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Optimal integration of D-STATCOM in distribution grids for annual operating costs reduction via the discrete version sine-cosine algorithm

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ABSTRACT

This paper deals with the problem of the optimal placement and sizing of distribution static compensators (D-STATCOM) in electrical distribution networks to reduce the total annual operative costs associated with the total costs of energy losses added with the investment costs in D-STATCOM. The metaheuristic sine-cosine optimization algorithm determines nodes with the location and optimal sizes of the D-STATCOM. A discrete-continuous codification represents the decision variables, where the discrete part is entrusted with the best candidate nodes selection. The continuous part deals with the optimal sizes assigned to the D-STATCOM. Numerical results in the IEEE 33- and IEEE 69-bus systems demonstrate the effectiveness of this approach since it helps to minimize the total grid operation costs compared with the solution of the mixed-integer nonlinear programming model in GAMS. All the numerical validations are carried out in the MATLAB programming environment.

1. Introduction

1.1. General context

Electrical distribution networks provide electricity to all end-users at medium- and low-voltage levels [1]; grids, typically created in radial topologies, minimize investment costs in conductors and protection schemes [2,3]. The radial system generates a new problem; it leads to high energy level losses that can oscillate between 6% and 18% of the total energy input [4–6]. This amount of energy losses is concerning compared with transmission networks where energy losses are not higher than 2.0% [3,7]. In order to minimize the energy losses in distribution networks [8,9] (mainly caused by its radial topology and voltage levels), several recent studies and researches have proposed multiple approaches that include: (i) optimal grid reconfiguration [10]; (ii) optimal placement and sizing of capacitor banks [11]; (iii) optimal placement and sizing of battery energy storage systems [12]; (iv) Optimal siting and sizing of dispersed generators [13,14]; and (v) optimal siting and sizing distribution-static compensators (D-STATCOM) [15]. Those methodologies indeed help with the reduction of the total grid energy losses. Nonetheless, in the case of the grid reconfiguration, there are crucial costs associated with the new lines' creation that are higher than the cost of total energy losses recovered in the planning period. In the case of the capacitors, they are low-cost devices with high reliability and durability; however, they inject reactive power in fixed steps while the grid demand is dynamic. This continuous dynamic implies that the energy loss reduction can be limited due to the discrete nature of the reactive power injection [15]. The optimal placement of batteries and capacitors effectively reduces the amount of grid power losses; notwithstanding the above, due to the investment and operating costs, these devices are used to minimize the energy grid purchasing and greenhouse gas emissions reduction and not only for energy losses reduction [12]. In the case of the D-STATCOM, there are reactive power compensators with similar reliability and durability compared with fixed-step capacitors. Nevertheless, their prices are superior to the capacitors since these include converters that allow the management of

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reactive power amounts [16]. Hence, the main advantage of D-STATCOM is that they compensate reactive power flow dynamically as a function of the grid requirements, with subsequent better reductions in the grid operative costs compared with conventional capacitor banks.

1.2. Contributions and scope

In this research, we explore the problem of the optimal location and sizing of D-STATCOM for minimizing the total annual grid operative costs. This problem has been widely studied in the current literature. Some of these approaches have presented below. This work does not propose a "new" metaheuristic optimization strategy nor establishes improvements over the respective original method [17,18]; the objective is to apply the Discrete Version Sine-Cosine Algorithm to a power system operation problem. The contribution and newness of this work is the adaptation and implementation of Discrete Version Sine-Cosine for the Annual Operating Costs Reduction process in distribution networks. The main advantage of the suggested SCA is that it requires a classical power flow methodology to explore and exploit the solution space. For the analysis of the results, taking into account the weaknesses of the comparative analysis between metaheuristics [17,19,20], we use the solvers of the GAMS software; for a fair comparison, in each of the scenarios, we use the same computational equipment. It is relevant to highlight that the analysis of this work focuses on cost reduction for the operation of distribution networks. For this reason, we implement two test feeders, the IEEE 33-node system and the IEEE 69-node system.

1.3. Literature review

Authors [21] present the optimal placement and sizing problem of the D-STATCOM solution in distribution and transmission networks. This result is achieved by solving the exact optimization model in the General Algebraic Modeling Systems (GAMS). Numerical results validated the proposed mixed-integer nonlinear programming model (MINLP) effectiveness in representing the dynamic injection of reactive power in electrical networks. Authors [22] present the simultaneous optimal placement and sizing of D-STATCOM, a modified sine cosine algorithm (MoSCA). The proposed method was validated on two standard test systems, the 33-bus, and the 69-bus systems. Authors [23] suggested the classical genetic algorithm application to locate and size D-STATCOM in radial; and meshed distribution networks considering daily residential, industrial, and commercial demand curves. Numerical results in the IEEE 33-bus system show the efficiency of the genetic algorithm compared with the GAMS solver. In Ref. [24], the authors have recommended the whale optimization algorithm application to locate and size D-STATCOM considering different load curves. This study formulates a multi-objective optimization problem considering voltage quality, non-supplied energy, and power losses. Numerical results in test feeders with 33 and 59 nodes are presented in the cited reference. Despite positive solutions illustrated in the papers, no comparisons are provided to show the effectiveness and robustness of the proposed algorithm. Authors [15] have combined the sine cosine algorithm to locate the D-STATCOM in distribution networks and a conic formulation to find their optimal size. Residential, industrial, and commercial curves are used in the optimization process of the test feeders composed of 33 and 69 nodes. Numerical results show the effectiveness and robustness of the proposed approach when compared with the GAMS solutions. Additional works regarding the optimal placement of D-STATCOM in distribution networks include gravitational search algorithm [25]; linear approximations [26]; bat algorithms [27]; vortex search algorithm [28], and crow search algorithm [29], among others.

Unlike previous literature reports, this research proposes a discrete-continuous version of the sine-cosine algorithm (SCA) application to locate and size D-STATCOM in distribution networks. The discrete part of the codification determines the places (i.e., nodes) where these will be installed; the continuous part defines their optimal sizes. The main advantage of the suggested SCA is that it requires a classical power flow methodology to explore and exploit the solution space. It also helps with the total time processing reduction necessary in solving the exact MINLP model that represents the studied problem. Numerical results in the IEEE 33- and IEEE 69-bus systems will demonstrate the effectiveness and robustness of the proposed approach when compared with the GAMS solvers.

1.4. Organization of the document

The remainder of this document is arranged as follows: Section 2 presents the MINLP formulation of the optimal siting and sizing of the D-STATCOM problem in distribution networks, considering their daily operation. In the same way, section 3 represents the main aspects of the solution methodology based on the discrete-continuous version of the SCA. Besides, Section 4 reveals the main characteristics of the IEEE 33- and IEEE 69-bus systems. This section also incorporates the complete information to evaluate the annual operative costs of the network, including the daily demand curve. Section 5 presents the optimization results obtained with the SCA and their comparisons with the GAMS solutions. Finally, Section 6 deals with the main concluding remarks derived from this research and some possible future developments.

2. Optimization model

The problem of the optimal placement and sizing of D-STATCOM in electrical distribution networks can be formulated as a mixed-integer nonlinear programming model (i.e., MINLP) where the continuous part of the optimization problem corresponds to the classical multi-period optimal power flow. The discrete part is associated with decision variables related to the location of a D-STATCOM in a particular node [15]. The optimization objective in this problem corresponds to the minimization of the total annual grid operative cost, which is the combination of the total energy losses costs and the investment costs in D-STATCOM. Equations (1)–(3) define the objective function.

$$\min A_{cost} = f_1 + f_2 \tag{1}$$

$$f_1 = C_{kWh} T \sum_{h\in H}\sum_{k\in N}\sum_{m\in N} Y_{km} V_{kh} V_{mh} \cos(\delta_{kmh} - \theta_{km}) \Delta_h \tag{2}$$

$$f_2 = T\left(\frac{k_1}{k_2}\right)\sum_{k\in N} y_k \left(\alpha y_k^2 + \beta y_k + \gamma\right) \tag{3}$$

Equation (1) establishes the annual operating cost function (A_{cost}); it is constructed through the sum of function f_1 and function f_2. Equation (2) characterizes f_1 and establishes the energy losses in the distribution network, where:

- C_{kWh} corresponds to the expected energy costs during the planning period; T is the number of days in an ordinary year, i.e., 365 days.
- Y_{km} is the admittance parameter connecting nodes k and m, respectively.
- V_{kh} is the voltage magnitude at node k during the period h.
- V_{mh} has the same interpretation applied to the node m.
- δ_{kmh} is the angular difference between the voltage angles at nodes k and m during the time h.
- θ_{km} corresponds to the admittance angle that relates nodes k and m.
- Δ_h is the time where all the electrical variables take constant values (i.e., 1 h for this study).
- H and N represent the sets that contain all the grid nodes and all the periods considering the planning period, respectively.

Equation (3) characterizes f_2 and establishes the total investment costs in D-STATCOM, where:

- k_1 and k_2 are positive constant concerning the annualization of the D-STATCOM investment costs.
- y_k is the size of the D-STATCOM installed at node k.
- α, β, and γ are coefficients associated with the D-STATCOM investment costs.

Note that the objective function defined in Equation (1) is nonlinear and non-convex due to the presence of products among variables along with trigonometric functions in Equation (2); due to the cubic nature of the investment costs in D-STATCOM as defined in Equation (3).

In addition, the constraints associated with the studied problem include power balance at each node of the network, voltage regulation bounds, and D-STATCOM capabilities, among others. Equation (4) to Equation (8) defines the considered constraints.

$$P_{kh}^{g} - P_{kh}^{d} = \sum_{m\epsilon N} {}_{km} V_{kh} V_{mh} \cos(\delta_{kmh} - \theta_{km}) \quad \left\{ \begin{array}{l} \forall k \in N \\ \forall h \in H \end{array} \right\} \tag{4}$$

$$Q_{kh}^{g} - Q_{kh}^{d} + y_k = \sum_{m\epsilon N} {}_{km} V_{kh} V_{mh} \sin(\delta_{kmh} - \theta_{km}) \quad \left\{ \begin{array}{l} \forall k| \in N \\ \forall h \in H \end{array} \right\} \tag{5}$$

$$V_{\min} \le V_{kh} \le V_{\max} \left\{ \begin{array}{l} \forall k \in N \\ \forall h \in H \end{array} \right\} \tag{6}$$

$$x_k Q_{\min}^{DS} \le V_{kh} \le x_k Q_{\max}^{DS} \{\forall k \in N\} \tag{7}$$

$$\sum_{k \in N} x_k \le N_{ava}^{DS} \tag{8}$$

Where P_{kh}^{g} and Q_{kh}^{g} represent the active and reactive power generation in the slack node connected at node k during the period h; P_{kh}^{d} and Q_{kh}^{d} are the active and reactive constant demands linked to the node k during the period h. V_{min} and V_{max} are the minimum and maximum voltage regulation bounds permitted for all the nodal voltages at any time. x_k corresponds to the binary variable that determines the location of a D-STATCOM at node k: this is; D-STATCOM is located if x_k takes the value $x_k = 1$; otherwise, this variable takes a zero value; Q_{min}^{DS} and Q_{max}^{DS} are the minimum and maximum reactive power generation capabilities of the D-STATCOM, and N_{ava}^{DS} is a constant parameter that defines the number of D-STACOMs available for installation along with the distribution network.

It is relevant to mention that Equation (4) and Equation (5) represent the active and reactive power equilibrium at each network node during each time. The inequality constraint described in Equation (6) portrays the voltage regulation limits for all network nodes at any period; this is a typical condition imposed by regulatory entities. The inequality constraint described in Equation (7) is a box-type restriction that defines the optimal size of the D-STATCOM located at node k; the one described in Equation (8) establishes the maximum number of D-STATCOM installed along the distribution grid.

It is worth emphasizing that the set of constraints described in Equation (4) to Equation (8) is addressed through the implementation of a power flow problem that simultaneously solves Equations (4) and (5). We, as authors, handle the voltage regulation constraint described in Equation (6) through the fitness function defined in the next section. We address Equations (7) and (8) through the proposed discrete-continuous codification.

The following section explains the details of the solution methodology based on the discrete-continuous version of the sine-cosine algorithm.

3. Solution methodology

Equations (1)–(8) define the problem of the optimal placement and sizing of D-STATCOM in distribution networks to minimize the total grid annual operative costs. Hence, through the previous formulation emerges a nonlinear non-convex optimization problem. Its solution requires the application of a master-slave optimization methodology that deals with the binary and continuous parts of the model. This research proposes the application of the discrete-continuous version of the sine-cosine algorithm. The sine cosine algorithm is a combinatorial optimization technique from the family of mathematical-inspired algorithms [30]; S. M [31].

The SCA has been proposed for multiple nonlinear and mixed-integer nonlinear programming models with excellent numerical results. Some solved optimization problems have been parametric estimation in single-phase transformers [32]. The optimal location of dispersed generation in electrical distribution grids [33]. Optimal power flow solution in transmission networks [30]. And optimization of multimodal constrained models [34], among others.

In this research, we proposed an adaptation of the SCA to locate and size D-STATCOM in electrical distribution networks by using a discrete-continuous codification. The main steps in the implementation of the SCA are described below.

3.1. Generation of the initial population

The exploration and exploitation of the solution space through the SCA are developed by applying trigonometric evolution rules to an initial population [30]. This initial population is generated and randomly distributed along with the solution space. To create this population, we use a Gaussian distribution in the center of the solution space calculated as the average of the maximum and minimum bounds of the decision variables. The initial population takes the form presented in Equation (9).

$$X^0 = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{r1} & X_{r2} & \dots & X_{rn} \end{bmatrix} \tag{9}$$

Where n denotes the number of variables in the problem; in this case, the studied problem is defined by the number of D-STATCOM available for installation, i.e., $n = 2N_{ava}^{DS}$. In addition, r in the initial population in Equation (9) is the number of potential solutions. Here, the matrix X^0 represents the initial population at 0 iterations. Note that each row in X^0 is generated with the structure of Equation (10).

$$X_j^0 = [10 \quad \cdots \quad k \quad \vdots \quad 0.8725 \quad \cdots \quad y_k], \forall j = 1, \dots, r \tag{10}$$

Note that the first N_{ava}^{DS} corresponds to discrete variables that define the nodes where the D-STATCOM will be installed (integer part of the codification). The second part of this vector represents the sizes assigned for these D-STATCOM.

To generate each component of the initial population in Equation (9), with the same structure defined in Equation (10), we implemented the rule presented in Equation (11).

$$X_{jl} = X_l^{\min} + r_1 \left(X_l^{\max} - X_l^{\min} \right) \quad \forall l = 1, \dots, n \quad \forall j = 1, 2, \dots, r \tag{11}$$

Here, l corresponds to the row, j corresponds to the matrix column X^0, and r_1 is a random number with normal distribution in the rank [0,1]. Now, X_l^{max} and X_l^{min} denote the maximum and minimum values for the variables. It is worth noting that for $l \le N_{ava}^{DS}$, the rates generated with Equation (11) are rounded to the nearest integer value.

3.2. Fitness function

The evolution of the initial population through the solution space is guided by a modification of the objective function known as the fitness function. This function allows the metaheuristic optimizer to explore inaccessible zones that conduce to promissory non-explored solution regions [32]. The fitness function proposed in this research takes the form presented in Equation (12).

$$\min f_f = \left\{ A_{\text{cost}} - \mu_1 \min_{\{k \in N, h \in H\}} \{0, V_{\max} - V_{kh}\} + \mu_2 \max_{\{k \in N, h \in H\}} \{0, V_{\min} - V_{kh}\} \right\} \tag{12}$$

μ_1 and μ_2 are positive penalty factors associated with the possible voltage deviations regardless of their regulation bounds.

It is worth emphasizing that the evaluation of the proposed fitness function in Equation (12) requires the solution of the power flow Equation (4) and Equation (5). The answer to these equations is addressed in this research through the successive approximation power flow method reported in Ref. [35].

Equation (7) and Equation (8) are not included in the fitness function as penalizations since those are fulfilled by Equation (10).

3.3. Evolution of the population

The SCA receives its name incurring its trigonometric sine and cosine functions studies to explore and exploit the solution space. For doing so, the initial population is evaluated in the fitness function (Equation (12) with the help of the successive approximation power flow method [35]. This evaluation allows for identifying the best current solution named X_{best}. This solution is used to update the offspring in the later iterations for X_j^t (Equation (13) and Equation (14).

$$Y_j^{t+1} = X_j^t + r_2 \sin(r_3) \left| r_4 X_{best} - X_j^t \right| \qquad j = 1, 2, \ldots, r \tag{13}$$

$$Z_j^{t+1} = X_j^t + r_2 \cos(r_3) \left| r_4 X_{best} - X_j^t \right| \qquad j = 1, 2, \ldots, r \tag{14}$$

Where r_3 and r_4 are random numbers with normal distribution in the rank [0,1] and $[-\pi, -\pi]$, respectively; besides, r_2 is a variable factor that guides the convergence of the SCA. This factor can be defined as the current iteration t and the maximum number of iterations, i.e., $t_{máx}$. Equation (15) defines the mathematical form of the r_2 coefficient.

$$r_2 = 1 - \frac{t}{t_{max}} \tag{15}$$

The SCA determines which offspring individuals, i.e., Y_i^{t+1} and Z_i^{t+1} will make part of the new population. The following evaluation defines which individual will replace the current solution X_i^t in X_i^{t+1}.

- Select $X_i^{t+1} = Y_i^{t+1}$ as the new solution if $Z_f(Y_i^{t+1}) < f_f(Z_i^{t+1})$, and if it is better than $f_f(X_i^t)$.
- Select $X_i^{t+1} = Z_i^{t+1}$ as the new solution if $Z_f(Z_i^{t+1}) < f_f(Y_i^{t+1})$, and if it is better than $f_f(X_i^t)$.
- Otherwise, choose $X_i^{t+1} = X_i^t$.

It is relevant to mention that before the evaluation of the individuals Y_i^{t+1} and Z_i^{t+1} in the fitness function (Equation (12)), they must be adjusted using Equation (11) to preserve the feasibility of the population [32].

3.4. Ending the searching process of the SCA

The SCA iterative process finishes the solution space exploration and

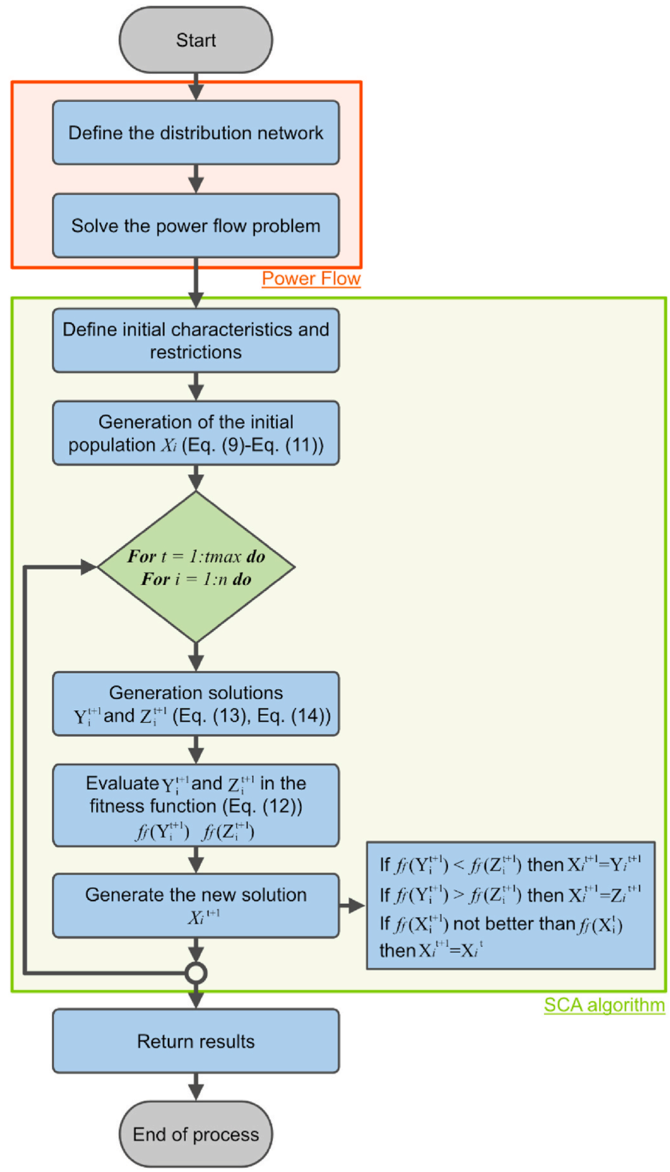

Fig. 1. Flowchart optimal integration of d-statcom with the discrete SCA algorithm.

exploitation if one of the following criteria is met [32].

- If the process reaches the maximum number of iterations $t_{máx}$.
- If, during k_{max} iterations, the best fitness function does not improve.

3.5. Algorithm

Following Algorithm 1, it is possible to solve the numerical implementation of the proposed SCA problem of the optimal placement and sizing of D-STATCOM in electrical distribution networks. Additionally, as an annex, Fig. 1 presents the respective flow diagram.

Algorithm 1
Application of the SCA to the problem of the optimal placement and sizing of D-STATCOM in distribution networks

Data: Parametrize the SCA, i.e., the number of iterations and repetitions.
Construct the initial population X^t;
Solve the power flow problem to fine the fitness function for each X_i^t to find X_{best};

(*continued on next page*)

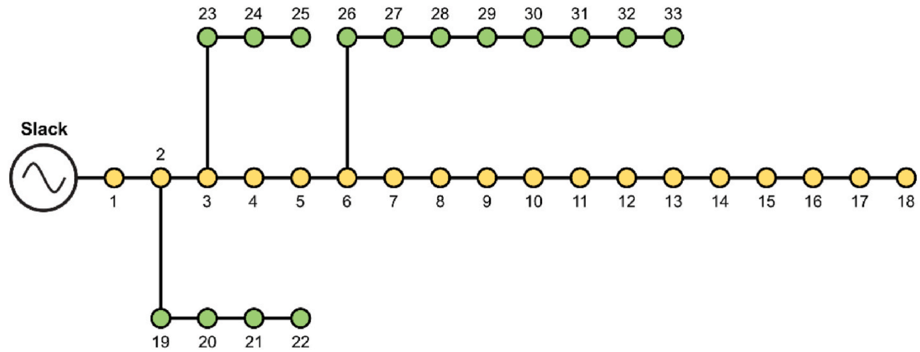

Fig. 2. IEEE 33-bus system configuration.

Table 1
Line and load parameters for the IEEE 33-bus system.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0477	100	60
2	3	0.4930	0.2511	90	40
3	4	0.3660	0.1864	120	80
4	5	0.3811	0.1941	60	30
5	6	0.8190	0.7070	60	20
6	7	0.1872	0.6188	200	100
7	8	1.7114	1.2351	200	100
8	9	1.0300	0.7400	60	20
9	10	1.0400	0.7400	60	20
10	11	0.1966	0.0650	45	30
11	12	0.3744	0.1238	60	35
12	13	1.4680	1.1550	60	35
13	14	0.5416	0.7129	120	80
14	15	0.5910	0.5260	60	10
15	16	0.7463	0.5450	60	20
16	17	1.2860	1.7210	60	20
17	18	0.7320	0.5740	90	40
2	19	0.1640	0.1565	90	40
19	20	1.5042	1.3554	90	40
20	21	0.4095	0.4784	90	40
21	22	0.7089	0.9373	90	40
3	23	0.4512	0.3083	90	50
23	24	0.8980	0.7091	420	200
24	25	0.8960	0.7011	420	200
6	26	0.2030	0.1034	60	25
26	27	0.2842	0.1447	60	25
27	28	1.0590	0.9337	60	20
28	29	0.8042	0.7006	120	70
29	30	0.5075	0.2585	200	600
30	31	0.9744	0.9630	150	70
31	32	0.3105	0.3619	210	100
32	33	0.3410	0.5302	60	40

Algorithm 1 (*continued*)

for $t = 1:\ t_{max}$ **do**
  for $i = 1 : n$ **do**
    Generate the solutions Y_i^{t+1} and Z_i^{t+1};
    Verify the feasibility of these individuals and correct them when necessary.
    Evaluate Y_i^{t+1} and Z_i^{t+1} in the fitness function, i.e., $f_f(Y_i^{t+1})$ and $f_f(Z_i^{t+1})$;
    Revise the replacing criteria for each individual to create the population for the following generation X^{t+1};
  end
    Evaluation of iterations without improvements of f_f;
  if $k \geq k_{max} \| t \geq t_{max}$ **then**
    Selection of the better solution in X^{t+1};
    Return decision variables and objective function;
    break;
  end
end
Result: Return results

4. Test cases

The evaluation of the proposed optimization methodology to locate and size D-STATCOM in electric distribution grids is made in two classical and well-known distribution grids composed of 33 and 69 nodes with radial structures. Below, we present the main details of these test.

4.1. IEEE 33-node test feeder

The IEEE 33-bus system is a radial distribution network composed of 33 nodes and 32 lines operated at the substation bus with a voltage magnitude of 12.66 kV [36]. Fig. 2 shows the electrical configuration of this test feeder.

Table 1 lists the peak active and reactive power consumptions as well as the resistances and reactances of the branches.

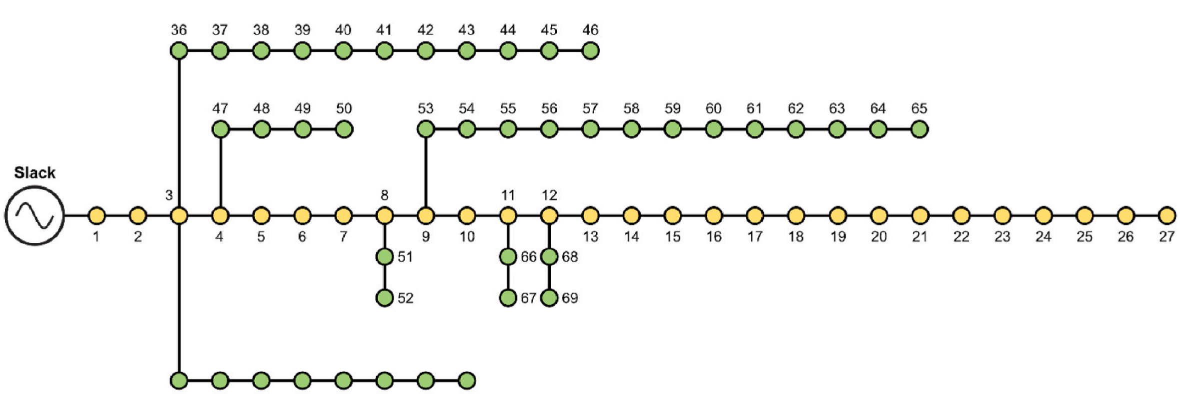

Fig. 3. IEEE 69-bus system configuration.

Table 2
Line and load parameters for the IEEE 69-bus system.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0005	0,0012	0	0
2	3	0.0005	0,0012	0	0
3	4	0.0015	0,0036	0	0
4	5	0.0251	0,0294	0	0
5	6	0.3660	0,1864	2.60	2.20
6	7	0.3810	0,1941	40.40	30
7	8	0.0922	0,0470	75	54
8	9	0.0493	0,0251	30	22
9	10	0.8190	0,2707	28	19
10	11	0.1872	0,0619	145	104
11	12	0.7114	0,2351	145	104
12	13	1.0300	0,3400	8	5
13	14	1.0440	0,3450	8	5,5
14	15	1.0580	0,3496	0	0
15	16	0.1966	0,0650	45.50	30
16	17	0.3744	0,1238	60	35
17	18	0.0047	0,0016	60	35
18	19	0.3276	0,1083	0	0
19	20	0.2106	0,0690	1	0.60
20	21	0.3416	0,1129	114	81
21	22	0.0140	0,0046	5	3.50
22	23	0.1591	0,0526	0	0
23	24	0.3463	0,1145	28	20
24	25	0.7488	0,2475	0	0
25	26	0.3089	0,1021	14	10
26	27	0.1732	0,0572	14	10
3	28	0.0044	0,0108	26	18.60
28	29	0.0640	0,1565	26	18.60
29	30	0.3978	0,1315	0	0
30	31	0.0702	0,0232	0	0
31	32	0.3510	0,1160	0	0
32	33	0.8390	0.2816	14	10
33	34	1.7080	0.5646	19.5	14
34	35	1.4740	0.4873	6	4
3	36	0.0044	0.0108	26	18.55
36	37	0.0640	0.1565	26	18.55
37	38	0.1053	0.1230	0	0
38	39	0.0304	0.0355	24	17
39	40	0.0018	0.0021	24	17
40	41	0.7283	0.8509	1.20	1
41	42	0.3100	0.3623	0	0
42	43	0.0410	0.0478	6	4,3
43	44	0.0092	0.0116	0	0
44	45	0.1089	0.1373	39.22	26.30
45	46	0.0009	0.0012	29.22	26.30
4	47	0.0034	0.0084	0	0
47	48	0.0851	0.2083	79	56.40
48	49	0.2898	0.7091	384.7	274.50
49	50	0.0822	0.2011	384.7	274.50
8	51	0.0928	0.0473	40.5	28.30
51	52	0.3319	0.1114	3.60	2,7
9	53	0.1740	0.0886	4.35	3.50
53	54	0.2030	0.1034	26.40	19
54	55	0.2842	0.1447	24	17.20
55	56	0.2813	0.1433	0	0
56	57	1.5900	0.5337	0	0
57	58	0.7837	0.2630	0	0
58	59	0.3042	0.1006	100	72
59	60	0.3861	0.1172	0	0
60	61	0.5075	0.2585	1244	888
61	62	0.0974	0.0496	32	23
62	63	0.1450	0.0738	0	0
63	64	0.7105	0.3619	227	162
64	65	1.0410	0.5302	59	42
11	66	0.2012	0.0611	18	13
66	67	0.0470	0.0140	18	13
12	68	0.7394	0.2444	28	20
68	69	0.0047	0.0016	28	20

4.2. IEEE 69-node test feeder

The IEEE 69-bus system corresponds to a radial distribution grid operated with 12.66 kV at the substation bus. Fig. 3 depicts the electrical configuration for this test feeder.

Table 3
Active and reactive power behavior in a typical working day.

Time	Act. (pu)	React. (pu)	Time	Act. (pu)	React. (pu)
1	0.3400	0.2954	25	0.9400	0.6764
2	0.2800	0.2238	26	0.9400	0.7228
3	0.2200	0.1964	27	0.9000	0.7754
4	0.2200	0.1666	28	0.8400	0.6868
5	0.2200	0.1478	29	0.8600	0.7542
6	0.2000	0.1654	30	0.9000	0.8538
7	0.1800	0.1662	31	0.9000	0.8448
8	0.1800	0.1274	32	0.9000	0.7294
9	0.1800	0.1404	33	0.9000	0.8452
10	0.2000	0.1750	34	0.9000	0.6162
11	0.2200	0.1456	35	0.9000	0.5988
12	0.2600	0.2428	36	0.9000	0.6672
13	0.2800	0.2462	37	0.8600	0.7086
14	0.3400	0.2780	38	0.8400	0.6798
15	0.4000	0.2820	39	0.9200	0.8468
16	0.5000	0.3996	40	1.0000	0.8122
17	0.6200	0.4994	41	0.9800	0.7640
18	0.6800	0.6448	42	0.9400	0.7640
19	0.7200	0.6526	43	0.9000	0.7774
20	0.7800	0.7322	44	0.8400	0.5502
21	0.8400	0.7170	45	0.7600	0.6766
22	0.8600	0.6632	46	0.6800	0.4710
23	0.9000	0.8374	47	0.5800	0.4602
24	0.9200	0.7304	48	0.5000	0.3636

Table 4
Parametrization of the objective function.

Parameter	Value	Unit	Parameter	Value	Unit
C_{kWh}	0.1390	USD/kWh	T	365	Days
Δ_h	0.50	h	α	0.30	USD/MVAr^3
β	−305.10	USD/MVAr^2	γ	127,380	USD/MVAr
k_1	6/2190	Days	k_2	10	Years

Table 2 classifies the peak active and reactive power consumptions and the resistances, reactances, and branches for the IEEE 69-bus system.

4.3. Active and reactive demand curves and D-STATCOM parameters

Table 3 details the active and reactive power curves that help to evaluate daily network energy losses.

Regarding the parametrization of the objective functions associated with the annual energy losses costs added with the investment costs in D-STATCOM, the parameters listed in Table 4 are employed.

4.4. characteristics assigned to the algorithm

The parametrization of the SCA to locate and size D-STATCOM in electrical distribution networks was developed through the implementation of multiple simulations varying the number of iterations and the population size. The number of iterations was between 500 and 2000, while the population sizes were between 5 and 30 individuals. After these simulations, results with 10 individuals in the population and 1000 iterations showed adequate convergence regarding the objective function value and processing times, for this reason, those values were set for the proposed SCA in this study.

5. Computational validations

As presented in Algorithm 1, the SCA implementation was made using MATLAB® 2021. The simulations were run on a PC with an AMD Ryzen 7 3700 2.3-GHz processor and 16.0 GB RAM, running on a 64-bit version of Microsoft Windows 10 (Single Language). As a different methodology, we consider the solution of the exact optimization model defined from Equation (1) to Equation (8) using the GAMS software with

Table 5
Comparative results between the proposed SCA and the GAMS solvers.

Method	Location (node) Size (MVAr)	A_{cost} (USD/year)
Benchmark case	–	112740.90
COUENNE	16(0.0109), 17(0.0224), 18(0.2065)	107589.50
BONMIN	17(0.0339), 18(0.0227), 30(0.2395)	102447.29
SCA	15(0.1557), 30(0.3364), 32(0.1282)	98550.08

the BONMIN and COUENNE solvers.

6. Results for IEEE 33-bus system

Table 5 presents the numerical results after implementing the SCA during 100 consecutive iterations and selecting the best optimal solution compared with the GAMS solvers.

Table 5 shows that the proposed SCA to locate and size D-STATCOM in distribution networks finds the best global optimum compared with the solvers COUENNE and BONMIM in the GAMS solvers. Concerning the benchmark case, the COUNNE solver reached a reduction of 4.57%. The BONMIM solver presented a cutback of 9.13%; for the proposed SCA, this reduction was 12.59%. Compared with the solution reported by the BONMIN solver, the proposed approach grant saved about 3897.21 USDper operation year.

Note that the solution reached by the SCA to located the D-STATCOM in nodes 15, 30, and 32 with a total reactive power capacity of 620.38 kvar; while the best solution provided by the GAMS solver located these devices at nodes 17, 18, and 30, with a nominal reactive power capability of 296.10 kvar; which is about 324.28 kvar less than the SCA. Even if this difference is high in installed capacity, the SCA prefers to invest more in D-STATCOM to have additional reductions in the total annual energy loss costs, which results in an objective function with a lower final value, as shown in Table 5.

To demonstrate the effectiveness of the proposed SCA after 100 consecutive evaluations in the IEEE 33-node test feeder, Fig. 4 depicts the relation between each solution concerning the solution reached by the BONMIN solver.

The relation between the SCA solutions and the BONMIN solution clearly shows that most of the 90% of the solutions found through the proposed discrete-continuous version of the SCA find objective functions with better numerical performance than the BONMIN solver. For instance, there are more than 90% possibilities of having a better numerical solution when the SCA is run once compared with the method available in the GAMS package. This result is especial since the multiple solutions found by the SCA will allow the distribution company to make the best decision regarding the optimal placement and sizing of their D-STATCOM as a function of the grid requirements. This distribution is not possible when only one solution is reached, as is the case of the BONMIN solver.

It is necessary to mention that between the first solution and the fiftieth solution reached by the SCA, the difference was about 870.22 USD per operation year, which implies that this solution will allow a reduction of 11.82% concerning the benchmark case, being at least 2.69% better than the solution reported by the BONMIN solver.

Regarding the total processing time, it is worth saying that the SCA takes about 47 s to solve the studied optimization problem. This result implies that, for planning purposes, the proposed approach is faster. Besides, it can carry out multiple evaluations in a short period allowing the distribution company to study numerous scenarios before deciding the solutions that will be implemented along with the distribution grid.

Table 6
Best five solutions reached by the proposed SCA in the IEEE 69-bus system.

Solution	Location (node) Size (MVAr)	A_{cost} (USD/year)
Benchmark case	–	119715.63
Solution 1	26(0.0735), 61(0.4945), 64(0.0816)	103049.30
Solution 2	23(0.0594), 61(0.4077), 64(0.1699)	103054.20
Solution 3	21(0.0634), 61(0.3514), 62(0.2277)	103128.06
Solution 4	19(0.1063), 61(0.3509), 63(0.2302)	103153.50
Solution 5	23(0.0873), 61(0.1741), 62(0.3949)	103165.25

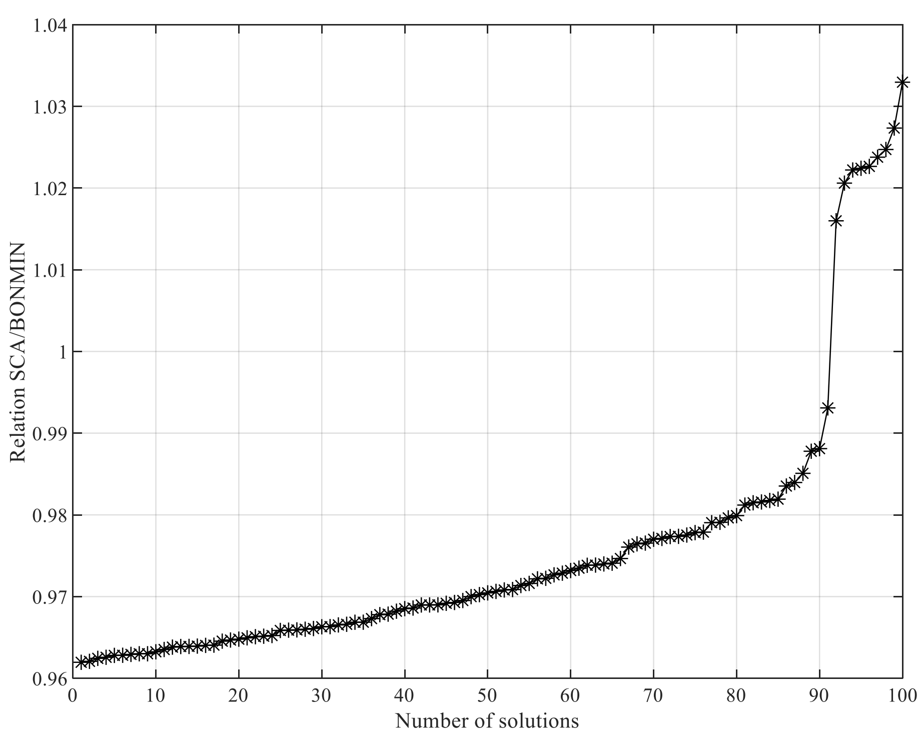

Fig. 4. Relation between the solutions reached by the SCA and the solution reported by the BONMIN solver.

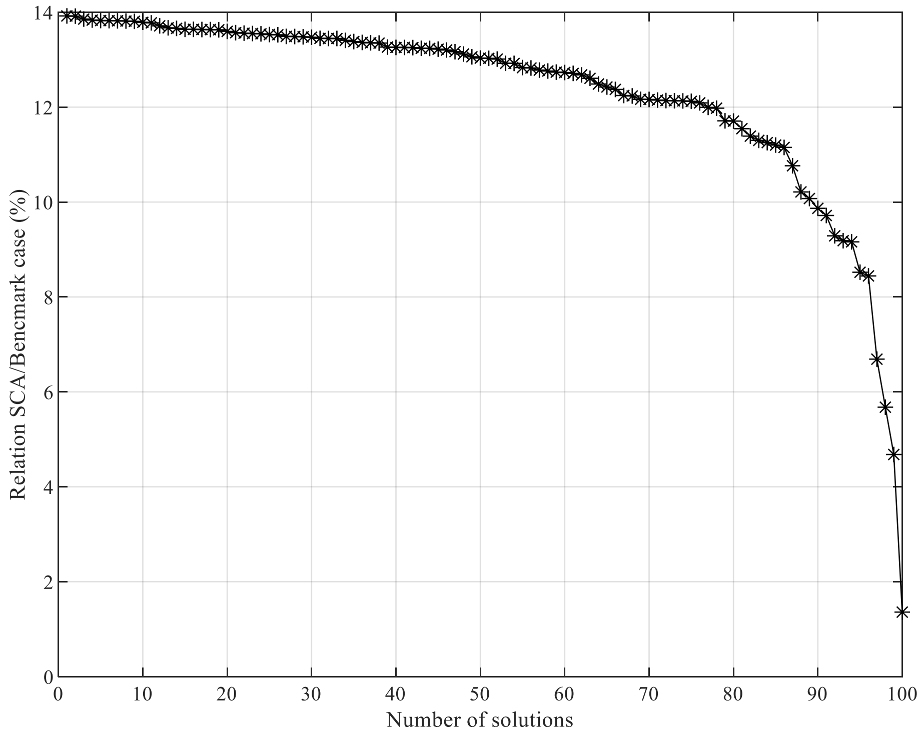

Fig. 5. Relation between the solutions reached by the SCA and the benchmark case.

6.1. Results for IEEE 69-bus system

In the IEEE 69-bus system, when is evaluated the exact optimization model (1)–(8), both solves, i.e., the BONMIN and the COUNNE, fail to reach a feasible solution. For this reason, Table 6 reports the best five solutions found by the SCA for this system.

Results in Table 6 show that the best solution reached by the SCA allows a reduction of 13.92% concerning the benchmark case, while the fifth solution represents a cutback of 13.82%. These reductions imply that the difference between both solutions is about 0.10%, which is about 115.95 USD per operation year.

Table 6 details the best solution by selecting nodes 26, 61, and 64 to locate all the D-STATCOM with a total installed capacity of 649.67 kvar. The fifth solution selects nodes 23, 61, and 62 with a total installed capacity of 656.25 kvar. In addition, for all the five answers in Table 6, node 61 is identified as part of the optimal solution. This outcome is relevant since the distribution company has confirmed that this node is indispensable in the final solution. This result is due to the high compromise of this node with the reactive power injection and the energy losses reduction.

Fig. 5 confirms the effectiveness of the proposed SCA in the solution of the optimal placement and sizing of the D-STATCOM problem in the IEEE 69-bus system, the relation between the 100 solutions found by the SCA and the benchmark case.

The percentage of reduction performance, when is achieved the cutback of each of the 100 solutions reached by the SCA, compared to the benchmark case, shows that more than 95% of the solutions find reductions higher than 8%; and 100% of solutions reduce the total costs of the grid when no D-STATCOM are considered. It is relevant to mention that about 70% of the solutions find a reduction concerning the benchmark case of about 12% or higher, which implies that in one execution of the proposed approach, there is a 70% possibility of improving the annual grid operative costs about 14,365.87 USD per operation year.

It is necessary to mention that the average processing time of the proposed SCA in the IEEE 69-bus system is about 223.66 s. This result is a minimum processing time considering that the SCA is solving a planning optimization problem that will improve the electrical performance of the grid for more than one decade.

7. Conclusions and future works

This study addresses the optimal placement and sizing of the D-STATCOM problem in distribution grids considering the reduction of the annual grid operative costs through the discrete-continuous version of the sine-cosine algorithm application. The main advantage of the discrete-continuous codification is that in just one stage, it is possible to solve the location (binary) and the size (continuous) part of the optimization model by employing only a power flow problem to calculate the objective function value. This strategy helps to reduce considerably the total processing time that requires master-slave methodologies that decouple the siting and sizing problems and solve these with different metaheuristics and embedded power flows.

Numerical results demonstrated that in the IEEE 33-bus system, the reduction for the benchmark case was about 12.59%. In the case of the IEEE 69-bus system, this reduction was around 13.92%. In addition, in the first test feeder, the BONMIN solution only reached about a 9.13% reduction, and the COUNNE only 4.57%. In the second test feeder, both solvers failed. In addition, for the IEEE 33-bus system, the SCA demonstrates an efficiency of about 90% to improve the BONMIN solver solution. In the IEEE 69-bus system, the SCA experienced 70% of possibilities to find a solution that allows a reduction of 12% or higher concerning the benchmark case.

In further studies, it is possible to develop the following research: (i) range the application of the discrete-continuous version of the SCA to the optimal siting and sizing problem of photovoltaic sources in distribution grids considering the investment, operating, and maintenance costs. (ii) Extend the application of the discrete-continuous version of the SCA to the integrated wind-solar-hydro-thermal power system considering the wind speed and solar radiation uncertainty. (iii) to combine the optimal reactive power injection problem with active power support from renewable sources to improve the electrical performance of the distribution networks considering technical, economic, and environmental objective functions.

Additionally, taking into account the characteristics of the Metaphor-based metaheuristics, other types of strategies can be used, such as DE/PSO/GSA/ABC/SCA, etc. However, it is relevant to highlight that if the objective is to compare results between metaheuristics, it is necessary to implement statistical tests, e.g.., Wilcoxon rank-sum/signed-rank test or *t*-test or ANOVA and parameter sensitivity methodologies. It is relevant to obtain several quality metrics to characterize the efficiency and effectiveness of the results; the work is permanent and requires a continuous research process.

Credit author statement

O.D. Montoya: Conception and design of study, acquisition of data, analysis and/or interpretation of data, Drafting the manuscript, revising the manuscript critically for important intellectual content. A. Molina-Cabrera: Conception and design of study, acquisition of data, analysis and/or interpretation of data, Drafting the manuscript, revising the manuscript critically for important intellectual content. D.A. Giral-Ramírez: Conception and design of study, acquisition of data, analysis and/or interpretation of data, Drafting the manuscript, revising the manuscript critically for important intellectual content. E. Rivas-Trujillo: Conception and design of study, acquisition of data, analysis and/or interpretation of data, Drafting the manuscript, revising the manuscript critically for important intellectual content. J.A. Alarcón-Villamil: Conception and design of study, acquisition of data, analysis and/or interpretation of data, Drafting the manuscript, revising the manuscript critically for important intellectual content.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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