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ORIGINAL PAPER

A quadratic convex approximation for optimal operation of battery energy storage systems in DC distribution networks

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Abstract
This paper proposes a quadratic convex model for optimal operation of battery energy storage systems in a direct current (DC) network that approximates the original nonlinear non-convex one. The proposed quadratic convex model uses Taylor's series expansion to transform the product between voltage variables in the power balance equations into a linear combination of them. Numerical simulations in the general algebraic modeling system (GAMS) for both models show small differences in the daily energy losses, which are lower than 3.00%. The main advantage of the proposed quadratic model is that its optimal solution is achievable with interior point methods guaranteeing its uniqueness (convexity properties of the solution space and objective function), which is not possible to guarantee with the exact nonlinear non-convex model. The 30-bus DC test feeder with four distributed generators (with power generation forecast via artificial neural networks with errors lower than 1% between real and predicted generation curves) and three batteries is used to validate the proposed convex and exact models. Numerical results obtained by GAMS show the effectiveness of the proposed quadratic convex model for different simulation scenarios tested, which was confirmed by the CVX tool for convex optimization in MATLAB.

Keywords Battery energy storage systems · Quadratic convex approximation · Economic dispatch · Taylor's series expansion · Direct current distribution networks · Artificial neural networks

List of symbols

Δt	Length of the period of time where the loads are constants (h)
$\mathcal{N}$	Set that contains all the nodes of the network
$\mathcal{T}$	Set that contains all the nodes of the periods of time

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CoE_t	Cost of the energy in period t (COP\$/Wh)
G_ij	Conductance value that associates nodes i and j (Ω^{-1})
$p_i^{\max}, t$	Maximum bound of the power generated in the conventional source connected at node i in the period t (W)
$p_i^{\min}, t$	Minimum bound of the power generated in the conventional source connected at node i in the period t (W)
$p_i^{\mathrm{b,max}}$	Maximum discharge capability of a battery connected at node i (W)
$p_i^{\mathrm{b,min}}$	Minimum charge capability of a battery connected at node i (W)
p_i^{b}, t	Power injected/absorbed in the battery connected at node i in the period t (W)
p_i^{dg}, t	Power generated in the distributed generator connected at node i in the period t (W)
p_i^{d}, t	Power demanded in node i in the period t (W)
$p_{i,t}$	Power generated in the conventional source connected at node i in the period t (W)
$SoC_i^{\mathrm{b,max}}$	Maximum bound of the state of charge of the battery in the ith node (pu)
$SoC_i^{\mathrm{b,min}}$	Minimum bound of the state of charge of the battery in the ith node (pu)
$SoC_i^{\mathrm{b,fin}}$	Final state of charge of the battery in the i^th node (pu)
$SoC_i^{\mathrm{b,ini}}$	Initial state of charge of the battery in the ith node (pu)
SoC_i^{b}, t	state of charge of the battery in the ith node at the t th time period (pu)
$v_i^{\max}$	Maximum voltage regulation bound at node i (V)
$v_i^{\min}$	Minimum voltage regulation bound at node i (V)
v_i, t	Voltage value at node i at the period of time t (V)
v_i, t	Voltage value at node j at the period of time t (V)
z	Value of the objective function regarding the costs of the energy losses (COP\$/day)
BESS	Battery energy storage system
GAMS	General algebraic modeling system
NLP	Nonlinear programming
$p_i^{\mathrm{dg,max}}, t$	Maximum bound of the power generated in the distributed generator connected at node i in the period t (W)
$p_i^{\mathrm{dg,max}}, t$	Minimum bound of the power generated in the distributed generator connected at node i in the period t (W)

1 Introduction

Electrical networks have been submitted to important changes during the last decade in their conceptions and paradigms due to the high challenges imposed by greenhouse effects [1, 2]. It is widely-known that electrical networks are the third high emitter of pollutants to the atmosphere, after transportation systems and cattle farms [3, 4], due to the usage of fossil fuels (i.e., coal, diesel, or natural gas) to produce energy in thermal plants [5, 6]. The signature of the Paris agreement has obligated the majority of the countries worldwide to propose policies that make a sustainable environment that makes possible the survival of the new generations [7, 8]. In this context, one of the main challenges is to transform the classical electrical networks with predominant fossil fuels into clean, renewable, and sustainable energy systems [9]. To deal with this objective, manufacturers of renewable energy systems, after years of continuous developments and researches, have reached competitive prices in the photovoltaic generation, wind power and geothermic generation that make it possible to substitute fossil generation with competitive prices in the electricity spot market [10, 11]. An additional fact, in this paradigm shift, is the importance of energy storage systems roll-out based on batteries [12–14], supercapacitors [15, 16], superconductors [17, 18], or flywheels [19, 20], among others; since these energy storage technologies compensate the intermittency of renewables when dispatching power sources based on clean and sustainable energies [21].

Additionally, due to the transformation of the energetic matrix and the emerging of distributed energy resources in distribution networks [22], these grids have passed from classical upright networks with active sources and passive users to active networks with dynamically interactions between energy producers and end-users; which have originated the concepts of active grids and microgrids [23]. The concept of microgrid comes with the opportunity of having electrical networks with the possibility of grid-connected or isolated operation [24]. This implies that microgrids are autonomous electrical networks capable of independent grid operation, suitable for rural or urban applications [25]. An important characteristic of microgrids is that these can be operated in ac [26], dc [21, 27] or hybrid configurations [28]. Notwithstanding, due to recent advancements, power electronic interfaces have promoted the usage of dc grids since these are more efficient and less complex than their ac counterparts.

In specialized literature, dc networks are being widely explored from static and dynamic points of view. The static analysis is related to power and optimal power flow analysis that are fundamental tasks in planning and operation environments [29, 30]. In the case of dynamical analysis, the efforts are approached to design efficient controllers for operating power electronic converters [31, 32]. Here, the main interest is to analyze dc networks from the tertiary control approaches, which corresponds to the optimization stage in hierarchical control models [33]. In [34], a distributed economic dispatch strategy for multiple energy storage in a microgrid was presented. The strategy satisfied all coupled challenges with the dispatch in a centralized dispatching formulation such as decision variables and stochastic variables. In addition, the strategy could also be used in the microgrid central controller. In [35], the impact on the economic dispatch of the integration of two types of battery technologies (lithium and lead-acid) in DC microgrids was studied. In addition, the economic dispatch modeled the operation of a DC microgrid for 24 h considering variable price schemes, distributed generation, and energy storage resources, and finally, the authors of [35] concluded that lithium batteries have a better technical-economical performance than lead-acid batteries. In [36], an economic dispatch for BESS operation in DC grids based on a master-slave methodology was proposed. This methodology was formulated using a parallel implementation of the particle swarm optimization algorithm and an hourly power flow method based on successive approximations. The results shown in [36] reached a performance in terms of solution quality better than other methodologies based on metaheuristic optimization methods. In [9], an optimal dispatch of BESS and renewables in dc grids considering voltage-dependent load models was developed. This dispatch solved the resulting optimization model using the general algebraic modeling system (GAMS) via large-scale nonlinear optimizers. Although these methodologies have good performance, none of them can guarantee the optimum global since they used an exact nonlinear non-convex model. In order to achieve the optimal global of the problem, this research proposes a quadratic convex optimization model for optimal coordination of batteries in dc distributed networks considering high penetration of renewables; the proposed model allows guaranteeing the existence and uniqueness in the solution, with minimum estimation errors compared to the nonlinear non-convex model [9]. It is important to highlight that the current literature has proposed one model for optimal dispatch of BESS and renewables in dc grids, which ensures the optimum global based on a semidefinite programming (SDP) model. This approach has been reported in [21], which used an SDP formulation to deal with the non-convexities of the original optimization problem; the obtained results are satisfactory, and the computational burden is acceptable for medium size distribution networks. The main lack of this approach is that the SDP approximation requires the n^2 square number of variables to solve the equivalent convex problem, which compromises the computational performance of this methodology, mainly in an electrical network with a high number of nodes [37].

The main differences between the convex quadratic model proposed in this research and the previous ones are that:

- The proposed model does not increase the number of variables since it continues working in the same domain of the voltage variables.
- Special assumptions are not required for positive definiteness or rank conditions in the matrices of variables as the case of the SDP.
- The solution of the resulting quadratic optimization model can be made in any optimization package such as **quadprog** and CVX for MATLAB or QP solvers in GAMS guaranteeing global minimum and uniqueness in the solution [29].

Note that the main contribution of our proposal regarding existing researches corresponds to the reformulation of the problem of the optimal operation of batteries in DC distribution networks through an approximate quadratic programming model with linear constraints by using Taylor's series approximation. The main advantage of this formulation lies in the size of the solution space since this remains equal to the exact NLP model since no new variables are introduced, which are not the cases of the SDP or conic programming models.

The remainder of this document is organized as follows: Sect. 2 presents the exact nonlinear formulation of the economic dispatch problem for BESS and renewable generators in dc networks considering constant power loads. In Sect. 3 is presented the convex reformulation of the power flow equations based on the product linearization of equations with multiple variables as reported in [29]. Section 4 presents the main aspects of renewable energy forecasting via artificial neural networks. Section 5 presents the main characteristics of the test feeder, while Sect. 6 presents all the simulation results, their analysis, and discussion. In Sect. 7 is presented the main concluding remarks derived from this work and some possible future works.

2 Nonlinear non-convex optimization model

The problem of optimal operation of battery energy storage systems in dc distribution networks corresponds to a nonlinear non-convex optimization problem due to the power balance constraint that generates a set of non-affine quadratic equalities [9]. The formulation of this problem generates a single-objective minimization problem where the total energy cost in the spot market is the objective to be minimized [14]. The complete nonlinear non-convex optimization model is presented as follows.

Objective function

$$\min z = \sum_{t\in\mathcal{T}} CoE_t\left(\sum_{i\in\mathcal{N}} v_{i,t}\left(\sum_{j\in\mathcal{N}} G_{ij}v_{j,t}\right)\right)\Delta t \tag{1}$$

where z represents the objective function value, CoE_t is the cost of the energy in period t, $v_{i,t}$ is the voltage value at node i at the period of time t, G_{ij} is the conductive value that relates nodes i and j which is obtained from the conductance matrix, and Δt is the length of the time period under analysis (e.g., 1 h or 15 min). $\mathcal{T}$ and $\mathcal{N}$ are the sets that contain all periods of time considered and the total number of nodes in the DC microgrid, respectively.

Set of constraints

$$p_{i,t} + p^{dg}_{i,t} + p^{b}_{i,t} - p^{d}_{i,t} = v_{i,t}\sum_{j\in\mathcal{N}} G_{ij}v_{j,t}, \quad \{\forall i\in\mathcal{N} \,\&\, \forall t\in\mathcal{T}\} \tag{2}$$

$$SoC^{b}_{i,t} = SoC^{b}_{i,t-1} - \varphi^{b}_{i}p^{b}_{i,t}\Delta t, \quad \{\forall i\in\mathcal{N} \,\&\, \forall t\in\mathcal{T}\} \tag{3}$$

$$SoC^{b}_{i,t_0} = SoC^{b,ini}_{i}, \quad \{\forall i\in\mathcal{N}\} \tag{4}$$

$$SoC^{b}_{i,t_f} = SoC^{b,fin}_{i}, \quad \{\forall i\in\mathcal{N}\} \tag{5}$$

$$p^{dg,\min}_{i,t} \le p^{dg}_{i,t} \le p^{dg,\max}_{i,t}, \quad \{\forall i\in\mathcal{N} \,\&\, \forall t\in\mathcal{T}\} \tag{6}$$

$$p^{b,\min}_{i} \le p^{b}_{i,t} \le p^{b,\max}_{i}, \quad \{\forall i\in\mathcal{N} \,\&\, \forall t\in\mathcal{T}\} \tag{7}$$

$$v^{\min}_{i} \le v_{i,t} \le v^{\max}_{i}, \quad \{\forall i\in\mathcal{N} \,\&\, \forall t\in\mathcal{T}\} \tag{8}$$

$$SoC^{b,\min}_{i} \le SoC^{b}_{i,t} \le SoC^{b,\max}_{i}, \quad \{\forall i\in\mathcal{N} \,\&\, \forall t\in\mathcal{T}\} \tag{9}$$

where $p_{i,t}$, $p^{dg}_{i,t}$, $p^{b}_{i,t}$, and $p^{d}_{i,t}$ are the power generation by conventional generators, renewable energy resources (i.e., distributed generation), the power delivered/

absorbed by the batteries, and the power demand at node i during the time period t, respectively. $SoC_{i,t}^{b}$ represents the state of charge of the battery in the ith node at the tth time period. $SoC_{i}^{b,ini}$ and $SoC_{i}^{b,fin}$ are the initial and final desired states of charge of the batteries, while $SoC_{i}^{b,\min}$ and $SoC_{i}^{b,\max}$ are the minimum and maximum state-of-charge bounds. $p_{i,t}^{\min}$, $p_{i,t}^{\max}$, $p_{i,t}^{dg,\min}$, and $p_{i,t}^{dg,\max}$ are the minimum and maximum bounds of admissible generation for conventional and renewable generators located in the ith node in time period t, while $p_{i}^{b,\min}$ and $p_{i}^{b,\max}$ represent the minimum and maximum charge/discharge capabilities of a battery connected at node i. $v_{i}^{\min}$ and $v_{i}^{\max}$ are the voltage regulation bounds of the DC microgrid. Finally, φ_{i}^{b} represents the coefficient of charge of a battery connected at node i.

Remark 1 Note that the power generation in the conventional and distributed generators, voltages, state of charge of the battery are contained in the set of the real positive numbers. However, the power injected/absorbed by the battery energy storage system is positive when the battery works as a generator (discharging stage) and negative when the battery is working as load (charging stage). This implies that $p_{i}^{b,\max}$ and $p_{i}^{b,\min}$ positive and negative parameters respectively.

The complete interpretation of the mathematical model defined in Eqs. (1)–(9) is the following: the objective function of the optimization model is defined by Eq. (1), and it represents the minimization of the energy losses costs of the network. This objective function is selected since this becomes the technical aspect of power losses into an economic aspect that directly affects the utility operator and its efficiency indicators. Equation (2) is known in the specialized literature as the power balance equilibrium at each node of the network. This expression is the most complicating constraint in the mathematical model for the optimal coordination of batteries in DC grids since this is nonlinear non-convex due to the product among nodal voltages. Equation (3) presents the linear relation between the power injected or absorbed by the batteries and their state of charges. This linear relation is widely accepted in literature to model the relationship between the energy stored in the battery and its rate of change [21, 38]. In constraints (4) and (5) are defined the initial (input reference) and final values (desired final states) of the batteries in order to ensure the correct operation in the next day by storing energy at the end of the current day. Box-type constraints (6) and (7) determine the lower and upper bounds of the distributed generation and battery power injection/absorption, respectively. Regulatory policies regarding voltage regulation bounds are defined by inequality constraint (8). The upper and lower voltage bounds of DC grids are typically assumed between $\pm 10\%$ for medium-voltage level and $\pm 5\%$ for low-voltage level. Inequality constraint (9) determines the minimum and maximum bounds of the state of charge variable for batteries; these bounds are recommended between 10% and 90% for lithium-ion batteries [38].

Observe that the objective function (1) is nonlinear; nevertheless, it is convex since it represents a quadratic expression as a function of the voltage variables. Taking into account that the components of the conductance matrix are symmetric and positive definite, the expression becomes into a convex function [39]. In addition, note that model (1)–(9) is nonlinear and non-convex due to the power balance constraints defined in (2), since they are non-affine quadratic function due to the product between voltages [29]. The main contribution of this work results of proposing a quadratic equivalent convex formulation as an alternative to the nonlinear non-convex one by using Taylor's series expansion method over the set of power balance constraints (2) by neglecting the high-order terms due to its small contribution to the model [40]. This reformulation is described in the next section.

3 Quadratic convex reformulation

The reformulation of the nonlinear non-convex model (1)–(9) into a convex one is possible under the following assumption:

Assumption 1 All the voltage profiles of the network are around to the unity when per unit representation is made considering the voltage rate of the network as the voltage base [41]. This implies that the linearization point of the network is assumed as 1.0 p.u [29].

Based on this assumption, let us define the following nonlinear continuous and soft function to be linearized

$$f(x,y) = xy, \tag{10}$$

being x and y two continuous variables.

Now, if we applied Taylor's series expansion for multi-variable equations as recommended in [29] considering the linearization point as $\left(x_0, y_0\right) = (1,1)$, and neglecting the high-order terms, then, the following expression is achieved

$$f(x,y) \approx y_0 x + x_0 y - x_0 y_0 = x + y - 1. \tag{11}$$

Figure 1 presents the percentage error between the exact function (see (10)) and the linearized one (see (11)) around (x_0, y_0), which confirms that under Assumption 1 the proposed linearization works well with an error lower than 1% [29].

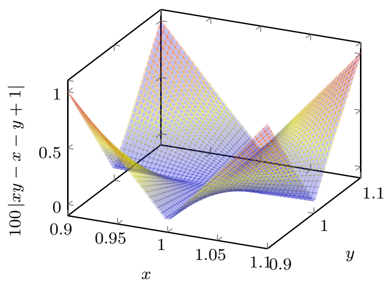

Fig. 1 Error behavior between the nonlinear and equivalent convex approximation of the product of two real and continuous variables around $(x_0, y_0) = (1, 1)$

On the other hand, if Expression (11) is substituted into (2), considering that $v_{i,t} = x$ and $v_{j,t} = y$, then the power balance equation becomes into a convex affine set of hyperplanes as follows

$$p_{i,t} + p_{i,t}^{dg} + p_{i,t}^{b} - p_{i,t}^{d} = \sum_{j \in \mathcal{N}} G_{ij}\left(v_{i,t} + v_{j,t} - 1\right). \tag{12}$$

Note that when power balance equation (2) is replaced by (12) a quadratic convex model for economic dispatch analysis is achieved, and it has the following structure

$$\begin{aligned} &\text{Minimize } z: \ \text{Equation } (1), \\ &\text{Subject to:} \\ &\qquad p_{i,t} + p_{i,t}^{dg} + p_{i,t}^{b} - p_{i,t}^{d} = \sum_{j \in \mathcal{N}} G_{ij}\left(v_{i,t} + v_{j,t} - 1\right), \\ &\qquad \text{Equations } (3)\text{–}(9) \end{aligned} \tag{13}$$

The solution of the equivalent convex model defined by (13) can be achievable with any optimization package that solves quadratic programming problems, here we select the GAMS optimization package and the CONOPT solver to reach the solution of the exact nonlinear and convex models [9].

4 Artificial neural networks for renewable energy generation forecasting

An important aspect for addressing economic dispatch problems in power systems with high penetration of renewable energy resources corresponds to power forecasting since renewable sources are sensitive to weather changes, especially when they are focused on wind, and photovoltaic plants [25]. Here we employ an artificial neural network (ANN) to predict power outputs in the wind and photovoltaic sources with the same structure reported in [38]. We have the ANN approach to predict the photovoltaic and wind generation output since this is a simple and efficient methodology largely used in literature to make predictions. Some of the successful applications of the ANN networks are: gross domestic product forecasting [42, 43]; model predictive control design for greenhouse ventilation systems [44]; weather forecasting [45]; greenhouse gas emissions prediction [46]; foreign-exchange-rate forecasting [47]; and renewable generation forecasting [38]. Here, we adopt the structure recommended in [21] for photovoltaic generation prediction using ANN. The structure of this ANN is depicted in Fig. 2.

The inputs and outputs selected of the ANN selected for renewable generation forecasting are listed in Table 1. Training and validation processes for the ANN are carried out in MATLAB software employing *ntstool*. The following items define the main steps of the ANN method implemented:

- ✓ It uses two inputs are used (see Table 1) to predict solar radiation where six delays and 18 hidden neurons are considered. While four inputs (see Table 1)

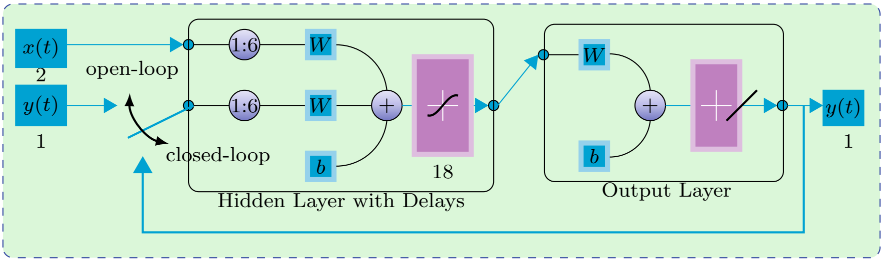

Fig. 2 ANN implemented for forecasting power output in PV sources

Table 1 Input and outputs parameters for ANN

Wind power					
Inputs					Output
Temperature	Humidity	Pressure	Time		Wind speed
PV power					
	Inputs			Output	
	Temperature	Time		Solar radiation	

are used to forecast wind speed where 12 hidden neurons and four delays are considered. The hidden neurons use sigmoid functions, while the output layer employs a linear transfer function.

- ✓ It receives feedback of the estimated output, i.e., y, for considering the recent behavior of the weather in the current prediction.
- ✓ For training, the ANN is used one-year information divided by periods of 1 h (8760 h/year), taking 70% for training and 30% for validation. The algorithm used during the training is the Levenberg-Marquardt available for the *nnstart* toolbox in MATLAB 2017*a*.

Figure 3 Depicts the information employed for renewable energy generation.

The precision of the ANN network for prediction of wind power and photovoltaic generation was tested using a year of information (i.e., 8760 h), where 2628 h were used for validation. When one of these days is selected for validation, the cumulative error of the ANN to predict the total power output is lower than 3% in all the simulation cases, which confirms its efficiency for this research. All data can be found in [48].

5 Test system

The computational validation of the proposed quadratic convex approximation to solve the problem of optimal operation of batteries in medium-voltage distribution grids is made in a distribution grid with 30 nodes operated with a voltage of 13.8 kV

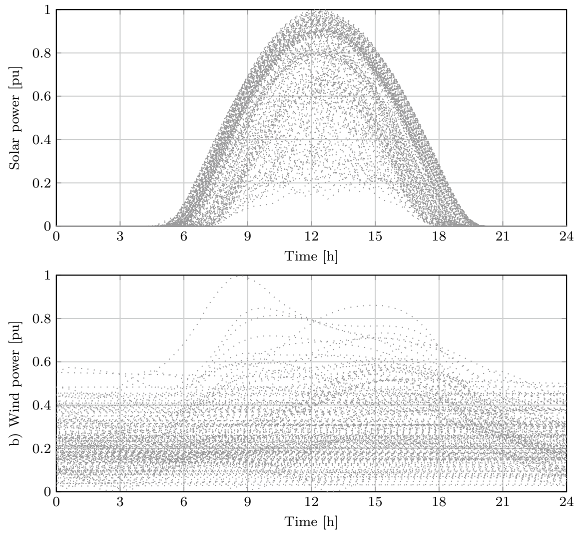

Fig. 3 Historic data for the ANN training process [9]

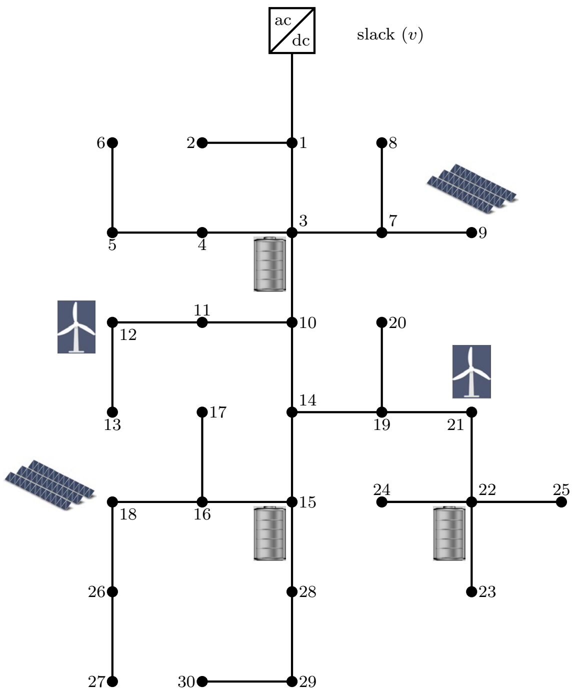

Fig. 4 Schematic connection among nodes in the 30-bus test system

at the substation node [9]. The electrical configuration of this test feeder is depicted in Fig. 4. The total load of this test feeder is 5.85 MW, which corresponds to a typical demand value for an electric distribution network in Colombia. The parametric information of the conductors of this test feeder is reported in Table 2.

Table 2 Type of calibers in the 30-bus test system

Type	Caliber AWG/kcmil	Resistance (Ω/km)	Max. current (A)
1	4	1.360	138
2	2	0.854	185
3	266.8	0.213	443

The electrical parameters of the conductors of the 30-bus system are reported in Table 3, where the peak load consumption of the receiving nodes is reported in the last column.

Table 3 Electrical information of the branches and demand nodes

Node i	Node j	Type of conductor	Length (km)	p_j^{d0} (kW)
1	2	1	1.75	100
1	3	3	1.25	0
3	4	1	0.75	500
4	5	1	0.25	350
5	6	1	0.40	150
3	7	1	0.50	0
7	8	1	0.45	400
7	9	1	0.80	300
3	10	3	1.85	0
10	11	1	0.75	400
11	12	1	1.00	175
12	13	1	0.40	225
10	14	3	0.85	0
14	15	3	1.70	0
15	16	1	0.52	0
16	17	1	0.15	200
16	18	1	0.42	150
14	19	2	0.28	0
19	20	2	0.35	250
19	21	2	0.45	150
21	22	2	0.75	0
22	23	2	0.26	600
22	24	2	0.34	500
22	25	2	0.17	300
18	26	2	0.85	450
26	27	1	0.42	200
15	28	1	1.40	100
28	29	1	0.75	150
23	30	1	0.82	200

Table 4 Energy storage capabilities and locations

Location	Energy (kWh)	Charge/discharge time (h)	Power inj./abs. (kW)
3	1500	3	500
15	2000	5	400
22	1200	4	300

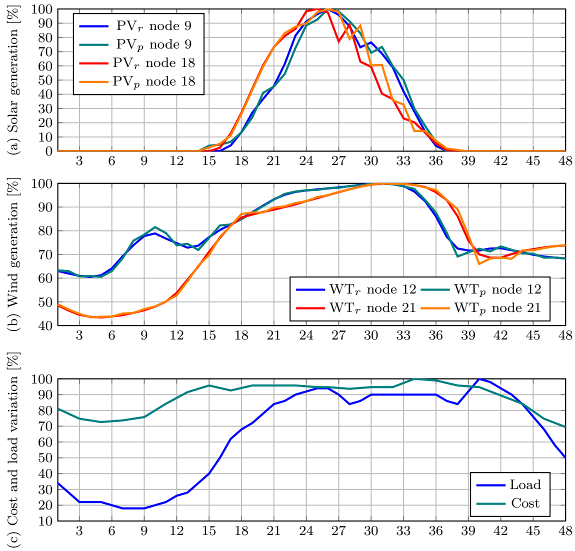

Fig. 5 Daily behavior of renewable generation, demand, and costs for the 30-bus test system: **a** real and projected photovoltaic generation; **b** real and projected wind generation; and **c** load and costs curves

The batteries considered in this study correspond to Lithium-Ion batteries, with which the current advances in energy storage technologies can operate continuously between 3 to 15 years, and the extensions of the lifespan in this period depends on the optimal operation methodology [9]. Here, we assume that the utility company has previously installed the batteries, and we are proposing a convex optimization model to optimize the total grid operative costs using a quadratic convex model. The information regarding the batteries considered in this research is presented in Table 4, where batteries with nominal energy storage capabilities of 1200, 1500, and 2000 kWh are considered.

To consult additional information regarding battery energy storage systems, such as installation costs, recommended maximum and minimum state of charges, refer to the following reports [49, 50].

Regarding the renewable generation information, Fig. 5 presents the real wind and generation curves which were obtained using online data for wind speed, pressure, solar radiation, and time for periods of 0.5 h. This information can be consulted in [51]. In addition, Fig. 5 presents the projected outputs with the proposed ANN methodology presented in Sect. 4.

To compute the daily operative costs of the network, we consider that the cost of the energy is US$/kWh 0.1390 as reported in [38]. In addition, the following simulation scenarios are taken into account:

S1 : A comparative study without batteries is made to demonstrate the effectiveness of the ANN to forecast renewable generation with exact model and the convex proposal.

S2 : The batteries are set with initial and final state of charges (see Eqs. (4) and (5)) of 50%; in addition, along the day, the state of charge can vary from 10 to 90%.

S3 : Reductions in the renewable generation from 0 to 100% are considered to model the impact of the weather conditions in the grid operation.

6 Simulation results

All simulations were carried out in a desk-computer INTEL(R) Core(TM) i7-7700, 3.60 GHz, 8 GB RAM with 64 bits Windows 10 Pro using GAMS 25.1.3 with the nonlinear large-scale solver CONOPT for the exact nonlinear non-convex model and the quadratic convex one as recommended in [9, 25].

Note that GAMS is a specialized package widely used in mathematical optimization for multiple problems as follows: optimal operation of batteries in ac and dc networks with nonlinear models [9, 52]; economic dispatch models in thermal power systems [53]; optimal location and sizing of distributed generators in ac and dc networks [25, 54]; multi-objective optimization of the stack of a thermoacoustic engine [55]; optimization of pump and valve schedules in complex large-scale water distribution systems [56]; and general nonlinear optimization problems [57], among others.

The general implementation of an optimization model in the GAMS software is depicted in the flow diagram presented in Fig. 6.

6.1 Scenario 1

This simulation case evaluates the capability of the ANN to predict the renewable energy production of the day-ahead considering data historic of the wind and power sources. In addition, this simulation scenario shows the effectiveness of the proposed quadratic convex model to approximate the solution of the exact nonlinear programming (NLP) model. Table 5 presents the comparative performance of the exact and quadratic convex model when real and predicted curves are analyzed.

Results in Table 5 show that: (1) the difference in the exact model for the real and estimated generation curves is US$/day 0.8119, and for the proposed quadratic convex model is about US$/day 0.7837; these values imply that the ANN allows reaching estimation errors lower than 1% in the daily operative costs when real and projected renewable generation curves are compared; which clearly confirms its efficiency and robustness for renewable generation forecasting and its applicability in

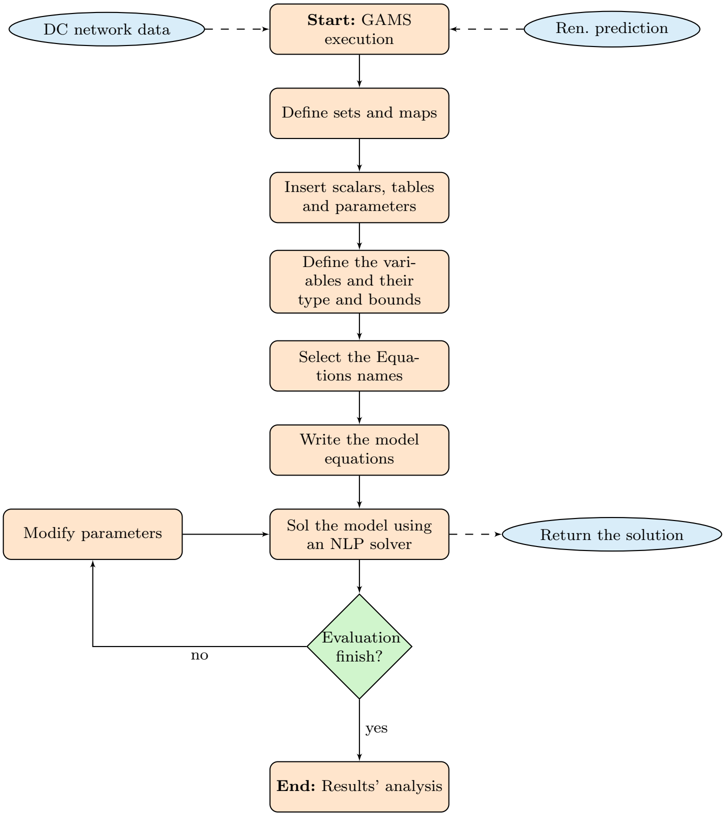

Fig. 6 Procedure to implement an optimization model in GAMS [38]

Table 5 Comparative results between the real and predicted curves using the exact and convex models

Model	Real (US$/day)	Estimated (US$/day)
Exact NLP	82.0880	81.2761
Quadratic convex	79.7523	78.9686

the optimal operation of batteries in distribution networks; and (2) the difference between the exact and convex model for both curves are US$/day 2.3357 and US$/day 2.3075; which imply that the estimation errors between the exact and convex proposal is lower than 3% in both simulation cases. This result confirms that the proposed quadratic convex model is accurate to solve the optimization associated with the optimal operation of batteries in DC grids.

6.2 Scenario 2

This simulation tests the effectiveness of the proposed quadratic model to operate batteries in DC networks. Here, we consider the predicted curves for the renewable generation since the real curves will be only known when the day of analysis ends. When the exact model is solved considering batteries, the cost of the energy losses is US$/day 73.9415, while the solution of the convex approximation is US$/day 72.0683. These results show that the difference between models is less than 2.6% confirming the effectiveness of the proposed approximation with the main advantage that the solution reached is global due to the convex properties of the solution space.

On the other hand, Fig. 7 presents the behavior of the state of charge variable at each node when the exact and convex models are used to solve the problem of the optimal operation of batteries in DC networks.

Results in the state of charge variables of the batteries in all the nodes for the exact and convex model follow the same behavior (see Fig. 7). This implies from the operating point of view; the proposed model will provide pretty similar charging/discharging profiles compared to the exact model, with the main advantage of ensuring the global optimum finding of the problem (i.e., convexity properties in optimization). However, the most important fact of this result is that the proposed quadratic convex model can define the charging/discharging characteristics of the batteries in a real application, considering that this solution will provide essential grid profits in terms of reductions in the total grid energy losses costs. Note that the reductions regarding the base case are 9.02% for the exact optimization model and 9.01% when the solution provided by the convex model is set in the exact nonlinear, i.e., the error between both solutions is lower than 0.12%.

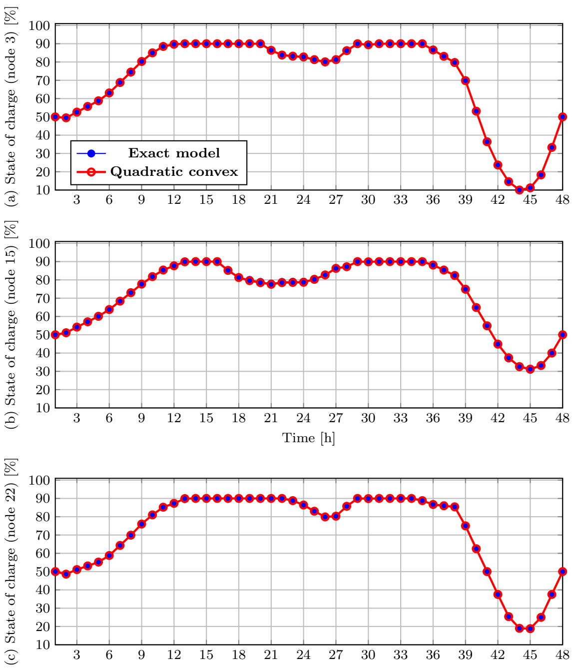

Fig. 7 Behavior of the state of charge in the batteries: **a** node 3, **b** node 15, and **c** node 22

6.3 Scenario 3

In this scenario, the effect of renewable energy changes in the expected objective function value is tested. Here, we only present the results of the exact model by replacing the profile of the state of charge in batteries provided by the proposed convex approach.

Results in Fig. 8 observe that: (1) the variation of renewable generation availability produces important variation in the expected costs of the energy losses. For example, if we compare the case with 60% of availability without batteries, the daily energy losses cost difference is about US$/day 24.0706. In addition, when batteries are added, this difference is about US$/day 23.5556; and (2) the usage of batteries in this test feeder allows reducing US$/day 10 for all the percentages of renewable energy availability; even if it is a small value, it helps with the improvement of the grid performance and the quality indicators for the utility company.

6.4 Additional comments

Based on the numerical results reported in the above scenarios, the following facts can be highlighted:

- ✓ The comparison of the daily energy cost using real and forecasted curves of the renewable generation demonstrates the efficiency of the ANN to provide accurate renewable generation outputs in day-ahead operation environments since the difference among these curves was less than 1% for the exact NLP model and the proposed convex formulation.
- ✓ The simulation times reported by the GAMS optimization package and the CONOPT solver is about 900 ms for the exact model and 400 ms for the convex proposal; even if these times are very short for a day-ahead operation scenario, this difference can be significant when the size of the DC grid and/or the number of batteries and renewable generator increase. However, the main

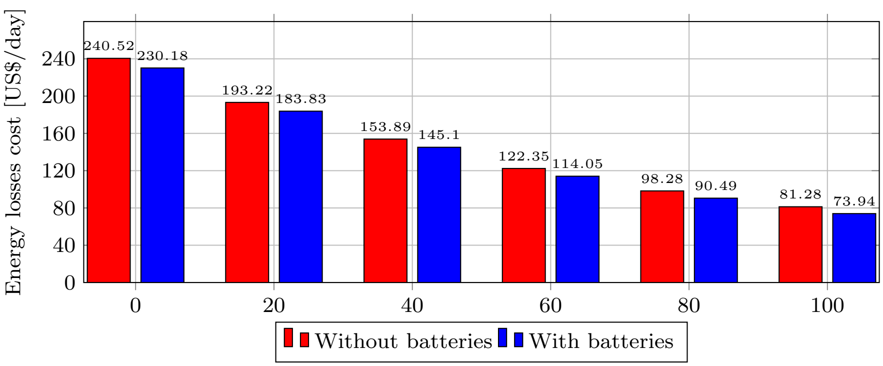

Fig. 8 Behavior of the daily energy losses costs with the variation in the availability of the renewable generation

characteristic of the proposed convex model corresponds to ensure the global optimum finding with any optimization package due to the affine characteristics of the Taylor-based approximation of the product among voltage variables in the power balance constraint; which is non-insurable for the exact NLP model.

✓ The most important variable in the optimal operation of distribution networks corresponds to the availability of renewable generation since the behavior of the battery as a generator or demand, becomes a notable benefit of the operation of the network. This is explained from the perspective of enough distributed generator availability to store in the batteries, to be able to return to the grid when the renewable generation is reduced, i.e., at night hours in the case of solar generation.

✓ In order to corroborate that the proposed convex model ensures the global optimum finding, numerical simulations in the CVX tool, i.e., convex optimization package, with the MOSEK and SeDuMi solvers demonstrate that the solution provided by GAMS and the CVX solvers is identical for the proposed quadratic convex approximations.

7 Conclusions and future works

This paper addressed the economic dispatch problem in DC networks with high penetration of renewable generation and battery energy storage systems. Using Taylor's series expansion on the power balance equations, a quadratic programming model was proposed to transform the exact nonlinear non-convex model into a convex one. Numerical results confirmed that the states of charges in the batteries are identical for both models and the objective function differed 3% as maximum due to the estimation error introduced by the linearization.

The main advantage of the proposed quadratic programming model is the warranty of finding the global optimum by using the interior point methods, which is not possible for the exact nonlinear formulation due to its intrinsic non-convexities.

The usage of artificial neural networks for renewable generation forecasting demonstrated its efficiency since the maximum error in the objective function calculation was less than 1% for the exact and convex optimization models.

As future work, it will be possible to develop the following researches: (1) transform the quadratic programming model into a mixed-integer quadratic programming model to solve the problem of the optimal location (reallocation) and operation of energy storage systems in DC networks by ensuring the global optimum finding with Branch and Bound methods; (2) compare different methodologies to transform the product between voltage variables with the proposed approach, i.e., McCormick envelopes or the difference between quadratic terms to determine an adequate formulation to be applied to optimal power flow studies; and (3) propose hybrid algorithms to locate and size renewable generators and batteries in DC distribution grids using the proposed quadratic optimization model to evaluate the continuous part of the optimization problem.

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