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Optimal Demand Reconfiguration in Three-Phase Distribution Grids Using an MI-Convex Model

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Abstract: The problem of the optimal load redistribution in electrical three-phase medium-voltage grids is addressed in this research from the point of view of mixed-integer convex optimization. The mathematical formulation of the load redistribution problem is developed in terminals of the distribution node by accumulating all active and reactive power loads per phase. These loads are used to propose an objective function in terms of minimization of the average unbalanced (asymmetry) grade of the network with respect to the ideal mean consumption per-phase. The objective function is defined as the l_1 -norm which is a convex function. As the constraints consider the binary nature of the decision variable, each node is conformed by a 3×3 matrix where each row and column have to sum 1, and two equations associated with the load redistribution at each phase for each of the network nodes. Numerical results demonstrate the efficiency of the proposed mixed-integer convex model to equilibrate the power consumption per phase in regards with the ideal value in three different test feeders, which are composed of 4, 15, and 37 buses, respectively.

Keywords: load redistribution; leveling power consumption per phase; three-phase asymmetric distribution networks; ideal power consumption; mixed-integer convex optimization

1. Introduction

Most of the electricity users are typically connected to medium- and low-voltage levels, corresponding to three-phase distribution system structures [1]. The main characteristics of these networks are: (i) the radial connection among nodes helps to reduce the investment costs in protective schemes [2]; (ii) the existence of multiple single-, two-, and three-phase loads produce current unbalances that increases the amount of power losses with respect to the perfectly balanced load scenario [3]; and the high grade of active and reactive power imbalances in terminals of the substation causes deterioration of voltage profile in the nodes located at the end of the feeder [4]. The importance of the three-phase distribution networks to supply medium- and low-voltage users shows the need of proposing optimization methodologies to improve their electrical performance when the connection of new loads is required and the consumption at industrial nodes is increased [5]. The most common methodologies to improve the operative performance of the distribution networks are: optimal placement and sizing of reactive power compensators, i.e., capacitor banks and static distribution compensators [6,7]; optimal placement and sizing of disperse generation [8,9]; optimal grid reconfiguration [10]; and optimal phase-balancing [3,11].

Note that the first two methodologies are based on the connection of new devices (shunt generators and compensators) to the network, which implies large amounts of investment to improve the quality of the grid, hardly recovered in short periods of time, i.e., 5 to 15 years [12–14]; the third methodology based on grid reconfiguration involves moderate investments in tie-lines and reconfiguration of protective devices [15]; whereas the phase-balancing method is the most simple strategy to reduce power losses in three-phase distribution networks with minimum investment efforts, since devices are not required to implement the phase-balancing plan and it is only necessary to send few working crews along with the grid infrastructure to interchange the phase connections in the required nodes [3,16,17].

In the current literature can be found multiple optimization strategies, most of them based on evolutionary optimization algorithms to address the problem of the phase-balancing in three-phase networks. Some of these works apply to the following optimization methods: genetic algorithms [18,19]; tabu search algorithm [20]; particle swarm optimization [21]; ant colony optimization [22], and the vortex search algorithm [3], among others. The main characteristic of these evolutionary algorithms is the master–slave optimization strategy, where the master stage is entrusted of defining the connection of the loads using an integer or binary codification, while the slave stage determines the amount of energy losses at each connection provided by the master stage. Even if the master–slave optimization approach is widely accepted in the current literature, its main problem arises with not ensuring the global optimum finding. Recently, authors of [23] have proposed a mixed-integer quadratic programming model that allows to ensure the global optimum finding of the phase-balancing problem in three-phase networks; however, the effectiveness of this methodology was only tested in a small low-voltage microgrid. This fact has reduced the real impact of the convex methodologies in the general operation improvement of the electrical networks.

Based on the aforementioned arguments, this research deals with a problem similar to the phase-balancing approach, which is known as the load redistribution at terminals of the substation, i.e., the problem studied corresponds to leveling the active power demands per phase, making these to be accumulated in the main bus of the network (without considering the effect of the three-phase lines). The main advantage of this optimization problem is that it can be formulated with a mixed-integer convex (MIC) model that allows to ensure the global optimum finding by combining the Branch & Bound method with the interior point method. The main contributions of this research are listed below:

- The formulation of a MIC to represent the problem of the load redistribution in terminals of the substation that guarantees the global optimum finding with convex optimization tools that deal with integer problems [23];
- The evaluation of each solution provided by the MIC in a three-phase asymmetric power flow method based on its matrix formulation by rotating all the loads connected at the nodes, to find and evaluate the load redistribution configuration with minimum power losses.

The remainder of this document is structured as presented below:

Section 2 presents the proposed mixed-integer convex optimization model to represent the problem of the load redistribution in the terminals of the main substation; Section 3 presents the main characteristics of the optimization methodology based on the combination of the branch and bound (B&B) method with linear programming methods, ensuring the global optimum finding for MIC optimization models; Section 4 presents the main characteristics of the test feeders composed of 4, 15, and 37 nodes used to validate the proposed MIC optimization model using the CVX optimization tool with the MOSEK solver in the MATLAB programming environment; Section 5 describes the main numerical achievements in the three test feeders under study regarding the minimization of the general average grade of unbalance in terminals of the substation and the grid power losses. Finally, Section 6 presents the main concluding remarks derived from this work

based on the main numerical results obtained after solving the proposed MIC model with the MOSEK solver.

2. Exact Mathematical Formulation

In general terms, the problem of the load redistribution in electrical three-phase medium-voltage grids is a mixed-integer non-linear programming model due to the presence of the power balance equations [23]; however, here, we propose a mixed-integer convex (MIC) model that allows redistributing the loads at all the nodes by the accumulation in the main substation (i.e., by neglecting the effect of the electrical distribution grid) [11]. The main objective of the MIC is to find the global optimum by combining the (B&B) method with linear programming search methods. Figure 1 presents a schematic model of the load redistribution in a particular node of the network before and after the solution of the proposed MIC model.

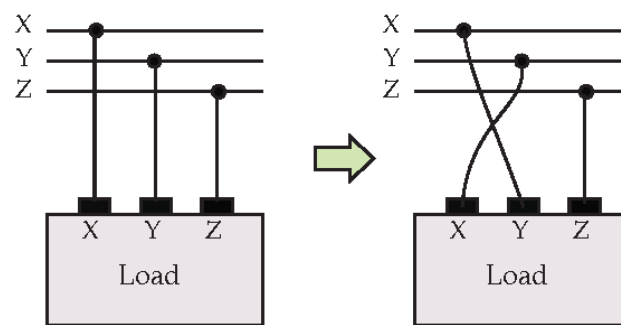


Figure 1. Redistribution of the load in a particular node of the network after solving the proposed MIC model.

Note that the main objective of the proposed MIC model is to reduce the level of asymmetry within all the loads of the network, analyzed at terminal of the main substation, i.e., redistribute all the loads in the nodes to reach the maximum level of balanced (symmetry) in the power consumption.

The complete optimization model for load redistribution in electrical asymmetric networks is fully described below.

2.1. Objective Function

The objective function of the problem is the minimization of general grid unbalance of the network with respect to the ideal consumption per phase. The objective function proposed is defined by Equation (1).

$$\min U_{\%} = \left(\frac{100}{3P_{ave}} \right) \sum_{f \in \mathcal{F}} |P_f - P_{ave}|, \quad (1)$$

where $U_{\%}$ represents the average grid unbalance; P_{ave} corresponds to the average active power consumption per phase, which is calculated as the total active power load divided by three; and P_f is the total active power consumption per phase. Note that the \mathcal{F} represents the set that contains all the phases of the network.

Remark 1. The main advantage of the objective function defined in (1) comes from the fact that this corresponds to the l_1 -norm (i.e., absolute value) which is a convex function. The convexity property is important since it is possible to ensure the global optimum finding of the optimization problem if, and only if, the set of constraints are also convex, or by using the MIC through the combination of the (B&B) method with the Simplex method.

2.2. Set of Constraints

The problem of the load redistribution is subject to linear constraints which correspond to the load reconfiguration at each phase, the binary nature of the decision variable, and the calculation of the total load per phase, among others. The complete list of constraints is defined as follows:

$$P_{kf}^d = \sum_{g \in \mathcal{G}} x_{kfg} P_{kg}^d, \quad \{\forall k \in \mathcal{N}, \forall f \in \mathcal{F}\}, \quad (2)$$

$$Q_{kf}^d = \sum_{g \in \mathcal{G}} x_{kfg} Q_{kg}^d, \quad \{\forall k \in \mathcal{N}, \forall f \in \mathcal{F}\}, \quad (3)$$

$$P_f = \sum_{k \in \mathcal{N}} P_{kf}^d, \quad \{\forall f \in \mathcal{F}\}, \quad (4)$$

$$Q_f = \sum_{k \in \mathcal{N}} Q_{kf}^d, \quad \{\forall f \in \mathcal{F}\}, \quad (5)$$

$$\sum_{g \in \mathcal{F}} x_{kfg} = 1, \quad \{\forall k \in \mathcal{N}, \forall f \in \mathcal{F}\}, \quad (6)$$

$$\sum_{f \in \mathcal{F}} x_{kfg} = 1, \quad \{\forall k \in \mathcal{N}, \forall g \in \mathcal{F}\}, \quad (7)$$

where P_{kf}^d and Q_{kf}^d are the active and reactive power connected at node k in phase f after the redistribution of the loads; P_{kg}^d and Q_{kg}^d correspond to the active and reactive power connected at the node k in the phase f before the redistribution of the loads; x_{kfg} is the binary variable that determines if the load connected in phase g is reassigned to the phase f in the node k ; P_f represents the total active power consumption of the network in the phase f after redistributing all the loads; Q_f defines the total reactive power consumption of the network in the phase f after redistributing all the loads. Note that \mathcal{N} represents the set that contains all the buses of the network.

The set of constraints defined from (2) to (7) are explained as follows: Equations (2) and (3) determine the amount of active and reactive power consumption at each phase and node after redistributing the loads in all the buses and phases of the network. Equations (4) and (5) determine the final equivalent active and reactive power consumption in the terminals of the substation after redistributing all the loads in order to minimize the average unbalance of the network; finally, Equations (6) and (7) ensure that each load is uniquely connected to one phase of the network.

Remark 2. The general structure of the set of constraints above presented show that the optimization model defined from (1) to (7) is indeed MIC, which implies that its optimal solution is achievable with conventional mixed-integer optimization methods [24].

Remark 3. The solution of the optimization model presented in (1) to (7) with conventional optimization techniques such as the (B&B) and interior point methods ensures the global optimum finding based on the mixed-integer convex theory [25]; however, it is not possible that the combination of the variables that produce the optimum value is unique. This behavior is observed in the numerical analysis presented in the Results' section.

To illustrate the effect of the three-dimensional characteristics of the decision variable x_{kfg} , let us consider the possible three-phase load connections presented in Table 1.

Table 1. Possible load distributions in a three-phase network [3].

Type of Connection	Phases	Sequence	Binary Variable x_{kfg}
1	XYZ		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2	ZXY	No change	$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
3	YZX		$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
4	XZY		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
5	YXZ	Change	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
6	ZYX		$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Note that the decision variables in Table 1 in the last column represent the six possible combinations for the connection of the three-phase loads [23], which clearly fulfills the requirements in equality constraints (6) and (7) associated with the uniqueness of the loads per phase.

It is worth mentioning that the exact formulation of the load redistribution problem in three-phase asymmetric networks is indeed a mixed-integer non-linear programming (MINLP) problem due to the power balance equations that relates the power injection at each node with the voltage and angle variables [18]; the main difficulty of the exact MINLP model lies in the non-convexity of the power balance equations that makes impossible to find the global optimum with exact or metaheuristic methods. To reduce this model complexity, here, we propose a reformulation of the load balancing problem in two stages which corresponds to the exact MIC model formulated from (1) to (7) in the first stage that helps with finding the optimal redistribution of the loads in all the nodes. The solution obtained in the first stage is evaluated at the second one to determine the final level of power losses of the grid. The two-level methodology proposed in this research is easily implemented at any optimization software that combines the (B&B) and interior point methods with the main advantage of ensuring the optimal solution as demonstrated in [25,26], some solvers that can deal with this type of problems are available in the GAMS and AMPL software. Numerical results that will be reported in the Results' section were corroborated with the CPLEX software in the GAMS software [27].

3. Methodology of Solution

To efficiently solve the MIC optimization model defined from (1) to (7) it is possible to use any programming language that deals with convex optimization. Here, we adopt the CVX optimization package in the MATLAB programming environment with the MOSEK solver. The main characteristic of the optimization model is that the objective function is defined as the l_1 -norm which is a convex function. The illustration of the objective function in a three-dimensional space is presented in Figure 2.

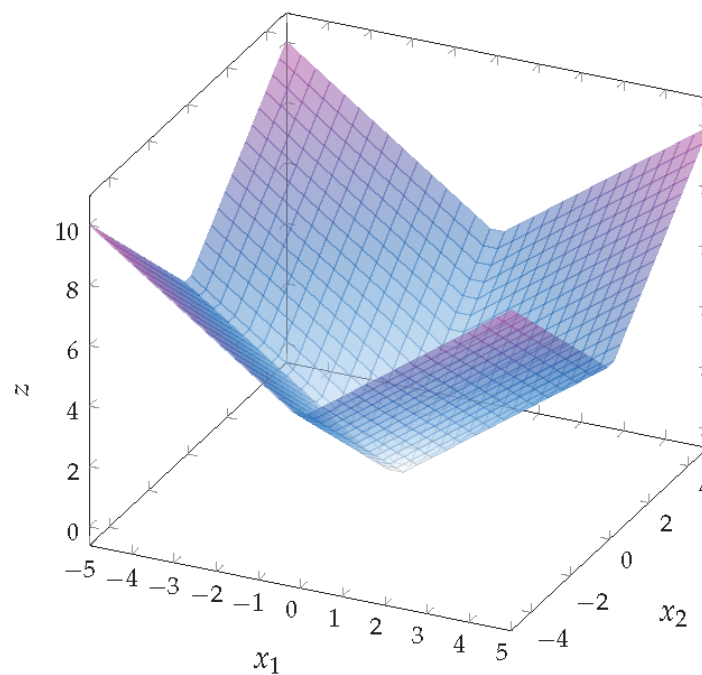


Figure 2. Representation of the objective function in a three-dimensional space $z = |x_1| + |x_2|$.

It is important to mention, that as with the most of the integer optimization models, an MIC can be solved with a modification of the (B&B) method as presented in Figure 3 [28]. Note that at each iteration, it is solved a linear programming model which ensures the optimal solution finding at each nodal exploration [29].

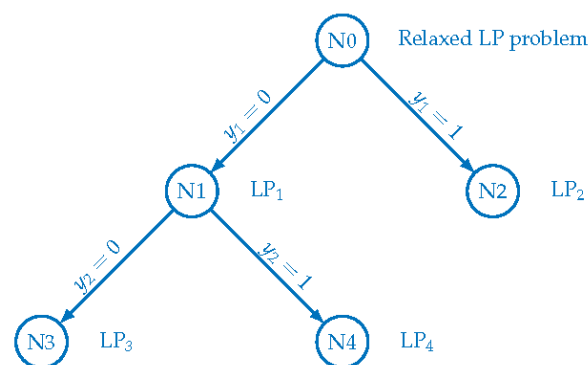


Figure 3. General application of the B&B method for addressing MIC problems.

Remark 4. Notice that the MIC model defined from (1) to (7) can be rewritten as a mixed-integer quadratic programming problem which is also convex, i.e., it is also possible to find its global optimum by combining the interior point and the (B&B) method [25].

To verify the efficiency of the optimization model to balance the total power consumption in the substation terminals and its positive effects on the minimization of the power losses, we evaluate the final load reconfiguration in an asymmetrical three-phase power flow method to determine the final power losses and compare with the initial state of the network. The power flow methodology used in this research corresponds to the matrix version of the backward–forward method reported in [3,30].

4. Electric Distribution Grids

The computational validation of the proposed MIC to redistribute loads in three-phase networks considering a simplified model in the substation terminals is made with three

different test feeders composed of 4, 15, and 37 buses, respectively. The information of these test feeders is presented below.

4.1. 4-Bus Test Feeder

The 4-bus system is a medium-voltage grid with 4 nodes and 3 lines with a nominal line-to-line voltage of 11.4 kV. The information of the loads and branches are listed in Tables 2 and 3, respectively. This information was obtained from [3].

Table 2. Parametric information of the 4-bus test system (kW and kvar units are used for all powers).

Line	Node <i>i</i>	Node <i>j</i>	Cond.	Length (ft)	P_{ja}	Q_{ja}	P_{jb}	Q_{jb}	P_{jc}	Q_{jc}
1	1	2	1	29,536	500	300	250	100	600	400
2	2	3	2	17,850	0	0	700	350	200	100
3	3	4	3	13,070	750	500	620	540	0	0

Table 3. Impedances' information for the conductors used in the 4-bus system.

Conductor	Impedance Matrix (Ω/mi)		
1		$0.3686 + j0.6852$	$0.0169 + j0.1515$
		$0.0169 + j0.1515$	$0.3757 + j0.6715$
		$0.0155 + j0.1098$	$0.0188 + j0.2072$
		$0.9775 + j0.8717$	$0.0167 + j0.1697$
2		$0.0167 + j0.1697$	$0.9844 + j0.8654$
		$0.0152 + j0.1264$	$0.0186 + j0.2275$
		$1.9280 + j1.4194$	$0.0161 + j0.1183$
3		$0.0161 + j0.1183$	$1.9308 + j1.4215$
		$0.0161 + j0.1183$	$1.9337 + j1.4236$

4.2. 15-Bus Test Feeder

This test feeder is composed by 15 buses and 14 branches, asymmetric three-phase nature network with 13.2 kV of nominal phase voltage at the node of the substation, which corresponds to the typical operating voltage in Colombian power distribution grids. In Figure 4, it is shown the electrical configuration of this test feeder.

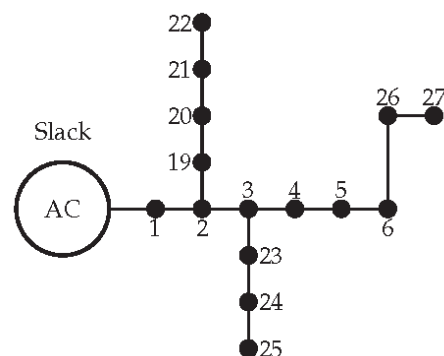


Figure 4. Nodal connections in the 15-bus test feeder.

Tables 3 and 4 present the parametric data of the 15-bus system.

Table 4. Parametric information of the 15-bus test system (kW and kvar units are used for all power values).

Line	Node i	Node j	Cond.	Length (ft)	P_{ja}	Q_{ja}	P_{jb}	Q_{jb}	P_{jc}	Q_{jc}
1	1	2	1	603	0	0	725	300	1100	600
2	2	3	2	776	480	220	720	600	1040	558
3	3	4	3	825	2250	1610	0	0	0	0
4	4	5	3	1182	700	225	0	0	996	765
5	5	6	4	350	0	0	820	700	1220	1050
6	2	7	5	691	2500	1200	0	0	0	0
7	7	8	6	539	0	0	960	540	0	0
8	8	9	6	225	0	0	0	0	2035	1104
9	9	10	6	1050	1519	1250	1259	1200	0	0
10	3	11	3	837	0	0	259	126	1486	1235
11	11	12	4	414	0	0	0	0	1924	1857
12	12	13	5	925	1670	486	0	0	726	509
13	6	14	4	386	0	0	850	752	1450	1100
14	14	15	2	401	486	235	887	722	0	0

4.3. IEEE 37-Bus Test Feeder

The IEEE 37-bus system is a three-phase unbalanced network that is a portion of a real power grid located in California, USA. This grid has 37 nodes with radial connection among them. The line-to-line voltage assigned to the substation bus is 4.8 kV. Note that the electrical configuration of this test feeder was taken from [18] where some variations to the grid topology were included. The single-phase equivalent diagram of the IEEE 37-bus system is presented in Figure 5.

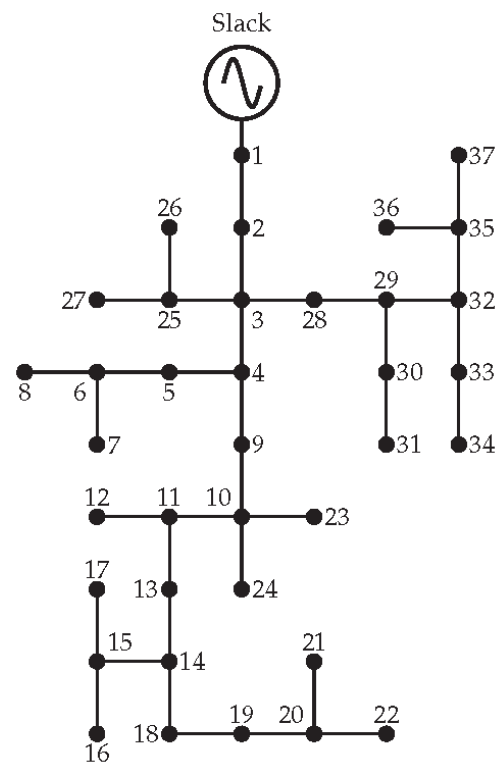


Figure 5. Nodal connection of the IEEE 37-bus system.

The complete parametric information for this test feeder is reported in Tables 5 and 6. Note that this information was taken from [3].

Table 5. Parametric information of the IEEE 37-bus test system (kW and kvar units are used for all power values).

Line	Node <i>i</i>	Node <i>j</i>	Cond.	Length (ft)	P_{ja}	Q_{ja}	P_{jb}	Q_{jb}	P_{jc}	Q_{jc}
1	1	2	1	1850	140	70	140	70	350	175
2	2	3	2	960	0	0	0	0	0	0
3	3	24	4	400	0	0	0	0	0	0
4	3	27	3	360	0	0	0	0	85	40
5	3	4	2	1320	0	0	0	0	0	0
6	4	5	4	240	0	0	0	0	42	21
7	4	9	3	600	0	0	0	0	85	40
8	5	6	3	280	42	21	0	0	0	0
9	6	7	4	200	42	21	42	21	42	21
10	6	8	4	280	42	21	0	0	0	0
11	9	10	3	200	0	0	0	0	0	0
12	10	23	3	600	0	0	85	40	0	0
13	10	11	3	320	0	0	0	0	0	0
14	11	13	3	320	85	40	0	0	0	0
15	11	12	4	320	0	0	0	0	42	21
16	13	14	3	560	0	0	0	0	42	21
17	14	18	3	640	140	70	0	0	0	0
18	14	15	4	520	0	0	0	0	0	0
19	15	16	4	200	0	0	0	0	85	40
20	15	17	4	1280	0	0	42	21	0	0
21	18	19	3	400	126	62	0	0	0	0
22	19	20	3	400	0	0	0	0	0	0
23	20	22	3	400	0	0	0	0	42	21
24	20	21	4	200	0	0	0	0	85	40
25	24	26	4	320	8	4	85	40	0	0
26	24	25	4	240	0	0	0	0	85	40
27	27	28	3	520	0	0	0	0	0	0
28	28	29	4	80	17	8	21	10	0	0
29	28	31	3	800	0	0	0	0	85	40
30	29	30	4	520	85	40	0	0	0	0
31	31	34	4	920	0	0	0	0	0	0
32	31	32	3	600	0	0	0	0	0	0
33	32	33	4	280	0	0	42	21	0	0
34	34	36	4	760	0	0	42	21	0	0
35	34	35	4	120	0	0	140	70	21	10

Table 6. Data of impedance for the conductors used in the IEEE 37-bus system.

Conductor	Impedance Matrix (Ω/mi)		
1	$0.2926 + j0.1973$	$0.0673 - j0.0368$	$0.0337 - j0.0417$
	$0.0673 - j0.0368$	$0.2646 + j0.1900$	$0.0673 - j0.0368$
	$0.0337 - j0.0417$	$0.0673 - j0.0368$	$0.2926 + j0.1973$
	$0.4751 + j0.2973$	$0.1629 - j0.0326$	$0.1234 - j0.0607$
2	$0.1629 - j0.0326$	$0.4488 + j0.2678$	$0.1629 - j0.0326$
	$0.1234 - j0.0607$	$0.1629 - j0.0326$	$0.4751 + j0.2973$
	$1.2936 + j0.6713$	$0.4871 + j0.2111$	$0.4585 + j0.1521$
3	$0.4871 + j0.2111$	$1.3022 + j0.6326$	$0.4871 + j0.2111$
	$0.4585 + j0.1521$	$0.4871 + j0.2111$	$1.2936 + j0.6713$
	$2.0952 + j0.7758$	$0.5204 + j0.2738$	$0.4926 + j0.2123$
4	$0.5204 + j0.2738$	$2.1068 + j0.7398$	$0.5204 + j0.2738$
	$0.4926 + j0.2123$	$0.5204 + j0.2738$	$2.0952 + j0.7758$

5. Computational Validation

The computational validation of the proposed MIC programming model is made in the MATLAB environment with the CVX optimization package and the MOSEK solver. In addition, we evaluate the power losses before and after solving the MIC model using the matrix version of the backward–forward asymmetrical three-phase power flow method [30].

5.1. 4-Bus System

This test feeder presents an initial power losses of 68.6292 kW with an average grid unbalance of 22.47%. After solving the load reconfiguration problem, total grid power losses is 62.5449 kW, i.e., a reduction with respect to the benchmark case is about 8.87%; in addition, the general grid unbalance is reduced until 0.74%. In Figure 6 is presented the comparison between the initial and the final grade of unbalance per phase.

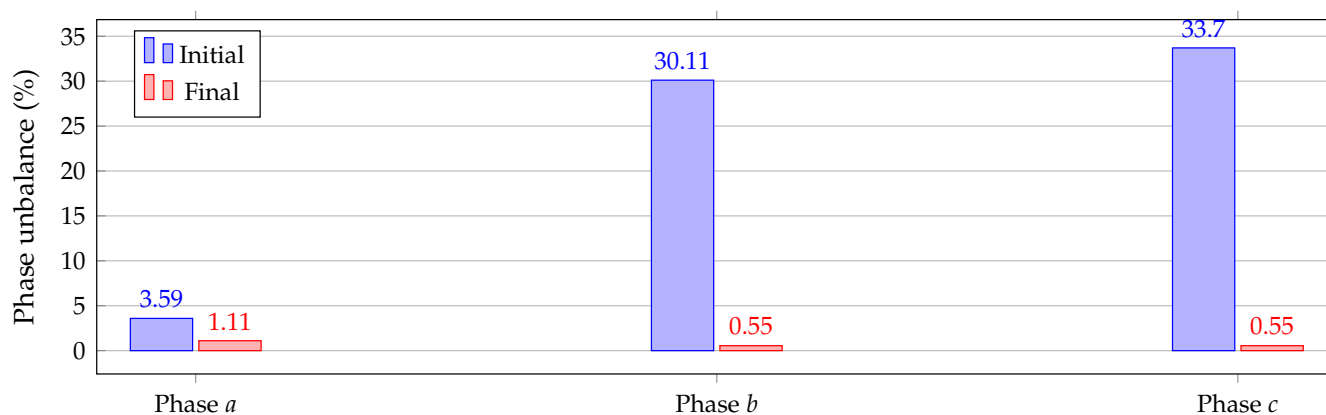


Figure 6. Initial and final unbalances in the 4-bus system.

Note that all the phases are effectively balanced with differences lower than 1.20% with respect to the ideal consumption case, i.e., P_{ave} ; in addition, the final active power load at phases a , b , and c in terminals of the substation are 1220 kW, 1200 kW, and 1200 kW, respectively; where the variations with respect to the initial case were 30 kW, 370 kW, and 400 kW, for phases a , b , and c , respectively.

Table 7 presents the final solution regarding load connections after solving the MIC model to redistribute all the loads. The most important result observed in this table corresponds to the existence at least of two possible solutions for the MIC model in the 4-bus system. This happens in this test feeder since two of the phases ends with 1200 kW of total load, which implies that some rotations in the phase connections will exhibit the same final active power losses.

Table 7. Optimal solutions reached by the MIC optimization model in the 4-bus system.

Scenario	Solution	Losses (kW)	Reduction (%)	$U_{\%}$ (%)
Benchmark case	{1, 1, 1, 1}	68.6292	0.00	22.47
Solution 1	{1, 6, 4, 5}	62.7868	8.51	0.74
Solution 2	{1, 2, 1, 3}	62.5449	8.87	0.74

It is important to mention that the solutions obtained with the MOSEK solver in the CVX environment were validated with the GAMS optimization package with the COUENNE solver. In addition, the average processing time in MATLAB including the power flow evaluations was about 1.83 s.

5.2. 15-Bus System

In this test feeder, previous to the application of the MIC model to redistribute all the loads among the phases of the network, we know that the initial power losses is 134.2472 kW, caused by a general unbalance of 20.48%, which is distributed as 2.68% for phase *a*, 30.72% pf phase *b*, and 28.04% for phase *c*. Once the MIC model defined from (1) to (7) is executed, we observe that the active power losses is uniformly distributed for all the phases with respect to the average value. In Table 8 is presented the load redistribution in the 15-bus system.

Table 8. Comparison between initial and final load distributions per phase (all the values in kW and kVar).

Scenario	P_a	Q_a	P_b	Q_b	P_c	Q_c	$U_{\%}$ (%)
Benchmark case	9605	5226	6480	4940	11,977	8778	28.04
Solution 1	9354	7112	9354	5794	9354	6038	0.00
Solution 2	9354	6038	9354	5794	9354	7112	0.00

Note that results in Table 8 show that: (i) there are at least two solutions of the optimization model (1)–(7) that present the same objective function performance which in this system is exactly zero; this implies that all the phases have the same active power load consumption per phase, i.e., 9354 kW; (ii) the general unbalance in the case of reactive power for this system in the benchmark case is 26.01% which is reduced to 8.42% after making the load redistribution; this result implies an important effect when redistributing the total consumption per phase in the substation terminals, since the modification of the active power load connection is directly connected with the total reactive power consumption; (iii) the final power losses for solutions 1 and 2 are 117.8982 kW and 115.1107 kW, with reductions respect to the benchmark case of about 12.18%, and 14.25%, respectively; and (iv) note that the main difference between solutions 1 and 2 corresponds to the rotation of the loads connected between phases *a* and *c* in all the nodes; this results important since 4 additional solutions can be obtained making possible the 6 load rotations presented in Table 1 with the same objective function of 0.00%, and power losses between 117.8982 kW and 115.1107 kW, respectively.

In regards with the total processing time the MOSEK solver using the MATLAB/CVX environment takes about 174.45 s; which is a quite small processing time taking into account that there are 6^{n-1} possible combinations of the loads, where n is the total number of nodes, i.e., 78,364,164,096, this is, more than 78,000 million of combinations.

5.3. IEEE 37-Bus System

The initial average unbalance in the IEEE 37-bus system is 22.14%, which is distributed in 11.23% for the phase *a*, 21.98% for the phase *b*, and 33.21 for the phase *c*, respectively. Once it is solved the proposed MIC model the final average unbalance in the network is 1.71%. Figure 7 depicts the initial and final unbalances per phase.

Numerical results per phase in Figure 7 show that phases *a*, *b*, and *c* are improved in about 9.16%, 19.41%, and 32.72%, respectively, with respect to the benchmark case of active power; which confirms the efficiency of the proposed approach for optimizing the general average grid unbalance, guaranteeing the global minimum of the problem.

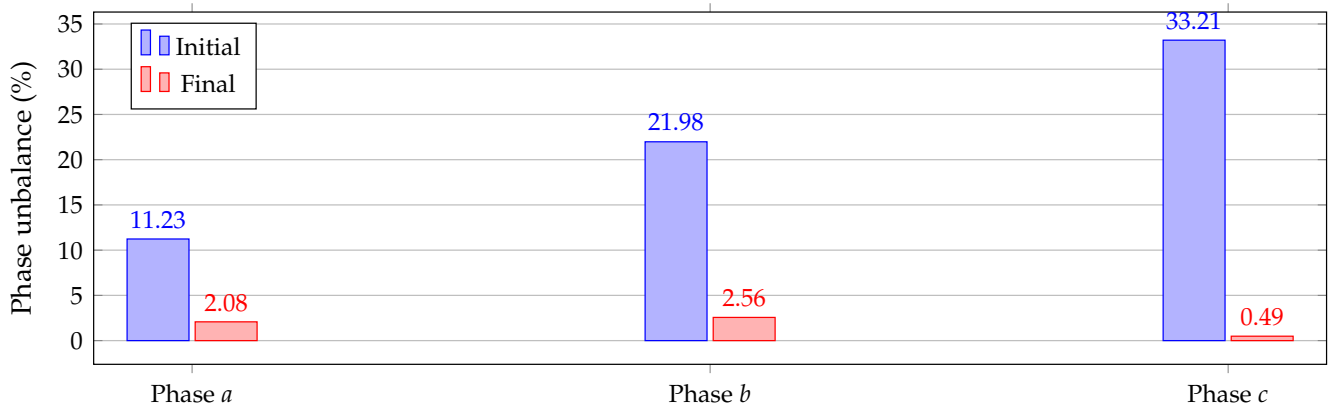


Figure 7. Initial and final unbalances in the IEEE 37-bus system.

In relation with the amount of power losses, the benchmark case presents the initial power losses of 76.1357 kW; however, for this test system after solving the MIC model there are six possible combinations that produce different levels of power losses reduction. Table 9 reports all the possible solutions in regards with power losses obtained by our proposed optimization approach.

Table 9. Optimal solutions reached by the MIC optimization model in the 15-bus system.

Scenario	Solution	Losses (kW)	Reduction (%)	$U_{\%}$ (%)
Ben. case	$\{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$	76.1357	0.00	22.14
Sol. 1	$\{1, 5, 1, 1, 2, 4, 1, 1, 4, 1, 1, 2, 5, 2, 4, 3, 5, 3\}$ $\{6, 4, 4, 6, 6, 4, 4, 1, 6, 4, 6, 4, 2, 1, 1, 1, 2, 5\}$	66.5829	12.55	1.71
Sol. 2	Rotation of Sol. 1 from XYZ to ZXY in all the nodes	67.2585	11.66	1.71
Sol. 3	Rotation of Sol. 1 from XYZ to YZX in all the nodes	67.3892	11.49	1.71
Sol. 4	Rotation of Sol. 1 from XYZ to XZY in all the nodes	67.4765	11.37	1.71
Sol. 5	Rotation of Sol. 1 from XYZ to YXZ in all the nodes	67.1325	11.83	1.71
Sol. 6	Rotation of Sol. 1 from XYZ to ZYX in all the nodes	66.6432	12.47	1.71

Results in Table 9 allow concluding that: (i) the first solution obtained by the MIC approach presents the best numerical performance regarding grid power losses with a reduction of 12.55% in comparison to the benchmark case; (ii) the worst solution regarding of power losses corresponds to solution 4, which is obtained by rotating solution 1 from XYZ to XZY in all the nodes since the reduction of the power losses in this case decreases until 11.37%; (iii) all the solutions in Table 9 are indeed the global optimum for the optimization model (1)–(7) since the general grid imbalance is 1.71% for all the solution cases; however, the calculation of the final power losses can be considered a decision criterion to select the most attractive solution from the point of view of the grid operator, which, in this context, is solution 1.

Finally, with respect to total processing time, the MOSEK solver using the MATLAB/CVX environment takes about 29,879.44 s; which is an acceptable processing time taking into account that there are 6^{n-1} possible combination of the loads, where n is the total number of nodes, i.e., $1.03144247984905 \times 10^{28}$.

5.4. Additional Comments

It is worth mentioning that all the numerical results reported with the CVX tool in the MATLAB environment for the 4-, 15-, and 37-bus systems were confirmed by the CPLEX solver in the GAMS environment with simulation times that do not get over 10 s [27]. These processing times confirm the effectiveness of the MIC model to solve the problem of the load redistribution in asymmetrical three-phase networks by ensuring the global optimum finding in the first stage of the two-level proposed optimization method [26]. The solutions provided in the first stage were evaluated in the triangular-based three-phase power flow which takes less than 10 ms to solve it and determine the final level of power losses in the grid at the second stage [3].

On the other hand, the proposed two-stage optimization methodology to redistribute the load connections in three-phase networks is suitable to be applied in the improvement of the resilience level of the electricity distribution activity [31]. This is in the context of the physical- or cyber-attacks to the distribution network or any electrical disturbance that implies the reconfiguration of the grid topology; since the proposed optimization model can be applied to each possible grid topology to ensure that after clarifying the disturbance the resulting electrical network has the minimum level of unbalance, in other words, minimum power losses.

6. Conclusions

The problem of the load redistribution in three-phase distribution networks was addressed in this research from the point of view of the mathematical optimization, by proposing a mixed-integer convex model that ensures the global optimum finding via (B&B) and linear programming methods. Numerical results in the 4-, 15-, and 37-bus systems demonstrate the effectiveness of the proposed optimization model to reduce the general average grid unbalance of the network by reducing from 22.47% to 0.74% for the 4-bus system, 20.48% to 0.00% for the 15-bus system; and 22.14% to 1.71% for the IEEE-37 bus system, respectively.

In addition, for each test feeder it was observed that when the final load reconfiguration is rotated for all the possible phase combinations, the amount of power losses in the final configuration changes with the same objective function value, which confirms the multi-modal behavior of the load redistribution optimization problem in terminals of the main substation. For the 4-, 15-, and IEEE 37-bus systems the maximum power losses reductions with respect to the benchmark case were 8.87%, 14.25%, and 12.55%, respectively. These reductions demonstrate the strong relationship between the load redistribution problem and the total grid power losses, which can be taken as an advantage of the grid owner to improve the quality of the electricity service at the same time that increases the net profit due to the reduction in the costs of the energy losses.

Regarding the processing times it was observed that for each test feeder the MOSEK solver in the CVX package using the MATLAB environment takes 1.73 s, 174.45 s, and 29,879.44 s, for the 4-, 15-, and IEEE 37-bus systems, respectively, which can be considered as short processing times due to the large dimension of the solution space. This latter can be calculated as 6^{n-1} , being 216, $7.8364164096 \times 10^{10}$, and $1.031442479849054 \times 10^{28}$ for the test feeders mentioned previously. As future work it will be possible to analyze the inclusion in the proposed MIC model of the power balance constraints with a second-order cone representation that will ensure the global optimum finding for the problem of the phase-balancing problem in three-phase networks.

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