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Recursive convex approximations for optimal power flow solution in direct current networks

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ABSTRACT

The optimal power flow problem in direct current (DC) networks considering dispersal generation is addressed in this paper from the recursive programming point of view. The nonlinear programming model is transformed into two quadratic programming approximations that are convex since the power balance constraint is approximated between affine equivalents. These models are recursively (iteratively) solved from the initial point v^t equal to 1.0 pu with t equal to 0, until that the error between both consecutive voltage iterations reaches the desired convergence criteria. The main advantage of the proposed quadratic programming models is that the global optimum finding is ensured due to the convexity of the solution space around v^t. Numerical results in the DC version of the IEEE 69-bus system demonstrate the effectiveness and robustness of both proposals when compared with classical metaheuristic approaches such as particle swarm and antlion optimizers, among others. All the numerical validations are carried out in the MATLAB programming environment version 2021b with the software for disciplined convex programming known as CVX tool in conjuction with the Gurobi solver version 9.0; while the metaheuristic optimizers are directly implemented in the MATLAB scripts.

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1. INTRODUCTION

In the last year the smart operation of electrical networks and the inclusion of distributed generation have been a topic highly studied in literature [1]–[3]. In this way, the direct current (DC) networks have rapidly increase their presence in modern power systems from high to low voltage applications due to their efficiency regarding power losses and voltage profiles when compared with classical alternating current (AC) networks [4], [5]. The main difference between AC and DC grids in that in the case of the latter the frequency and reactive power are non-existing concepts, which make these efficient and ease controllable when compared with their AC counterparts [6], [7]. The analysis of DC distribution systems can be addressed from the dynamic and static points of view; being the former mainly entrusted with control designs on power electronic converters to regulate voltages and maximize power usage in renewable energies and batteries, among others [8]–[11]. The latter approach is focused on the optimization stage that allows determining the optimal references that must follow the controllers in the former stage [12]. In this research, we pay attention to the optimization stage in DC distribution networks through the solution of the optimal power flow (OPF) problem considering dispersed generation [13], [14]. The OPF problem, as well as the economic dispatch model in power systems, are nonlinear programming models from the family of non-convex optimization problems, due to the presence of products among variables in the power equilibrium constraint at each bus of the network [15], [16]. In the current literature different optimization approaches, most of them based on combinatorial optimization, have proposed to solve the OPF in DC networks. Some of these approaches are listed in Table 1.

The main characteristic of the metaheuristic optimization methods in Table 2 is that all of them works based on the master-slave optimization concept. The master optimizer is the stage entrusted with defining the power outputs of the constant power sources (distributed generators). The slave optimization component is the stage that solves the power flow problem to determine the total grid power losses for each power input [21], [22].

Even if this methodologies are widely-known and accepted in the current literature to address the OPF problem, the main problem with these methodologies is the impossibility of ensuring the global optimum finding owing the random processes during the exploration of the solution space [23]. An alternative way to solve the OPF problem in DC networks correspond to the exact optimization techniques that allow ensuring the global optimum finding with convex approximations [24]. The most known exact optimization methods are semidefinite programming; second-order cone programming [25]; sequential quadratic programming models [26]; and quadratic non-iterative approximations [27]; among others. The main characteristic of these OPF approaches is that these work directly with the non-linearities present in the power equilibrium equation to propose convex equivalent formulation based on semidefinite programming, conic or linear equivalents, which ensure the existence of the optimal solution of the relaxed model and also in some cases with zero gap when compared with the nonlinear model [24].

In this research, we focus on the family of the exact optimization methods by proposing two alternative methodologies to solve the OPF problem via recursive convex optimization. These recursive approaches deal with the power equilibrium equations, where the first approach consists in the application of the McCormick envelopes to the product between the continuous variables $v_k v_j$ around the operative point $\left(v_k^0, v_j^0\right)$; while the second approach directly linearizes this product by using through as $v_k^0 v_j$. Both equivalent voltage products are updated through a recursive solution by assigning an iterative counter t that allows updating the voltage profile from the initial value $v^t = 1.0$ pu (with $t = 0$) until that the error between both consecutive voltages $\max\{\left|v^{t+1} - v^t\right|\}$ reaches a minimum convergence parameter ε. Numerical results demonstrate that both recursive approximations guarantee the global optimum finding since the equivalent models are from the family of the quadratic programming, i.e., convex optimization equivalents. In addition, the main advantage of the proposed recursive quadratic programming models when compared with classical combinatorial optimizers is the fact that no statistical tests are required to validate their efficiency since from convexity theory the global optimization properties are ensured.

The remainder of this paper is structured as follows: section 2 presents the nonlinear programming model associated with the OPF problem in DC distribution networks; section 3 presents the proposed recursive approximations which are based on the McCormick envelopes and the product approximation [28]; in addition, the general solution algorithm is presented. Section 4 presents the main characteristics of the IEEE 69-bus system in its DC version, and all the numerical validations of the proposed convex and their comparison with classical combinatorial methods. Finally, the main concluding remarks and possible future works are listed in section 5.

Table 1. Metaheuristic methods proposed for solving the OPF problem

Method	Acronym	Reference	Year
Particle swarm optimization	PSO	[17]	2017
Black hole optimization	BH	[18]	2019
Continuous genetic algorithm	CGA	[9]	2020
Antlion optimizer	ALO	[13]	2020
Multi-verse optimization	MVO	[19]	2021
Sine-cosine algorithm	SCA	[20]	2022

2. EXACT POWER FLOW FORMULATION

DC distribution networks can be modeled by applying the nodal voltage method in all the nodes of the network (except the reference node) by considering that most of these nodes includes constant power loads [5]. The application of the nodal voltage method produces a nonlinear programming (NLP) model, where the objective function is typically associated with the minimization of the amount of power losses [13]. Here, we assume that the DC grid, contains one slack source and some distributed generators that optimally dispatched allows reducing the total grid power losses [9]. The complete optimization model is presented below.

2.1. Objective function

The problem of the optimal power flow problem in DC networks has as the objective function the minimization of the total grid power losses in all the branches of the network, which can be formulated mathematically as (1):

$$\min p_{\text{loss}} = \sum_{j\in\mathcal{N}} \sum_{k\in\mathcal{N}} G_{kj} v_k v_j, \tag{1}$$

where p_{loss} is the objective function value associated with the total grid power losses; G_{kj} is the component of the conductance matrix that relates nodes j and k; and v_k and v_k represent the voltage magnitudes at nodes j and k, respectively. Note that $\mathcal{N}$ represent the set that contains all the buses of the network. The objective function defined in (1) corresponds to a quadratic convex function, since the nodal conductance matrix (G) is positive semidefinite and symmetric matrix; which imply that the objective function can be rewritten as $p_{\text{loss}} = v^\top G v$, where v is the vector that contain all the nodal voltages.

2.2. Set of constraints

The optimal power flow problem in DC grids is typically constrained by the power balance equation at each node as well as by restrictions associated with devices capabilities and voltage regulation bounds [9], among others. The complete list of constraints is shown in (2)-(6).

$$p_k^g + p_k^{gd} - P_k^d = \sum_{j\in\mathcal{N}} G_{kj} v_k v_j,\ \forall k \in \mathcal{N} \tag{2}$$

$$p_k^{g,\min} \le p_k^g \le p_k^{g,\max},\ \forall k \in \mathcal{N} \tag{3}$$

$$p_k^{gd,\min} \le p_k^{gd} \le p_k^{gd,\max},\ \forall k \in \mathcal{N} \tag{4}$$

$$v_k^{\min} \le v_k \le v_k^{\max},\ \forall k \in \mathcal{N} \tag{5}$$

$$v_j = V_{\text{nom}},\ j = \text{slack}, \tag{6}$$

Where p_k^g is the total power injection in the conventional source connected to the node k, p_k^{gd} is the total power injection in the dispersed generator connected at node k, and P_k^d represents the total constant power consumption at node k; $p_k^{g,\min}$ and $p_k^{g,\max}$ represent the lower and upper bounds assigned to the power generation in the slack source connected at bus k, respectively; $p_k^{gd,\min}$ and $p_k^{gd,\max}$ are the minimum and maximum power bounds associated with the dispersed generation connected at bus k, respectively; $v_k^{\min}$ and $v_k^{\max}$ correspond to the lower and upper voltage regulation limits allowed for each node of the network. V_{nom} is the operative voltage output at the substation bus, which in per unit representation, corresponds to 1 pu. Note that the power balance constraint defined in (2) is a nonlinear non-convex constraint due to the presence of the products among voltages in the right-hand side of this equation [5]. In addition, the main challenge in the power flow analysis of DC networks corresponds to convexify this constraint to ensure the global optimum finding via specialized convex optimizers [27].

2.3. Model interpretation

The optimal power flow formulation defined from (1) to (6) has the following interpretation: as shown in (1) corresponds to the objective function that is associated with the minimization of the power losses in all the branches of the network; (2) is the constraint related with the power equilibrium at each node of the network. Box-type constraints (3) and (4) represent the lower and upper bounds of the decision variables associated with the power injection of the conventional and disperse generators. Inequality constraint (5) is a box-type constraint associated with the voltage regulation limits allowed for all the nodes of the network. This constraint is typically related with regulatory policies. Finally, (6) defines the output voltage profile at the substation bus, which is typically constant in grid connected networks.

3. RECURSIVE CONVEX APPROXIMATIONS

The mathematical formulation related with the problem of the optimal power flow problem in DC grids presented from (1)-(6) is a nonlinear non-convex optimization problem due to the power equilibrium constraint defined by (2). To convexify this model, two possible recursive formulations which are based on the approximation of the product among the voltage variables can be implemented. The main advantage of these approximaitons is that the size of the solution space remains constant, which are not the cases of the second-order cone and semidefinite programming models.

3.1. Recursive formulation based on the McCormick approximation (RMA)

The McCormick approximation of the voltage variables is a recent developed approximation approach to reduce the complexity of the power balance equation in electrical networks. To obtain the McCormick approximation, let is consider a function of two variables, i.e., $f(z_1, z_2) = z_1 z_2$, which will be linearized using the first approximation of the Taylor's series expansion around (z_{10}, z_{20}), then, the function $f(z_1, z_2)$ takes the following linear form:

$$f(z_1, z_2) \approx z_1 z_{20} + z_{10} z_2 - z_{10} z_{20}. \tag{7}$$

note that if $z_1 = v_k$ and $z_2 = v_j$, then, the power balance constraint can be approximated around the point $\left(v_k^0, v_j^0\right)$, as (8):

$$p_k^g + p_k^{gd} - P_k^d = \sum_{j \in \mathcal{N}} G_{kj} \left(v_k v_j^0 + v_k^0 v_j - v_k^0 v_j^0\right). \; \forall k \in \mathcal{N} \tag{8}$$

observe that (8) is now a linear affine constraint, i.e., a convex constraints that allows transforming the optimization model (1)-(6) into a quadratic convex model by replacing the nonlinear power equilibrium constraint (2) by its linear equivalent (8).

To obtain a recursive optimization model for the optimal power flow problem, let us add an iterative counter t to the power balance equilibrium restriction, which allows having a recursive optimization model with the following structure:

Obj. Func.

$$\min p_{\text{loss}} = \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} G_{kj} v_k v_j$$

Subject to:

$$\begin{aligned} p_k^g + p_k^{gd} - P_k^d = \sum_{j \in \mathcal{N}} G_{kj} \left(v_k v_j^t + v_k^t v_j - v_k^t v_j^t\right), \; \forall k \in \mathcal{N} \\ p_k^{g,\min} \leq p_k^g \leq p_k^{g,\max}, \; \forall k \in \mathcal{N} \\ p_k^{gd,\min} \leq p_k^{gd} \leq p_k^{gd,\max}, \; \forall k \in \mathcal{N} \\ v_k^{\min} \leq v_k \leq v_k^{\max}, \; \forall k \in \mathcal{N} \\ v_j = V_{\text{nom}}, \; j = \text{slack} \end{aligned} \tag{9}$$

The solution of the quadratic programming model (9) is solved recursively until the convergence criteria meets which is defined as the difference between both consecutive voltage iterations, i.e.,

$$\max\left\{\left\|v^{t+1}-v^{t}\right\|\right\}\leq\varepsilon, \tag{10}$$

where ε is the error of convergence, which is typically defined as $\varepsilon = 1\times 10^{-10}$ for power flow studies [13].

3.2. Recursive formulation based on the product relaxation (RPR)

The product relaxation is a new approximation proposed in this research for the power equilibrium equation in (2) that allows simplify its right-hand-side part as (11).

$$p_k^{g}+p_k^{gd}-P_k^{d}=\sum_{j\in\mathcal{N}}G_{kj}v_k^{0}v_j.\ \forall k\in\mathcal{N} \tag{11}$$

The main characteristic of this approximation is that the voltage in the current node of analysis, i.e., bus k, is relaxed to obtain a linear relation among the remainder voltages and the power generations and demands. With this relaxation, the following quadratic programming model is obtained.

Obj. Func.

$$\min p_{\text{loss}}=\sum_{j\in\mathcal{N}}\sum_{k\in\mathcal{N}}G_{kj}v_kv_j$$

Subject to:

$$\begin{aligned}
p_k^{g}+p_k^{gd}-P_k^{d}&=\sum_{j\in\mathcal{N}}G_{kj}v_k^{t}v_j.\ \forall k\in\mathcal{N}\\
p_k^{g,\min}&\leq p_k^{g}\leq p_k^{g,\max},\ \forall k\in\mathcal{N}\\
p_k^{gd,\min}&\leq p_k^{gd}\leq p_k^{gd,\max},\ \forall k\in\mathcal{N}\\
v_k^{\min}&\leq v_k\leq v_k^{\max},\ \forall k\in\mathcal{N}\\
v_j&=V_{\text{nom}},\ j=\text{slack}.
\end{aligned} \tag{12}$$

Note that the solution of the recursive model (12) is recursively implemented until the difference between the voltages in two consecutive iterations fulfills (10).

3.3. General implementation

The general implementation of the proposed recursive quadratic programming model for solving the optimal power flow problem in DC distribution networks is reported in algorithm 1. The main advantage of the general recursive approach reported in algorithm 1 is that each problem is a convex quadratic programming approach that ensures the global optimum finding; and the improvement of the v^t at each iteration reduces the estimation error between the nonlinear and the proposed approximation. This interactive improvement allows ensuring with low number of iterations, the same numerical solution of the nonlinear programming model (1)-(6).

Algorithm 1 Recursive implementation of the OPF using quadratic programming models

Data: Recursive implementation of the proposed quadratic programming models for solving the OPF problem in DC grids.
Define the DC network under study;
Define the iterative counter $t = 0$;
Define the initial voltages in per-unit $v_k^t = 1.0,\ \forall k = 1, 2, ..., n$
Select the convergence error $\varepsilon = 1\times 10^{-10}$;
Implement the optimization model (9) or (12) in the CVX tool for MATLAB;
for $k \leftarrow 1$ *to* N **do**
  Solve the optimization model using the SDPT3 solver;
  Evaluate the maximum voltage error $\rho=\max\left\{\left\|v^{t+1}-v^{t}\right\|\right\}$;
  if $\rho\leq\varepsilon$ **then**
    Report the final power losses p_{loss} and the final voltages v^t
    BREAK
  else
    Make $v^{t+1} = v^t$
Result: Return the optimal solution of the OPF problem.

4. TEST SYSTEM AND NUMERICAL VALIDATIONS

This section presents the main characteristics of the test feeder and the main numerical results. Note that these results are reached through the comparison between the proposed convex reformulations and the most common metaheuristic approaches reported in the current literature. The selected comparative metaheuristics are the following: MVO [19], PSO [17], BH [18], CGA [9], and ALO [13].

4.1. Test feeder

To validate the effectiveness and robustness of the proposed quadratic programming approximations in this work, it was used the DC 69 bus test system reported in [19]. This test systems presents 69 buses, 68 branches, an unique slack generator and multiple constant power loads connected in the different buses. The benchmark case presents a total power demand of 3889.25 kW and a total power losses equal to 153.84 kW. Furthermore, the maximum current allowed in all the branches is 335 A, which corresponds to the 400-kcmil conductor (this value was founded through the power flow evaluation); fixing as voltage limits for +/- 10% of the nominal voltage. Finally, the base values used were 100 kW and 12.66 kV.

4.2. Numerical results

In order to compare the proposed solution methods with other works reported in literature, we have been selected the following optimization methods: MVO, PSO, BH, CGA, and ALO. Observe that the selection of these as comparison methods was based on the excellent results reported by the authors of the works aforementioned were taken the location of the distributed generators into the 69 bus systems; as well as the maximum distributed generation penetration, that considering 60% of the power supplied by the slack bus in base case (the test systems in an environment without DGs). All simulations were executed 100 times for each solution methodology, with the aim to evaluate the repeatability of the solution obtained (standard deviation), and analyzing the minimum and average power losses, and required average processing times. Note that all simulation were carried out in a personal computer with processor Intel(R) Core(TM) i5-2410M CPU @ 2.30 GHz, 2301 Mhz, 6 Gb RAM and the Microsoft Windows 10 Home x64 operative system, by using the software MATLAB 2021b version. The implementation of the convex model is made in the CVX Software 2014 Version 2.2 with the Gurobi solver version 9.0.

Table 2 presents the simulation results obtained for each optimization method, by presenting from left to right: the optimization method, the power supplied by each distributed generator (DG), the power losses and the reduction obtained in percent with respect to the base case, the standard deviation obtained in percent (STD), the average processing times in seconds, the worst voltage profile and the bus in that this is presented, and finally it is shown the maximum branch current achieved by each solution. In this table, the first row presents the results obtained by the base case which considers the operation of the electrical grid without DGs dispatched, by presenting the voltages and branch currents bounds allowed for the 69 bus test system.

Table 2. Simulation results for the proposed and comparative methods to solve the OPF problem in DC networks

Method	DGs power kW	Power loss (kW) Reduction (%)	STD (%)	Avg. Time (s)	Worst voltage (pu)/bus	Imax (A)
Base case (without DGs)	Bus 26/Bus 61/Bus 66	153.8476/- - -	- - -	- - -	[0.9-.1]	335
RMA	375.11/1588.40/245.78	5.55579/96.39	0	4.833	0.995/12	133.13
RPR	373.72/1587.89/245.65	5.55579/96.39	0	10.115	0.995/12	133.13
MVO [19]	375.11/1588.50/245.73	5.55579/96.39	6.7e-06	120.066	0.995/12	133.49
PSO [17]	375.11/1588.47/245.74	5.55579/96.39	5.9e-07	13.680	0.995/12	133.13
BH [18]	401.27/1417.44/343.43	5.88404/96.18	21.543	13.739	0.995/12	136.89
CGA [9]	373.60/1589.01/245.74	5.55645/96.38	0.298	17.388	0.995/12	133.49
ALO [13]	380.38/1584.26/250.89	5.55774/96.38	6.332	9.686	0.995/12	132.64

Note that considering the numerical information in Table 2 it is possible to observe that the RMA and RPR methods achieved the best solution for the optimal power flow problem, as the optimization methods MVO and PSO. However, the exact solution methods proposed obtained a standard deviation equal to zero, which guarantee that every time the solution methods are executed going to be achieved the best solution (global optimization properties). Furthermore and not less important, due to the that the proposed methods belong to the convex optimization group, it is possible to affirm that this solution corresponds to the optimal global solution of the problem [27]. By analyzing the results of Table 2 was possible building the Figure 1, in which can be appreciated that the optimization method RMA is the best solution for solving the optimal power flow problem in DC grid, due to this achieved the best solution in terms of power loss, presented an STD of 0 and required the shorter processing time when is compared with the other optimization methods, with an average reduction of 57.14%. The RPR obtained the same results in terms of power loss and STD, but this solution method required more time for solving the problem. By occupying the third position in relation to the processing time, being surpassed by the RMA and ALO and presenting an average reduction in processing times of 22.19% with respect to the other optimization methods.

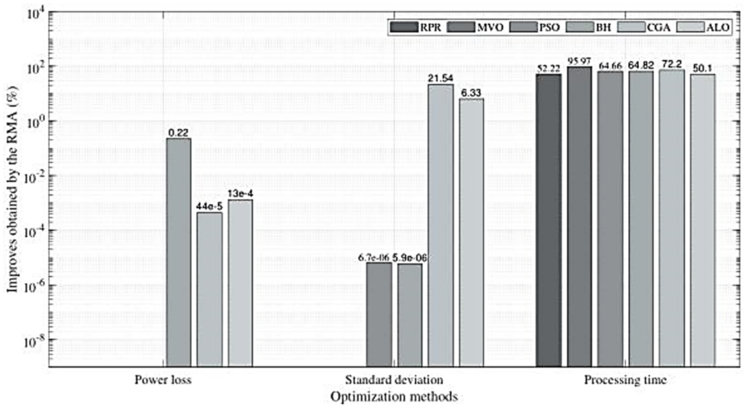

Figure 1. Improves obtained by the RMA in percent with respect to the other optimization methods used for 69 bus test system

5. CONCLUSION AND FUTURE WORK

In this work were proposed two quadratic programming approximations models for reformulated the optimal power flow problem in DC grids by considering the reduction of power loss as objective function and considering the set of constraints that represents this kind of electrical grids. This was possible by using a RMA and a RPR; being both mathematical formulation convex since the power balance constraint is approximated between affine equivalents. For solving the mathematical formulations proposed was used the CVX optimization tool, specifically its GUROBI solver. As test systems was used the 69 bus DC system, by using all considerations that represents the DC grids under an environmental of distributed generation. For evaluating the effectiveness and robustness of the proposed solutions were used five optimization methods, in addition were carried out 100 execution of the solution methods with the aim to evaluate the standard deviation associated to the solution obtained and the average processing times required.

The results obtained demonstrating that the RMA formulation solved by using the CVX tool obtained the best results in terms of quality solution (reduction of power loss), standard deviation and average processing time required. This results showed the effectiveness of the quadratic formulation based on the McCormick approximation, and as this converges to the global optimal solution of the problem studied, each time that the solution method is executed. With respect the RPR quadratic formulation proposed, this achieved the optimal global each time that it is executed, however this solution method required more time that the RMA due to the simple mathematical approximation used. This demonstrating that to use simple mathematical formulation to reformulate the problem produced an increment in the processing times. Finally, when we consider the results obtained in Table 2, then, it is important important to highlight the excellent results found by the MVO and PSO metaheuristics in terms of quality solution and standard deviation, due to the fact that using random variables inside the iterative process, their standard deviation values are low. As future works for this work, can be considered the following: (i) to use the proposed methodologies for optimal location and sizing of distributed generation in DC grids, by including economical and environmental index inside the objective functions; (ii) to solve the studied problem additional intelligent based optimization methods such as monarch butterfly optimization (MBO), earthworm optimization algorithm (EWA), elephant herding optimization (EHO), moth search (MS) algorithm, slime mold algorithm (SMA), and Harris hawks optimization (HHO), could be implemented in future researches; and (iii) to extend the application of the recursive convex optimal power flow approximations studied in this research to bipolar DC networks.

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