the voltage variables. This decomposition can transform the balance power flow equations from a nonlinear non-convex set of constraints into a set of affine hyperplanes, which permits obtaining a convex optimal power flow approximation. Sequential quadratic programming was also used for reducing the estimation error between power losses and voltage values in comparison to the solution of the exact non-convex OPF model. Furthermore, the relaxation of the binary variables associated to the optimal location of the power sources for obtaining a continuous convex formulation allowed identifying the most important nodes in terms of power injection. This relaxation permitted reducing the solution space by more than 80% for both studied DC test feeders.

This method could be combined with combinatorial optimization techniques for exploring the reduced solution space in order to reach the global optimal solution of the problem. In this context, the metaheuristic optimization technique could be used as a master search algorithm entrusted with defining the locations of the power sources, while the proposed convex optimal power flow problem can be used for determining their optimal sizes.

Table 4
Comparison with combinatorial methods.

Method	Nodes	σ [p.u.]	μ [p.u.]	min [p.u.]	t_{ave} [s]
21-node test system					
GA-BHO	{9,12,16}	2.2761×10^{-03}	0.0368	0.0318	111.9440
GA-CGA	{9,12,16}	1.3537×10^{-03}	0.0329	0.0311	33.1974
GA-PSO	{9,12,16}	1.8437×10^{-03}	0.0319	0.0306	58.0934
Proposed	{9,12,16}	1.0257×10^{-16}	0.0306	0.0306	8.9688
69-node test system					
GA-BHO	{23,61 67}	2.5207×10^{-03}	0.1633	0.1593	713.7193
GA-CGA	{21,61,64}	3.4801×10^{-04}	0.1648	0.1603	218.0169
GA-PSO	{21,61,64}	5.4023×10^{-04}	0.1689	0.1588	378.4731
Proposed	{21,61,64}	1.0520×10^{-16}	0.1573	0.1573	24.1868

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