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ORIGINAL PAPER

Production System in a Collaborative Supply Chain Considering Deterioration

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Abstract

This research presents a mathematical model for a collaborative planning of the supply chain involving four echelons (supplier, production plants, distribution, retails, or clients). The model seeks to maximize profit (utility) when all members of the chain share information related to demand. It is developed for the aggregate consolidation of different raw materials in cement production. The novelty of the model is the consideration of products that deteriorate in the process and thus it has effect on the production times in the plant and lead time. In this supply chain, quality and compliant products and the return of deteriorated products are two flows. The considerations are lead time, inventories with shortages and excesses, production times in normal and extra days, and subcontracting, among others. A mixed integer linear programming with demand scenario analysis is used to optimize and analyze the uncertainty that is consistent with the performance of the construction sector. The model is formed considering two suppliers, two production plants, two distributors, two retailers and two end customers. Four manufacturing inputs (raw materials) are considered for the manufacture of two types of products. A case study of the cement production supply chain of Cartagena (Colombia) is illustrated. The shared benefit is generated around 5 billion pesos (COP) for all members of the chain in a period of 6 months.

Keywords Collaborative supply chain · Deteriorating products · Reverse flow

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Introduction

The supply chain refers to the set of efficiently integrated companies that seek the best strategies to deal with goods and products in a timely manner at the lowest cost, satisfying the requirements of consumers [14]. The supply chain process integrates organizations and customer–supplier relationships. An integrated, coordinated, and synchronized management provides effective solutions for decision-making in all actors in the chain. The optimality of the supply chain allows profits in inventory, purchasing, transportation, information flow, customer service, and delivery times [2, 56].

The global dynamics of the economy and competitiveness require new challenges to interact and satisfy customers. Efficient analysis of the supply chain for the search for competitive advantages must consider all interactions and relationships among suppliers, manufacturers, distributors, and customers. A common goal shared by these actors in the chain is to maximize profits and customer satisfaction which translates into greater profitability and competitiveness. Strategies such as cooperation and collaboration of the different actors generate synergies and multiply the efforts of the logistics processes. Collaboration/cooperation in the supply chain guide processes to be more dynamic and competitive based on customer demand, eliminating barriers in the network, and seeking to simplify and make activities more efficiently.

This research seeks to develop a collaborative mathematical model in the supply chain for production planning.Problems such as the loss of sales due to low or missing inventories, obsolescence and deterioration of products, high transportation, and inventory costs, and uncertainty in the demand information motivate the approach of this mathematical model. The proposed model considers a case study problem of a company in the mining sector to produce cement in the city of Cartagena in Colombia. The productive chain of the Colombian mining sector deals with the exploration, exploitation, and commercialization of non-metallic minerals such as sand, limestones and clays, gypsum that are used to supply materials to cement or concrete in industrial production processes, housing construction and infrastructure [47]. Hence the study of this chain is important in the social and economic development of Colombia [8]. The use of mathematical models becomes an essential tool for the design and implementation of supply chains [57]. Supply chain modeling helps to capture the complexity and integrate the resources and mathematical programming represents the best way to approach the methodology for solving the problems of mining supply chains. Next, a literary review is developed in the field of collaboration/cooperation where mathematical modeling is applied in the supply chain.

Literature review

The concept of collaborative supply chains has taken on importance over the years [6, 19, 23, 31, 38, 40, 62, 64]. The collaborative supply chain is characterized to integrate, relate to the long term, and define common targets and benefits [45, 57]. Joint efforts to achieve the same objective and economic benefits within the chain become a comprehensive solution [60]. Collaboration/cooperation avoid inefficiencies and the bullwhip effect produced by a lack of coordination in the supply chain [26]. Table 1 shows the review of the literature on collaborative supply chain models, based on the structure, approaches to the solution, and their characteristics.

In the literary review on models of collaboration/cooperation of logistics chains with different approaches, no models are found that contemplate an integration in the supply

Table 1 Review of the literature on collaborative supply chain models: approaches and considerations

Model Class	Authors	Approaches of solution					Levels of SC				Considerations
		GT	MS	SE	SD	Others	2	3	M	Others	
Analytical model AHP	[51]					X		X			Collaborative information
Artificial intelligence-based model	[33]		X							X	Uncertainty in demand and supply
Conceptual model	[46]			X			X				Performance measures
	[41]			X				X			Development of new products and inter and intra business relationships
	[52]			X						X	CPFR model based on associations, analysis using structural equation models and confirmatory factor analysis
	[1]					X	X				SCOR model. Performance measurement
	[65]					X	X				SCOR model. Benchmarking techniques and performance measurement
	[67]					X				X	SCOR model. Collaboration of external partners and performance measurement (KPI's) of the logistics service
Inventory management model	[16]	X					X				Stochastic demand and life cycle analysis in green supply chains
	[57]	X						X			Scenarios with uncertainty, quantities of products in inventories
	[27]	X						X			Coordination of prices and inventory decisions
Stochastic programming model	[43]	X					X				Cooperative and non-cooperative scenarios for supplier selection
	[10]	X					X				Production costs for technical advice and discounts

Table 1 continued

Model Class	Authors	Approaches of solution					Levels of SC				Considerations
		GT	MS	SE	SD	Others	2	3	M	Others	
	[30]	X					X				Cooperative global scenario
	[61]	X					X				Vertical cooperation
	[54]	X					X				Shared capacity
	[35]	X					X				Non-cooperative games in assembly supply chains, decentralized
	[36]	X					X				Supply shortage
	[74]	X					X				Contract options
	[53]	X					X				Sharing information and forecasts
	[17]	X					X				Decreased Bullwhip Effect
	[73]	X					X				Simultaneous product distribution with infinite capacity considerations
	[71]	X					X				Optimal price, and advertising strategies in four-game scenarios
	[21]	X					X				Market demand, selling price, and marketing expenses
	[34]	X						X			The problem of allocation of savings in the exchange costs of demand information
	[25]	X					X				Stochastic demand and delivery times
	[44]	X						X			Benefits of allocation and stability in cooperative chains
	[55]	X							X		The price transfer problem
	[69]	X					X				Two scenarios, one with sufficient supply and the other insufficient from the supplier

Table 1 continued

Model Class	Authors	Approaches of solution					Levels of SC				Considerations
		GT	MS	SE	SD	Others	2	3	M	Others	
	[28]	X					X				Manufacturer dominance over retailers
	[59]						X			X	Co-op advertising among the supplier, manufacturer, and retailers with a variable demand driven by selling price and advertising costs (fuzzy)
Simulation model	[37]		X						X		Multi-flow Supply Chain, and inter-company relationships
	[13]					X			X		Knowledge-based personalization and sharing information for supply chain integration
	[9]					X	X				Collaborative transport, based on the activation of capacity restrictions
	[18]						X				CPFR model Accuracy in estimating forecasts
	[32]				X		X				Decision-making processes and their impact on CS performance
	[22]						X				Collaborative offer
Mathematical model/optimization	[66]		X							X	Operations costs, information sharing costs, and information processing costs
	[70]					X	X				Quantity discounts and incremental quantity discounts
	[5]					X	X				Collaborative forecasting between manufacturer and supplier
	[62]					X	X				Joint planning and problem solving
	[63]					X				X	Shared capacity

Table 1 continued

Model Class	Authors	Approaches of solution					Levels of SC				Considerations
		GT	MS	SE	SD	Others	2	3	M	Others	
	[12]					X	X				Joint replacement
	[4]					X	X				Demand forecast and joint replenishment in uncertain scenarios
	[3]					X	X				Collaborative forecasting between retailer and supplier
	[50]					X	X				Non-stationary demand
	[11]					X				X	A multi-product, multi-stage production and distribution planning model, and multi-period which maximizes the benefits, the level of services, and the inventory level
	[39]					X				X	Mathematical model with uncertainty, whose objective is to minimize the costs of the aggregate plan of the supply chain
	[49]				X					X	A four-echelon integrated production–distribution in supply chain. The chain is studied in competitive conditions (Cournot, Stackelberg, and Quality and competition-free condition. Two mixed-integer linear programs (MILPs) are developed, which are proved to be unimodular
	[20]		X							X	Proposing a multi-objective model for a multi-period multi-product multi-site aggregate production planning (APP) problem in a green supply chain

Table 1 continued

Model Class	Authors	Approaches of solution					Levels of SC				Considerations
		GT	MS	SE	SD	Others	2	3	M	Others	
	[15]					X				X	A mixed-integer linear programming model that can handle general supply chains and production processes that require multiple resources
	[72]					X	X				The model developed an aggregate production-pricing problem with multiple demand classes. Developing a new demand response function with cannibalization
	[58]					X			X		The objective is to minimize the total cost of the entire supply chain model (SCM) by simultaneously optimizing setup cost, process quality, deliveries, and lot size
	[24]					X	X				The model determines the production plans of the suppliers to minimize the sum of raw material purchasing costs, production costs, setup costs of the suppliers, transportation costs, and costs for outsourcing semi-finished products. The model is solved with a mixed-integer programming model and a two-step heuristic algorithm
Mathematical optimization by scenarios	This research					X				X	Demand uncertainty, process failure uncertainty, material arrival uncertainty, reverse flow of defective materials/products

GT game theory; *MS* multi-agent system; *SE* structural equations; *SD* system dynamics; *M* multilevel chain of non-metallic minerals such as cement production. The proposed model helps to plan the production of final products, the distribution of materials, and integrates functions from suppliers to customers, synchronizing, and collaborating among all the actors in the chain. Additionally, the model considers uncertainty in demand, uncertain delays due to process failures, and even uncertainty in material quantities. The model contributes to planning within the medium and short term at the tactical and operational levels of the supply chain.

The present research develops a collaborative supply chain planning model where the main objective is to maximize the benefits of all the members in the four-echelon supply chain comprising of suppliers, manufacturers, distributors, and retailers. The novelty of the model allows considering a mechanism for detecting products with minor defects or deterioration. These products are purchased at a lower price, avoiding return, which favors less use of transport and environmental impact. Additionally, it is considered a mechanism for detecting products with major defects in buyers (plants and distributors). The model considers returns and changes in delivery times. Likewise, the lead time is considered at the supplier/producer and producer/distributor. The model considers production with work on regular and extra days, and subcontracting also.

The content of this research is structured as follows: The introduction in Sect. 1 indicates the context of the problem. The methods in Sect. 2 present the elements for mathematical formulation. Sections 3 and 4 provide the case study and its results. Sections 5 and 6 provide managerial implications and conclusions of the proposed model respectively.

Method

The proposed model is of a mixed-integer linear programming approach with analysis by scenarios for demand according to the economic performance of the construction sector. The model allows to plan the transport in each echelon, plan the purchases with the suppliers, plan the production of the plant, the sales, the inventory levels, and calculation of the total cost. The chain integrates suppliers, production plants, distribution centers, retailers or customers.Suppliers deliver raw materials or items to the production plants, which transform them into finished products. The distribution centers receive the products from the production plants and deliver them to the retailers, who sell the products to the final client. This model maximizes the benefits of all members of the supply chain, considering parameters related to the production process, transport (times and capacities), inventories (capacities and shortage), and costs.

Fundamental Assumptions

The following assumptions are used to formulate the model:

The objective is to guarantee the maximization of the profit margin of all the entities of the supply chain (Revenues-Costs).

Multiple suppliers, production plants, and multiple distribution centers are considered.

Suppliers and production plants store raw material.

There is a fraction of raw materials with deterioration/defect that can be purchased by the plants at a lower price.

The lead times of plants and distribution centers are deterministic.

There is a fraction of products with deterioration/defect that generate a non-rejection condition, which are bought by the next echelon at a lower price.

At the beginning of each period new settings are given for production.
Defective/deteriorated products purchased at plants from distributors are reprocessed and sold to the retailer as compliant products.
Plants, distribution centers, and retailers store finished products.
Three scenarios are proposed for the representation of the model under uncertainty, high, medium, and low scenario.

2.2. Notation

The following notations are used to develop the model.
Declaration of Indices

$s \in S$: Suppliers
$p \in P$: Plants
$d \in D$: Distribution Centers
$r \in R$: Retailers
$c \in C$: Clients
$t \in T$: Periods
$e \in E$: Scenarios
$m \in M$: Raw Materials
$j \in J$: Products
$q \in Q$: Production Resources

Declaration of Sets

S^m: Set of suppliers s that provide raw material m ($S^m \subseteq S$)
S^p: Set of suppliers s that provide production plant p ($S^p \subseteq S$)
Q^p: Set of production resources q of the plants p ($Q^p \subseteq Q$)
D^p: Set of distribution centers d that receive finished products j ($D^p \subseteq D$)
R^d: Set of retailers r that receive finished products from distribution centers d ($R^d \subseteq R$)
C^r: Set of clients that receive finished products from retailers r ($C^r \subseteq CJ^q$.
J^q: Set of finished products j produced with the production resource q ($J^q \subseteq J$)
J^m: Set of finished products j produced with the raw material m ($J^m \subseteq J$).

2.3. Statement of Parameters

Suppliers' Parameters

$CC^S_{s,m,t}$: Cost per unit of raw material, component, or item m at supplier s in period t ($/Ton)
$H^S_{s,m}$: Handling cost per unit at supplier s for the raw material m ($/Ton)
$\pi^+_{s,m}$: Inventory excess cost at supplier s for the raw material m ($/Ton)
$\pi^-_{s,m}$: Inventory shortage cost at supplier s for the raw material m ($/Ton)
$CP^S_{s,m,t}$: Production cost per unit at supplier s for the raw material m in period t($/Ton)
$CD^S_{s,m}$: Disposal cost per unit defective item at supplier s for raw material m ($/Ton)
$CFT^S_{s,p,m}$: Fixed transportation cost at supplier s for the raw material m for the production plant p($)
$CT^S_{s,p,m}$: Transportation cost per unit from the supplier s to production plant p for the raw material m ($/Ton)

$CT1^S_{p,s,m}$: Transportation cost per unit from the production plant p to supplier s for the raw material m (\$/Ton)
$\alpha^S_{s,m}$: Expected percentage of defective items at supplier s for the raw material m $\left(0 \leq \alpha^S_{s,m} \leq 1\right)$
$\beta^S_{s,m}$: Screening rate (%) of defective items at supplier s for the raw material m
$CapT^S_{s,p,m}$: Transport capacity for the raw material m from the supplier s to the production plant p (Ton)
$TT^S_{s,p}$: Transportation time from supplier s to production plant p (Hours)
$Io^S_{s,m}$: Initial inventory level at supplier s for the raw material m (Ton)
$Imax^S_s$: Maximum inventory capacity at the supplier s (Ton)
$Fmax^S_{s,m}$: Maximum production capacity at the supplier s for the raw material m (Ton)
$PV^S_{s,p,m}$: Selling price of raw material, component, or item m at the production plant p (\$/Ton)
$PVD^S_{s,p,m}$: Selling price per unit defective item at supplier s for the raw material m for the production plant p (\$/Ton)

Production Plants' Parameters

$CF^P_{p,q,j,t}$: Fixed cost of changing material at plant p with the production resource q for the finished product j in period t (\$)
$CPR^P_{p,q,j,t}$: Production cost per unit at production plant p for the finished product j working in regular time with the production resource q in period t (\$/Ton)
$CPE^P_{p,q,j,t}$: Production cost per unit at production plant p for the finished product j working in extra time with the production resource q in period t (\$/Ton)
$CSub^P_{p,j}$: Purchasing cost per unit subcontracted in the production plant p for the finished product j (\$/Ton)
$\varphi_{p,q,j,t}$: Fixed handling cost at production plant p for the finished product j with the production resource q in period t(\$)
$H^P_{p,j}$: Handling cost per unit at production plant p for the finished product j (\$/Ton)
$\varphi^+_{p,j}$: Inventory excess cost at production plant p for the finished product j (\$/Ton)
$\varphi^-_{p,j}$: Inventory shortage cost at production plant p for the finished product j (\$/Sack)
$CFT^P_{p,d,j}$: Fixed transportation cost for finished product j from production plant p to distribution center d (\$)
$CT^P_{p,d,j}$: Transportation cost per unit for finished product j from production plant p to distribution center d (\$/Ton)
$MA^P_{m,j}$: Combination of raw material m necessary to produce the finished product j (Sack)
$CD^P_{p,j}$: Disposal cost per unit defective item at production plan p for the finished product j (\$/Sack)
$\beta^P_{p,m}$: Screening rate (%) of defective items at production plant p for the raw material m.
$\alpha^P_{p,j}$: Expected percentage of defective items at production plant p for the finished product $j\left(0 \leq \alpha^P_{p,j} \leq 1\right)$
$\mu^P_{p,j}$: Screening rate (%) of defective items at production plant p for the finished product j
$CapPR^P_{p,q,t}$: Maximum production capacity at production plant p working in regular time on the production resource q in period t (Sacks)
$CapPE^P_{p,q,t}$: Maximum production capacity at production plant p working in extra time on the production resource q in period t (Ton)

$CapTI_p^P$: Maximum capacity of transport input in the plant p (Ton)
$CapTO_p^P$: Maximum capacity of transport output in the plant p. (Sacks)
$CapT_{p,d,j}^P$: Transport capacity for finished product j from production plant p to distribution center d (Ton)
$TT_{p,d}^P$: Transportation time from the production plant p to the distribution center d (Hours)
$Io_{p,j}^P$: Initial inventory level at production plant p for the finished product j (Ton)
$Imax_p^P$: Maximum inventory capacity at the production plant p (Ton)
xpected percentage (%) of finished product j for subcontracting in the production plant p.
$PV_{p,d,j}^P$: Selling price per unit good item at production plant p to distrsibution center d for the finished product j ($/Ton)
$PVD_{p,d,j}^P$: Selling price per unit defective item at production plant p to distribution center d for the finished product j ($/Ton)

Distribution Centers' Parameters

$H_{d,j}^D$: Handling cost per unit at the distribution center d for the finished product j ($/Sack)
$\gamma_{d,j}^+$: Excess inventory cost at distribution center d for the finished product j ($/Sack)
$\gamma_{d,j}^-$: Shortage inventory cost at distribution center d for the finished product j ($/Sack)
$CFT_{d,r,j}^D$: Fixed transportation cost from distribution center d to the retailer r for the finished product j($)
$CT_{d,r,j}^D$: Fixed transportation cost per unit from distribution center d to the retailer r for the finished product j ($/Sack)
$CD_{d,j}^D$: Disposal cost per unit defective item at distribution center d for the finished product j ($/Sack)
$\delta_{d,j}^D$: Screening-rate (%) of defective items at distribution center d for the finished product j.
$CapTO_d^D$: Maximum capacity of transport output in the distribution center d (Sacks)
$CapT_{d,r,j}^D$: Transportation capacity of finished product j from distribution center d to the retailer r (Sacks)
$TT_{d,r}^D$: Transportation time from the distribution center d to the retailer r (Hours)
$Io_{d,j}^D$: Initial inventory level at distribution center d for the finished product j (Sacks)
$Imax_d^D$: Maximum inventory capacity at distribution center d (Sacks)
$PV_{d,r,j}^D$: Selling price per unit good item at distribution center d to the retailer r for the finished product j ($/Sack)

Retailers' Parameters.

$H_{r,j}^R$: Handling cost per unit at retailer r for the finished product j ($/Sack)
$\vartheta_{r,j}^+$: Excess inventory cost at retailer r for the finished product j ($/Sack)
$\vartheta_{r,j}^-$: Shortage inventory cost at retailer r for the finished product j ($/Sack)
$Io_{r,j}^R$: Initial inventory level at retailer r for the finished product j (Sacks)
$Imax_r^R$: Maximum inventory capacity at retailer r (Sacks)
$CapTO_r^R$: Maximum capacity of transport input in the retailer r (Sacks)
$Dem_{c,j,t,e}^C$: Demand rate of client c for the finished product j over scenario e in period t (Sacks)
$PV_{r,c,j}^R$: Selling price per unit good item at retailer r to the client c for the finished product j ($/Sack)
$Prob_e$: Probability over scenario e.

2.4. Statement of Variables

Continuous Variables

$I^{S}_{s,m,t,e}$: Inventory level of raw material m at supplier s in period t (Ton)
$I^{S+}_{s,m,t,e}$: Excess inventory level of raw material m at supplier s in period t (Ton)
$I^{S-}_{s,m,t,e}$: Shortage inventory level of raw material m at supplier s in period t (Ton)
$I^{P}_{p,j,t,e}$: Inventory level of the finished product j at the plant p in period t (Sacks)
$I^{P+}_{p,j,t,e}$: Excess inventory level of the finished product j at the plant p in period t (Sacks)
$I^{P-}_{p,j,t,e}$: Shortage inventory level of the finished product j at plant p in period t (Sacks)
$I^{D}_{d,j,t,e}$: Inventory level of the finished product j at the distribution center d in period t (Sacks)
$I^{D+}_{d,j,t,e}$: Excess Inventory level of the finished product j at the distribution center d in period t (Sacks)
$I^{D-}_{d,j,t,e}$: Shortage inventory level of the finished product j at the distribution center d in period t (Sacks)
$I^{R}_{r,j,t,e}$: Inventory level of the finished product j at the retailer r in period t (Sacks)
$I^{R+}_{r,j,t,e}$: Excess inventory level of the finished product j at retailer r in period t (Sacks)
$I^{R-}_{r,j,t,e}$: Shortage inventory level of the finished product j at the retailer r in period t (Sacks)
$Q^{S}_{s,m,t,e}$: Raw material m to purchase from the supplier s in period t (Ton)
$Q^{P}_{p,j,t,e}$: Production of the finished product j at the plant p in period t (Sacks)
$QP^{P}_{p,q,j,t,e}$: Product j to be produced in Plant p on Production Resource q in period t. (Sacks)
$QPR^{P}_{p,q,j,t,e}$: Production quantity of the finished product j at the plant p working in regular time on the production resource q in period t (Sacks)
$QPE^{P}_{p,q,j,t,e}$: Production quantity of the finished product j at the plant p working in extra time on the production resource q in period t (Sacks)
$QSub^{P}_{p,j,t,e}$: Subcontracting quantity of the finished product j at the plant p in period t (Sacks)
$QT^{S}_{s,p,m,t,e}$: Raw material m to transport from the supplier s to the plant p in period t (Ton)
$QT^{P}_{p,d,j,t,e}$: Product j to transport from the plant p to the distribution center d in period t (Sacks)
$QT^{D}_{d,r,j,t,e}$: Product j to transport from the distribution center d to the retailer r in period t (Sacks)
$QT^{R}_{r,c,j,t,e}$: Product j to transport from the retailer r to the client c in period t (Sacks)
$QTT^{S}_{s,p,t,e}$: Total quantity to transport from the supplier s to the plant p in period t (Ton)
$QTT^{P}_{p,d,t,e}$: Total quantity to transport from the plant p to the distribution center d in period t (Sacks)
$QTT^{D}_{d,r,t,e}$: Total quantity to transport from the distribution center d to the retailer r in period t (Sacks)
$QTT^{R}_{r,c,t,e}$: Total quantity to transport from the retail r to the customer c in period t (Sacks)
$B^{S}_{s,e}$: Expected benefits of the supplier s in period t (\$)
$B^{P}_{p,e}$: Expected benefits of the plant p in period t (\$)
$B^{D}_{d,e}$: Expected benefits of the distribution center d in period t (\$)
$B^{R}_{r,e}$: Expected benefits of the retailer r in period t (\$)

BT_e: Total expected benefit of the supply chain in the scenario e ($)
Z: Total expected benefits of the supply chain ($)

Binary Variables

$X_{s,p,m,t}$: 1 if $CFT^S_{s,p,m} > 0$, otherwise 0 (1 if the transport capacity level from supplier *s* to plant *p* is used for raw material, component, or item *m* in period *t*)
$Y_{p,d,j,t}$: 1 if $CFT^P_{p,d,j} > 0$, otherwise 0 (1 if the level of transport capacity from plant *p* to distribution center *d* is used for product *j* in period *t*)
$W_{d,r,j,t}$: 1 if $CFT^D_{d,r,j} > 0$, otherwise 0 (1 if the transport capacity level from distribution center *d* to retailer *r* is used for product *j* in period *t*)
$V_{p,q,j,t}$: 1<if $CF^P_{p,q,j,t} > 0$, otherwise 0 (1 if the enlistment time in plant *p* is used on the Production Resource *q* for product *j* in period *t*)
$U_{p,q,j,t}$: 1 if $CF^P_{p,q,j,t} > 0$, otherwise 0 (1 if a starting change is generated in Plant *p* on the Production Resource *q* of product *j*).

Optimization Model

Once the variables have been defined, the formulation is as follows:

Objective function and constraints: The expected profit of the whole chain is:

$$Z = \sum_{e\in E}\sum_{sS} Prob_e B^S_{s,e} + \sum_{e\in E}\sum_{pP} Prob_e B^P_{p,e} + \sum_{e\in E}\sum_{dD} Prob_e B^D_{d,e} + \sum_{e\in E}\sum_{rR} Prob_e B^R_{r,e} \tag{1}$$

subject to:

$$\begin{aligned} B^S_{s,e} = &\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\left[\left[PV^S_{s,p,m}\left(1-\beta^P_{p,m}\right) + PVD^S_{s,p,m}\beta^P_{p,m}\right]\left(1-\beta^S_{s,m}\right)QT^S_{s,p,m,t,e}\right] \\ &-\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\left[CFT^S_{s,p,m}X_{s,p,m,t} + \left(CT^S_{s,p,m} + H^S_{s,m}\right)QT^S_{s,p,m,t,e}\right] \\ &-\sum_{t\in T}\sum_{m\in M}\left[\left(CC^S_{s,m,t} + CD^S_{s,m}\alpha^S_{s,m}\right)Q^S_{s,m,t,e} + \pi^+_{s,m}I^{S+}_{s,m,t,e} + \pi^-_{s,m}I^{S-}_{s,m,t,e}\right] \quad \forall s \\ &\in S, e\in E \end{aligned} \tag{2}$$

$$\begin{aligned} B^P_{p,e} = &\sum_{t\in T}\sum_{d\in D}\sum_{q\in Q}\sum_{j\in J}\left[\left[PV^P_{p,d,j}\left(1-\delta^D_{d,j}\right) + PVD^P_{p,d,j}\delta^D_{d,j}\right]\left(1-\mu^P_{p,j}\right)QT^P_{p,d,j,t,e}\right] \\ &-\sum_{t\in T}\sum_{s\in S}\sum_{m\in M}\left[\left[PV^S_{s,p,m}\left(1-\beta^P_{p,m}\right) + PVD^S_{s,p,m}\beta^P_{p,m}\right]\left(1-\beta^S_{s,m}\right)QT^S_{s,p,m,t,e}\right] \\ &-\sum_{t\in T}\sum_{q\in Q}\sum_{j\in J}\left[\varphi_{p,q,j,t} + CF^P_{p,q,j,t}V_{p,q,j,t} + CPR^P_{p,q,j,t}QPR^P_{p,q,j,t,e} + CPE^P_{p,q,j,t}QPE^P_{p,q,j,t,e}\right] \\ &\times\sum_{t\in T}\sum_{d\in D}\sum_{j\in J}\left[CFT^P_{p,d,j}Y_{p,d,j,t} + CT^P_{p,d,j}QT^P_{p,d,j,t,e}\right] \\ &-\sum_{t\in T}\sum_{s\in S}\sum_{j\in J}\left[H^P_{p,j}\left(\sum_{m\in M}\left(MA^P_{m,j}\sum_{q\in Q_p}QP^P_{p,q,j,t-TT^S_{s,p},e}\right) + \sum_{d\in D}QT^P_{p,d,j,t,e}\right)\right] \\ &-\sum_{t\in T}\sum_{j\in J}\left[CD^P_{p,j}\alpha^P_{p,j}Q^P_{p,j,t,e} + CSub^P_{p,j}QSub^P_{p,j,t,e} + \varphi^+_{p,j}I^{P+}_{p,j,t,e} + \varphi^-_{p,j}I^{P-}_{p,j,t,e}\right] \quad \forall p\in P, e\in E \end{aligned} \tag{3}$$

$$B^D_{d,e} = \sum_{t\in T}\sum_{r\in R}\sum_{j\in J}\left(PV^D_{d,r,j}QT^D_{d,r,j,t,e}\right) - \sum_{t\in T}\sum_{p\in P}\sum_{j\in J}\left[\left[PV^P_{p,d,j}\left(1-\delta^D_{d,j}\right) + PVD^P_{p,d,j}\beta^P_{p,m}\delta^D_{d,j}\right]\left(1-\mu^P_{p,j}\right)QT^P_{p,d,j,t,e} + CD^D_{d,j}\delta^D_{d,j}QTPD_{dpjt}\right] - \sum_{t\in T}\sum_{d\in D}\sum_{j\in J}\left[CFT^D_{d,r,j}W_{d,r,j,t} + CT^D_{d,r,j}QT^D_{d,r,j,t,e}\right] - \sum_{t\in T}\sum_{j\in J}\left[H^D_{d,j}\left(\sum_{p\in P}QT^p_{p,d,j,t-TT^P_{p,d,e}} + \sum_{r\in R}QT^D_{d,r,j,t,e}\right) + \gamma^+_{d,j}I^{D+}_{d,j,t,e} + \gamma^-_{d,j}I^{D-}_{d,j,t,e}\right] \quad \forall d \in D, e\in E \tag{4}$$

$$B^R_{r,e} = \sum_{t\in T}\sum_{c\in C}\sum_{j\in J}\left(PV^R_{r,c,j}QT^R_{r,c,j,t,e}\right) - \sum_{t\in T}\sum_{d\in D}\sum_{j\in J}\left[PV^D_{d,r,j}QT^D_{d,r,j,t,e}\right] - \sum_{t\in T}\sum_{j\in J}\left[H^R_{r,j}\left(\sum_{d\in D}QT^D_{d,r,j,t-TT^D_{d,r,e}} + \sum_{r\in R}QT^R_{r,c,j,t,e}\right)\right] - \sum_{t\in T}\sum_{j\in J}\left[\vartheta^+_{r,j}I^{R+}_{r,j,t,e} + \vartheta^-_{r,j}I^{R-}_{r,j,t,e}\right] \quad \forall r\in R, e\in E \tag{5}$$

$$QP^P_{p,q,j,t,e} = QPR^P_{p,q,j,t,e} + QPE^P_{p,q,j,t,e} \quad \forall p\in P, q\in Q, j\in J, t\in T, e\in E \tag{6}$$

$$CapPR^P_{p,q,t}kk \quad \forall p\in P, q\in Q, t\in T, e\in E \tag{7}$$

$$\sum_{j\in J} QPE^P_{p,q,j,t,e} \le CapPE^P_{p,q,t} \quad \forall p\in P, q\in Q, t\in T, e\in E \tag{8}$$

$$\sum_{q\in Q}\sum_{j\in J^q} V_{p,q,j,t} = 1 \quad \forall p\in P, t\in T \tag{9}$$

$$U_{p,q,j,t} \ge V_{p,q,j,t} - V_{p,q,j,t-1} \quad \forall p\in P, q\in Q, j\in J, t\in T \tag{10}$$

$$QSub^P_{p,j,t,e} \le Sub^P_{p,j}Q^P_{p,j,t,e} \quad \forall p\in P, j\in J, t\in T, e\in E \tag{11}$$

$$QT^S_{s,p,m,t,e} = \sum_{j\in J^m}\left(MA^P_{m,j}\sum_{q\in Q^p}QP^P_{p,q,j,t-TT^S_{s,p},e}\right) \quad \forall s\in S, p\in P, m\in M, t\in T, e\in E \tag{12}$$

$$QT^S_{s,p,m,t,e} \le CapT^S_{s,p,m}X_{s,p,m,t} \quad \forall s\in S, p\in P, m\in M, t\in T, e\in E \tag{13}$$

$$QT^P_{p,d,j,t,e} \le CapT^P_{p,d,j}Y_{p,d,j,t} \quad \forall p\in P, d\in D, j\in J, t\in T, e\in E \tag{14}$$

$$QT^D_{d,r,j,t,e} \le CapT^D_{d,r,j}W_{d,r,j,t} \quad \forall d\in D, r\in R, j\in J, t\in T, e\in E \tag{15}$$

$$QT^R_{r,c,j,t,e} \le Dem^C_{c,j,t,e} \quad \forall r\in R, c\in C, j\in J, t\in T, e\in E \tag{16}$$

$$QTT^S_{s,p,t,e} = \sum_{m\in M}QT^S_{s,p,m,t,e} \quad \forall s\in S, p\in P, t\in T, e\in E \tag{17}$$

$$QTT^{P}_{p,d,t,e} = \sum_{j \in J} QT^{P}_{p,d,j,t,e} \quad \forall p \in P, d \in D, t \in T, e \in E \tag{18}$$

$$QTT^{D}_{d,r,t,e} = \sum_{j \in J} QT^{D}_{d,r,j,t,e} \quad \forall d \in D, r \in R, t \in T, e \in E \tag{19}$$

$$QTT^{R}_{r,c,t,e} = \sum_{j \in J} QT^{R}_{r,c,j,t,e} \quad \forall r \in R, c \in C, t \in T, e \in E \tag{20}$$

$$\sum_{s \in S^{p}} QTT^{S}_{s,p,t,e} \leq CapTI^{P}_{p} \quad \forall p \in P, t \in T, e \in E \tag{21}$$

$$\sum_{d \in D^{p}} QTT^{P}_{p,d,t,e} \leq CapTO^{P}_{p} \quad \forall p \in P, t \in T, e \in E \tag{22}$$

$$\sum_{r \in R^{p}} QTT^{D}_{d,r,t,e} \leq CapTO^{D}_{d} \quad \forall d \in D, t \in T, e \in E \tag{23}$$

$$\sum_{c \in C^{p}} QTT^{R}_{r,c,t,e} \leq CapTO^{R}_{r} \quad \forall r \in R, t \in T, e \in E \tag{24}$$

$$I^{S}_{s,m,t,e} = I^{S}_{s,m,t-1,e} + \left(1 - \alpha^{S}_{s,m}\right) Q^{S}_{s,m,t,e} - \sum_{p \in P} QT^{S}_{s,p,m,t,e} \quad \forall s \in S, m \in M, t \in T, e \in E \tag{25}$$

$$I^{P}_{p,j,t,e} = I^{P}_{p,j,t-1,e} + \left(1 - \alpha^{P}_{p,j}\right) Q^{P}_{p,j,t,e} - \sum_{d \in D} QT^{P}_{p,d,j,t,e} \quad \forall p \in P, j \in J, t \in T, e \in E \tag{26}$$

$$I^{D}_{d,j,t,e} = I^{D}_{d,j,t-1,e} + \sum_{p \in P} QD^{D}_{p,d,j,t-TT^{P}_{p,d},e} - \sum_{r \in R} QT^{D}_{d,r,j,t,e} \quad \forall d \in D, j \in J, t \in T, e \in E \tag{27}$$

$$I^{R}_{r,j,t,e} = I^{R}_{r,j,t-1,e} + \sum_{d \in D} QR^{R}_{d,r,j,t-TT^{D}_{d,r},e} - \sum_{c \in C} QT^{R}_{r,c,j,t,e} \quad \forall r \in R, j \in J, t \in T, e \in E \tag{28}$$

$$\sum_{m \in M} I^{S}_{s,m,t,e} \leq Imax^{S}_{s} \quad \forall s \in S, t \in T, e \in E \tag{29}$$

$$\sum_{j \in J} I^{P}_{p,j,t,e} \leq Imax^{P}_{p} \quad \forall p \in P, t \in T, e \in E \tag{30}$$

$$\sum_{j \in J} I^{D}_{d,j,t,e} \leq Imax^{D}_{d} \quad \forall d \in D, t \in T, e \in E \tag{31}$$

$$\sum_{j \in J} I^{R}_{r,j,t,e} \leq Imax^{R}_{r} \quad \forall r \in R, t \in T, e \in E \tag{32}$$

$$I^{S}_{s,m,t,e} = I^{S+}_{s,m,t,e} - I^{S-}_{s,m,t,e} \quad \forall s \in S, m \in M, t \in T, e \in E \tag{33}$$

$$I^{P}_{p,j,t,e} = I^{P+}_{p,j,t,e} - I^{P-}_{p,j,t,e} \quad \forall p \in P, j \in J, t \in T, e \in E \tag{34}$$

$$I^{D}_{d,j,t,e} = I^{D+}_{d,j,t,e} - I^{D-}_{d,j,t,e} \quad \forall d \in D, j \in J, t \in T, e \in E \tag{35}$$

$$I^{R}_{r,j,t,e} = I^{R+}_{r,j,t,e} - I^{R-}_{r,j,t,e} \quad \forall r \in R, j \in J, t \in T, e \in E \tag{36}$$

The variables ($Q^{S}_{s,m,t,e}$, $Q^{P}_{p,j,t,e}$, $QP^{P}_{p,q,j,t,e}$, $QPR^{P}_{p,q,j,t,e}$, $QPE^{P}_{p,q,j,t,e}$, $QSub^{P}_{p,j,t,e}$, $QT^{S}_{s,p,m,t,e}$, $QT^{P}_{p,d,j,t,e}$, $QT^{D}_{d,r,j,t,e}$, $QT^{R}_{r,c,j,t,e}$, $QTT^{S}_{s,p,t,e}$, $QTT^{P}_{p,d,t,e}$, $QTT^{D}_{d,r,t,e}$, $QTT^{R}_{r,c,t,e}$,, $I^{S+}_{s,m,t,e}$, $I^{S-}_{s,m,t,e}$, $I^{P+}_{p,j,t,e}$, $I^{P-}_{p,j,t,e}$, $I^{D+}_{d,j,t,e}$, $I^{D-}_{d,j,t,e}$, $I^{R+}_{r,j,t,e}$, $I^{R-}_{r,j,t,e}$) are nonnegative here.

Equations 1–5 contain the general objective function of the model. Equation 2 represents the benefits obtained by raw material suppliers, because of the sales of products (materials) and the recoverable fraction of defective products sold at a lower price to the plants. This equation also deducts handling costs (reception and dispatch), purchase costs of raw materials, cost of excess and shortage inventory, costs of inventory of the recoverable defective products, and costs of transporting materials. Equation 3 represents the benefits obtained by the plants because of the income from the sale of conforming products and with minor defects that are accepted by the distribution centers at a lower price. This equation also deducts fixed manufacturing costs, material handling costs, total manufacturing costs (normal and overtime), subcontracting costs, material reception and dispatch costs, raw material purchase costs, cost of excess and shortage inventory, including the costs of defective product inventory and the cost of transportation.

Equation 4 represents the benefits obtained by the distribution centers because of the income from the sale of products to retailers, deducting the costs of receipt and dispatch, the total cost of purchases from the plants, cost of excess and shortage inventory, defective product inventory costs, and transportation costs. Equation 5 represents the benefits obtained by retailers because of the income from the sale of products to customers, deducting the costs of receipt and dispatch, the total cost of purchases from the plants, cost of excess and shortage inventory, including the inventory of defective products detected and transportation costs.

Equation 6 represents the amount of total production considering regular hours and overtime products. Equations 7 and 8 denote the maximum production capacity available during regular working hours and overtime. Equations 9 and 10 specify that the manufacturing plant prepares for production or a possible change in a period. Equation 11 specifies the maximum amount of outsourcing. Equations 12–16 specify the quantity to be produced in a period.

Equations 17–20 specify the total quantities transported for each product in a period. Equation 21 specifies the maximum inbound transportation capacity to the plant from suppliers. Equations 22–24 specify the maximum outbound transportation capacity from the plant to the distribution center and from the distribution center to the retailer, and from the retailer to the customer, respectively. Equations 25–28 correspond to the inventory balance equations at each stage of the supply chain. Equations 29–32 represent the maximum inventory capacity in each of the stages of the chain. Equations 33–36 regulate the level of total inventory as the occurrence of excess inventory or shortage inventory at each stage of the supply chain.

Case Study

This section presents a numerical example to illustrate the proposed model, considering a cement company that produces and distributes cement and concrete. Figure 1 shows the schematic diagram of the cement manufacturing process includingthe grinding limestone, homogenization, preheating, clinkerization, cooling, clinker storage, cement grinding, cement storage, packing, and delivery.

Cement is produced from raw materials likelimestone or calcium carbonate, chert, iron ore, gypsum, coal, and slag. The limestone is extracted in quarries and transported in trucks to the grinding limestone section,where the size of the rock is reduced and send to raw material storage with a capacity of 50.000 tons. The raw materials are transported and dosed to the crude mill with the iron ore to adjust the levels of silica and iron oxide in the mix, which is discharged into the ponds with a capacity of 8000 tons (Fig. 1). The mix produced in the mills is homogenized in each pond and send to the preheater tower. In the rotary kiln, physical and chemical reactions allow the formation of clinker. The heat exchange occurs through heat transfers between the homogenized crude and the hot gases that are obtained from the preheater tower at high temperatures. The clinker obtained is subjected to a rapid cooling process, then it is taken to the clinker silos.

The clinker is conducted to the cement mills where it is ground together with the aggregates or additives, gypsum, and slag. At this point, the cement is ready to be transported and deposited in storage silos. The cement delivery process includes the extraction of cement

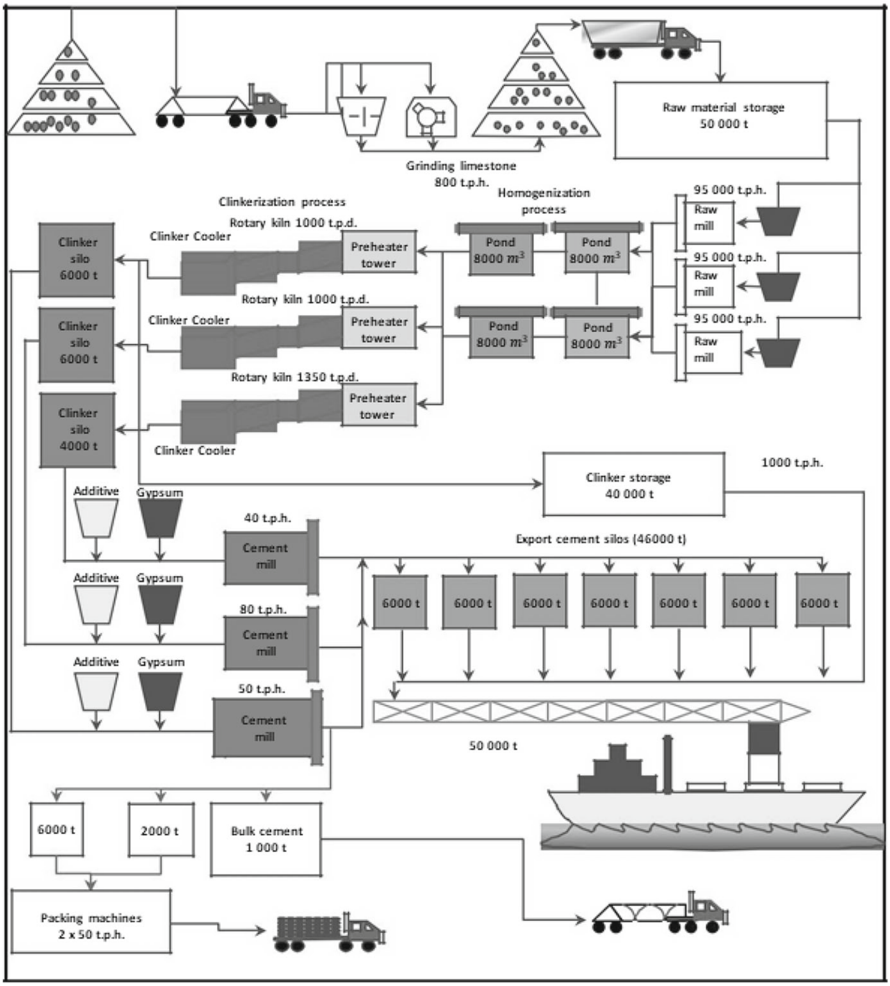

Fig. 1 Benefits of the supply chain per month. (echelons vs. Profit ($COP Million))

from the silos, transport, and loading.Modeling is assumed and validated in the supply chain of cement production. In the supplier stage, there are two (S1 and S2). It has two plants (P1 and P2). In distribution, two independent national distributors (D1 and D2) are considered. In the stages of retailers, they include the national companies that market the largest volumes of products to small builders and hardware stores (R1 and R2). The physical architecture of the stages of the supply chain to be applied is shown in Fig. 2. Two production resources (Q1, Q2) and two final products (A, B) are considered. The demand is assumed for six months.

The data used to validate the modelis provided by the cement company. The data contains the parameters of suppliers, production plants, distribution centers, and retailers considering selling prices, handling costs, inventory excess costs, inventory shortage costs, production costs, disposal costs, fixed transportation costs, purchasing costs and transportation costs. Also, the percentage of defective items, screening rates, transport capacities, transportation times, inventory levels, the combination of materials, production capacities and demand rates

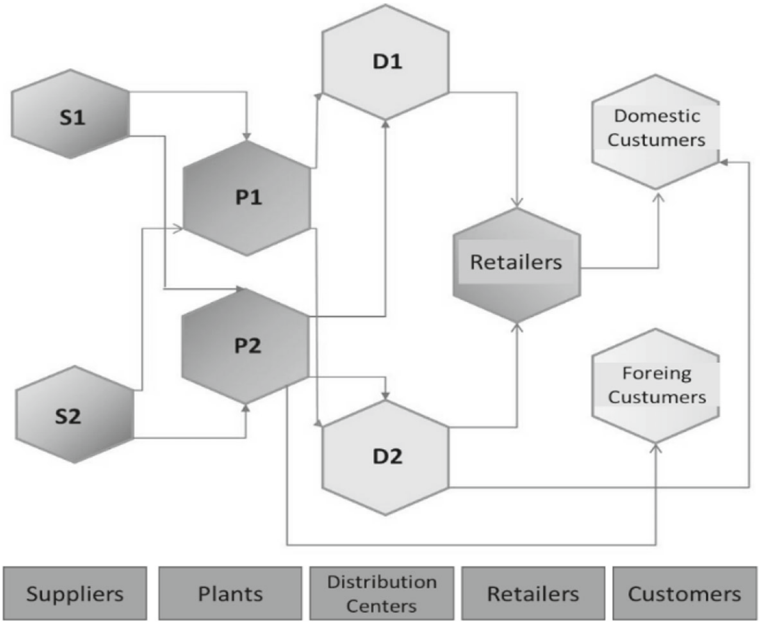

Fig. 2 Physical architecture of the stages of the supply chain

Table 2 Sets of the model

Sets		
s	Suppliers	1,2
m	Raw materials	Gypsum, slag, clinker, sacks
j	Products	A, B
p	Plants	1,2
q	Production resources	Q1, Q2
d	Distribution centers	1,2
r	Retailers	1,2
c	Clients	1,2
e	Scenarios	High, medium, low
t	Periods	1,2,3,4,5,6

are included. The sets of the model are presented in Table 2. The demand rate of client is provided in Table 3.

The proposed model considers three scenarios for the demand rate of the client: high, medium, and low (See Table 4) and the probability of each scenario is 33.33%. The parameters related to incomes and costs of suppliers, production plants, distribution centers and retailers are presented in Tables 4–8. The parameters of production and transportation capacities, inventory levels, and defective items of distribution centers are presented in Appendices A–D.

Table 3 Demand rate of clients

		Demand rate of client (tons)					
c	*j*	1	2	3	4	5	6
1	A	17,000	1,000	15,000	10,000	8,000	15,000
1	B	8,000	14,000	8,000	8,000	7,000	9,000
2	A	10,000	14,400	8,000	9,000	10,000	8,000
2	B	8,000	9,000	9,000	3,000	4,000	1,000

Table 4 Demand rate of clients for each scenario

	Demand rate of client (tons)							
c	*j*	*e*	1	2	3	4	5	6
1	A	Low	15,000	800	13,000	8,000	6,000	13,000
1	B	Low	6,000	10,000	5,800	9,000	10,000	9,000
2	A	Low	9,000	12,400	6,000	7,000	6,000	5,000
2	B	Low	1,500	2,000	3,200	2,000	10,000	800
1	A	Medium	17,000	1,000	15,000	10,000	8,000	15,000
1	B	Medium	8,000	14,000	8,000	8,000	2,000	9,000
2	A	Medium	10,000	14,400	8,000	9,000	8,000	8,000
2	B	Medium	8,000	9,000	9,000	3,000	7,000	1,000
1	A	High	19,000	1,200	17,000	12,000	10,000	17,000
1	B	High	10,000	20,000	8,500	8,500	8,000	11,000
2	A	High	17,000	18,400	10,000	11,000	11,000	10,000
2	B	High	9,000	10,000	10,800	4,800	5,000	1,200

Results

The proposed model for the master planning of operations in the supply chain with uncertainty seeks to maximize the profit margin of all the actors in the supply chain and it is validated through the data of a cement company. The model is solved with GAMS (General Algebraic Modeling System) under the XPRESS solver and likewise, the data is processed in Neos Solvers web support. In Table 9, the solution of the model under uncertainty is shown in the three established scenarios. Profit in high scenario is \$ 5,146,132,791 COP. Profit in the middle scenario is \$ 5,048,779,281 COP. Profit in low scenario is \$ 4,532,596,643 COP. Additionally, the expected total benefit value is \$ 4,909,164,663 COP. Likewise, the benefits derived for each actor in the supply chain are shown in Table 10.

Plant P1 is the largest producer, mainly producers of type A products. This product has the highest demand, and its manufacturing costs are lower. The plants distribute their production between resource Q1 and Q2, manufacturing larger quantities in Q2 for periods 3 and 4. Plant 1 is transporting the largest number of products, directing it to distribution center D2 in a higher proportion than D1. Products are transported from distributors to retails, with product A being the most marketed compared to product B. Retailer R2 receives a greater quantity of products than the retailer R1. Product A (61%) is supplied to the customer from the retailers in greater quantity. Retailer R1 is the one that supplies the largest quantities of product B.

Retailer R1 is the one that supplies the largest quantities of product B. Retailer R2 supplies a greater proportion of product A. The most significant costs at the suppliers are the

Table 5 Parameters of suppliers

$PV^{S}_{s,p,m}$ Selling price of raw material ($- COP)

s	p	Gypsum	Slag	Clinker	Sacks
1	1	20,825	20,298	59,358	2,958,501
1	2	20,825	20,298	59,358	2,958,501
2	1	20,630	20,546	59,849	2,988,086
2	2	20,630	20,546	59,894	2,988,086

$PVD^{S}_{s,p,m}$ Selling price per unit defective item ($-COP)

s	p	Gypsum	Slag	Clinker
1	1	13,101	11,979	44,712
2	1	12,958	12,158	44,809

$\mathrm{CP}^{\mathrm{S}}_{\mathrm{s,m,t}}$ Production cost per unit of raw material ($-COP)

s	m	1	2	3	4	5	6
1	Gypsum	1,101	1,101	1,101	1,101	1,101	1,101
1	Slag	1,979	1,979	1,979	1,979	1,979	1,979
1	Clinker	4,712	4,712	4,712	4,712	4,712	4,712
1	Sacks	218,876	218,876	218,876	218,876	218,876	218,876
2	Gypsum	1,101	1,101	1,101	1,101	1,101	1,101
2	Slag	1,979	1,979	1,979	1,979	1,979	1,979
2	Clinker	4,712	4,712	4,712	4,712	4,712	4,712
2	Sacks	218,876	218,876	218,876	218,876	218,876	218,876

$\mathrm{CT}^{\mathrm{S}}_{\mathrm{s,p,m}}$ Transportation cost per unit from the supplier to production plant ($- COP)

s	p	Gypsum	Slag	Clinker	Sacks
1	1	3,374	3,099	1,200	69,344
1	2	3,585	3,148	1,150	70,037
2	1	3,189	3,281	1,230	69,691
2	2	3,368	3,292	1,180	68,994

$H^{S}_{s,m}$ Handling cost per unit at supplier for the raw material ($-COP)

s	Gypsum	Slag	Clinker	Sacks
1	641	500	483	6,693
2	565	630	579	6,760

$\pi^{+}_{s,m}$ Inventory excess cost at supplier for the raw material ($-COP)

s	Gypsum	Slag	Clinker	Sacks
1	367	276	430	8,284

Table 5 continued

$\pi^{+}_{s,m}$ Inventory excess cost at supplier for the raw material ($-COP)				
s	Gypsum	Slag	Clinker	Sacks
2	251	269	383	8,367

$\pi^{-}_{s,m}$ Inventory shortage cost at supplier for the raw material ($-COP)				
s	Gypsum	Slag	Clinker	Sacks
1	5,348	4,889	16,607	887,550
2	5,289	4,964	16,408	887,550

$CD^{S}_{s,m}$ Disposal cost per unit defective item at supplier for raw material ($-COP)				
s	Gypsum	Slag	Clinker	Sacks
1	367	276	430	284
2	251	269	383	367

manufacturing costs of the raw material. The costs associated with detecting defective raw material at the supplier are higher in S2 since a greater quantity is handled. The costs of purchasing raw materials represent an important part of the costs. Manufacturing costs are also important within the total costs since the cement manufacturing process involves high energy consumption.

Figure 2 shows the benefits of the supply chain per month. The results show that suppliers participate in 47% of the total profits, plants receive 30%,distribution centers and retailers receive 23%. Distributors and retailers have a lower contribution. In addition to inventory costs(these assume the costs for defective or deterioration of the products), these results show the positive effects of the chain in a collaborative system and, if not, the total percentages of profits between retailers and distributors would be less than 10%.

The results of the model include the optimal values of the variables' production quantity of raw material to purchase $\left(Q^{S}_{s,m,t,e}\right)$ and to transport $\left(QT^{S}_{s,p,m,t,e}\right)$ from the suppliers to the plants and the total quantity to transport $\left(QTT^{S}_{s,p,t,e}\right)$. The production plantspresent thenumber of finished items to produce $\left(Q^{P}_{p,j,t,e}, QP^{P}_{p,q,j,t,e}, QPR^{P}_{p,q,j,t,e}, QPE^{P}_{p,q,j,t,e}, QT^{P}_{p,d,j,t,e}, QTT^{S}_{s,p,t,e}\right)$ andtransport from the plants to the distribution centers $\left(QT^{P}_{p,d,j,t,e}, QTT^{P}_{p,d,t,e}\right)$. Also, the quantitiesto transport from the distribution centers to the retailers $\left(QT^{D}_{d,r,j,t,e}, QTT^{D}_{d,r,t,e}\right)$, from retailers to clients $\left(QT^{R}_{r,c,j,t,e}, QTT^{R}_{r,c,t,e}\right)$, and the inventory levels $\left(I^{P-}_{p,j,t,e}, I^{D+}_{d,j,t,e}, I^{R+}_{r,j,t,e}, I^{R-}_{r,j,t,e}\right)$ are presented in Appendix E.

The optimal production quantity of finished products in each production plant is presented in Table 11 and the optimal values of total quantities to transport from suppliers, production plants, distribution centers, and retailers are presented in Tables 12–15.

Table 6 Parameters of production plants

$PV^{P}_{p,d,j}$ Selling price per unit good item at production plant ($-COP)

p	d	A	B
1	1	12,907	13,022
1	2	12,907	13,022
2	1	12,505	13,210
2	2	12,505	13,210

$PVD^{P}_{p,d,j}$ Selling price per unit defective item at production plant ($-COP)

p	d	A	B
1	1	9,680	9,766
1	2	9,680	9,766
2	1	9,449	9,684
2	2	9,449	9,684

$CPR^{P}_{p,q,j,t}$ Production cost per unit at the production plant for the finished product working in regular time ($-COP)

p	q	j	1	2	3	4	5	6
1	Q1	A	4,374	4,577	4,741	4,316	4,697	4,588
1	Q1	B	4,340	4,250	4,843	4,078	4,514	4,472
1	Q2	A	4,505	4,848	4,104	4,282	4,591	4,087
1	Q2	B	4,751	4,508	4,446	4,972	4,824	4,269
2	Q1	A	4,737	4,201	4,693	4,351	4,031	4,525
2	Q1	B	4,710	4,224	4,940	4,739	4,383	4,660
2	Q2	A	4,241	4,320	4,068	4,738	4,033	4,724
2	Q2	B	4,972	4,449	4,779	4,706	4,402	4,409

$CPE^{P}_{p,q,j,t}$ Production cost per unit at the production plant for the finished product working in extra time ($-COP)

p	q	j	1	2	3	4	5	6
1	Q1	A	14,374	14,577	4,741	4,316	4,697	4,588
1	Q1	B	14,340	14,250	4,843	4,078	4,514	4,472
1	Q2	A	14,505	14,848	4,104	4,282	4,591	4,087
1	Q2	B	14,751	14,508	4,446	4,972	4,824	4,269
2	Q1	A	14,737	14,201	4,693	4,351	4,031	4,525
2	Q1	B	14,710	4,224	4,940	4,739	4,383	4,660
2	Q2	A	14,241	4,320	4,068	4,738	4,033	4,724
2	Q2	B	14,972	4,449	4,779	4,706	4,402	4,409

$CSub^{P}_{p,j}$ Purchasing cost per unit subcontracted by the production plant ($-COP)

p	A	B
1	15,000	15,000
2	15,000	15,000

Table 6 continued

$H^{P}_{p,j}$ Handling cost per unit at production plant ($-COP)

p	A	B
1	397	397
2	338	338

$CT^{P}_{p,d,j}$ Transportation cost per unit for the finished product from production plant to distribution center ($-COP)

p	d	A	B
1	1	517	517
1	2	562	562
2	1	595	595
2	2	598	598

$\varphi^{+}_{p,j}$ Inventory excess cost at production plant for the finished product ($-COP)

p	A	B
1	444	444
2	482	482

$CD^{P}_{p,j}$ Disposal cost per unit defective item at production plan for the finished product ($-COP)

p	A	B
1	244	244
2	215	215

$\varphi^{-}_{p,j}$ Inventory shortage cost at production plant for the finished product ($-COP)

p	A	B
1	3,872	3,907
2	3,890	3,937

Managerial Implication

- In the operational context, this model presents a significant contribution to the management of the production system, as it supports decision-making related to the integration of the chain actors, the programming and control of production and distribution, efficient management of resources, and optimization of the level of customer service. Likewise, it guarantees the best use of capacities and optimizes inventory management in each of the chain's echelons, reducing the impact of the "bullwhip effect" on the chain.
- In the economic context, the model allows the maximization of collective benefits, considering the collaboration between the actors in the chain. There is a possibility to purchase of products with minor defects at lower price which not only allows the generation of adjacent income but also reduces the costs associated with transportation because of returns, and environmental consideration.

Table 7 Parameters of distribution centers

$PV^{D}_{d,r,j}$ Selling price per unit good item at distribution center to the retailer for the finished product ($-COP)

d	*r*	A	B
1	1	17,216	17,365
1	2	17,216	17,365
2	1	17,296	17,500
2	2	17,296	17,500

$CT^{D}_{d,r,j}$ Fixed transportation cost per unit from distribution center d to the retailer r for the finished product ($-COP)

d	*r*	A	B
1	1	314	314
1	2	338	338
2	1	315	315
2	2	342	342

$H^{D}_{d,j}$ Handling cost per unit at the distribution center for the finished product ($-COP)

d	A	B
1	22	22
2	23	23

$\gamma^{+}_{d,j}$ Excess inventory cost at distribution center for the finished product ($-COP)

d	A	B
1	69	69
2	69	69

$\gamma^{-}_{d,j}$ Shortage inventory cost at distribution center for the finished product ($-COP)

d	A	B
1	5,165	5,209
2	5,189	5,250

$CD^{D}_{d,j}$ Disposal cost per unit defective item at distribution center for the finished product ($-COP)

d	A	B
1	840	840
2	820	820

- Another fundamental aspect provided by the model is related to quality management by considering factors for detecting defects in the materials and products received, which guarantee customer satisfaction throughout the downstream chain.
- Finally, the model presents a good approximation of the behavior of demand in the real world.

Table 8 Parameters of retailers

$PV^R_{r,c,j}$ Selling price per unit good item at retailer to the client for the finished product ($-COP)

r	c	A	B
1	1	21,813	22,007
1	2	21,813	22,007
1	1	21,913	22,178
1	2	21,913	22,178

H^R_{rj} Handling cost per unit at the retailer for the finished product ($-COP)

r	A	B
1	13	13
2	13	13

$\gamma^+_{r,j}$ Excess inventory cost at retailer for the finished product ($-COP)

r	A	B
1	87	88
2	88	89

$\vartheta^-_{r,j}$ Shortage inventory cost at retailer for the finished product ($-COP)

r	j	A	B
1	A	6,544	6,602
2	B	6,574	6,653

Table 9 Total benefits in the supply chain by scenarios

	BT(r) ($ COP)	BT exp(r) ($ COP)
Low	5,146,132,792	1,715,375,882
Medium	5,048,779,282	1,682,924,744
High	4,532,596,643	1,510,864,037
		4,909,164,663

Table 10 Expected benefits in the supplier, plant, distribution center, and retailer

Scenario	S	BS(s) Million ($ COP)	P	BP(p) Million ($ COP)	D	BD(d) Million ($ COP)	R	BR(r) Million ($ COP)
Low	S1	1,063	P1	716.8	D1	222.4	R1	206.1
Medium	S1	1,151	P1	716.4	D1	228.6	R1	341.3
High	S1	1,151	P1	716.4	D1	228.6	R1	438.9
Low	S2	1,068	P2	704.9	D2	225.7	R2	324.7
Medium	S2	1,157	P2	744.4	D2	256.1	R2	453.2
High	S2	1,157	P2	744.4	D2	256.1	R2	452.9

Table 11 Optimal values of production quantities of finished products

$Q^{P}_{p,j,t,e}$ Production quantity of the finished product j at the plant p in period t (sacks)

p	j	e	1	2	3	4	5	6
1	A	High	21,120	7,751	11,080	15,962	9,187	
1	A	Medium	21,120	7,751	11,080	15,962	9,187	
1	A	Low	21,120	9,571	11,080	11,711	9,187	
1	B	High		2,811		6,768		
1	B	Medium		2,811		6,768		
1	B	Low		991		11,019		
2	A	High	5,755	216	10,589		9,620	
2	A	Medium	5,755	216	10,589		9,620	
2	A	Low	3,919	216	11,194		9,620	
2	B	High	10,522	22,018	605	5,960	12,714	
2	B	Medium	10,522	22,018	605	5,960	12,714	
2	B	Low	4,881	22,018		1,718	12,714	

Table 12 Optimal values of total quantities to transport from suppliers

$QTT^{S}_{s,p,t,e}$ Total quantity to transport from the supplier s tothe plant p in period t (ton)

s	p	e	1	2	3	4	5	6
1	1	High	1,098	549	576	1,182	478	
1	1	Medium	1,098	549	576	1,182	478	
1	1	Low	1,098	549	576	1,182	478	
1	2	High	846	1,156	582	310	1,161	
1	2	Medium	846	1,156	582	310	1,161	
1	2	Low	458	1,156	582	89	1,161	
2	1	High	1,098	549	576	1,182	478	
2	1	Medium	1,098	549	576	1,182	478	
2	1	Low	1,098	549	576	1,182	478	
2	2	High	846	1,156	582	310	1,161	
2	2	Medium	846	1,156	582	310	1,161	
2	2	Low	458	1,156	582	89	1,161	

Discussion and Conclusions

The mathematical model has been developed under the approach of collaboration and uncertainty in the demand in a supply chain of the cement sector which is proved to be a valuable tool for making decisions towards the maximization of profits in all actors of the supply chain. The proposed model considers critical decision variables, and it isused in investigations such as defective products, excesses, and shortages. Validating the model in a real scenario, common benefit exceeds 5 billion for all members in a period of six months.

The present model identifies the benefits of collaborative planning [68] and it allows the reduction in cycle times, greater flexibility in the processes associate with orders and deliveries, and the decrease in inventory levels for maximizing the profit of channel members.

Table 13 Optimal values of total quantities to transport from plants

$QTT^{P}_{p,d,t,e}$ Total quantity to transport from the plant p to the distribution center d in period t (sacks)								
p	d	e	1	2	3	4	5	6
1	A	High	10,453	5,562	10,026	14,174	9,000	10,090
1	A	Medium	10,453	5,562	10,026	14,174	9,000	10,090
1	A	Low	10,453	7,270	10,026	14,174	9,000	10,090
1	B	High	11,116	4,876	921	8,290	76.62	11,290
1	B	Medium	11,116	4,876	921	8,290	76.62	11,290
1	B	Low	11,116	3,166	921	8,466	76.62	11,290
2	A	High	7,325	10,910	400		6,200	10,910
2	A	Medium	7,325	10,910	400		6,200	10,910
2	A	Low	4,177	10,910	172		6,200	10,910
2	B	High	9,264	11,124	10,683	5,906	15,923	10,910
2	B	Medium	9,264	11,124	10,683	5,906	15,923	10,910
2	B	Low	5,004	11,124	10,910	1,703	15,923	10,910

Table 14 Optimal values of total quantities to transport from distribution centers

$QTT^{D}_{d,r,t,e}$ Total quantity to transport from the distribution center d to the retailer r in period t (sacks)								
d	r	e	1	2	3	4	5	6
1	1	High	6,800	6,800	6,800	6,800	6,800	6,800
1	1	Medium	6,800	6,800	6,800	6,800	6,800	6,800
1	1	Low	5,470	6,690	6,400	6,800	6,800	6,800
1	2	High	8,400	8,400	8,400	8,400	8,400	8,400
1	2	Medium	8,400	8,400	8,400	8,400	8,400	8,400
1	2	Low	8,400	8,400	8,400	8,400	8,400	8,400
2	1	High	8,200	8,200	8,200	8,200	8,200	8,200
2	1	Medium	8,200	8,200	8,200	8,200	8,200	8,200
2	1	Low	4,400	7,690	6,800	4,600	8,200	8,200
2	2	High	7,800	7,800	7,800	7,800	7,800	7,800
2	2	Medium	7,800	7,800	7,800	7,800	7,800	7,800
2	2	Low	7,340	7,800	7,800	7,800	7,800	7,800

Additionally, the model allows the flexibility of the processes as indicated by Binder and Clegg [7]. If resources are required, and when these are not available, proper planning will support the exchange of resources between agents in the supply chain. The approach of the model that allows validating the defective products within the chain, helps to improve the quality within the processes as mentioned by Sarkar et al. [58]. In this model, the collaboration between suppliers and customers will help to reduce poor quality costs and thus efficiently impact customer service. Finally, product inventories are kept in their economic balance as stated by Huiskonen [29]. As a result, collaborative planning improves the availability of products for customers.

Table 15 Optimal values of total quantities to transport from retailers

$QTT^{R}_{r,c,t,e}$ Total quantity to transport from the retailer r to the client c in period t (Sacks)								
r	c	e	1	2	3	4	5	6
1	1	High	8,000	200		3,100	2,600	6,800
1	1	Medium	10,740	2,740		2,600	1,600	6,800
1	1	Low	13,800	6,600	2,600	3,600	7,400	9,200
1	2	High	7,450	14,800	15,000	12,700	13,200	9,000
1	2	Medium	5,060	13,060	15,800	11,200	11,200	9,000
1	2	Low	1,500	2,400	9,200	6,200	5,200	5,800
2	1	High	14,650	2,600	10,400	13,100	13,400	14,000
2	1	Medium	8,500	5,860	15,000	15,400	13,400	16,200
2	1	Low	7,200	4,200	16,200	13,400	8,600	12,800
2	2	High	1,550	13,600	5,800	3,100	2,800	2,200
2	2	Medium	7,700	10,340	1,200	800	2,800	
2	2	Low	9,000	12,000		2,800	4,800	

In addition to the previously mentioned benefits, the proposed collaborative model provides a competitive advantage focused on generating transparency in production processes, reducing response times, and minimizing potential conflicts between chain actors as established by Sarkar et al. [59]. Optimization shares the benefits, giving importance to all the actors in the chain, and generating confidence so that everyone cooperates as indicated by Zhang and Huang [73]. Finally, among other novelty, the reverse flow of defective products is studied in the proposed model. Another novelty of this proposed model is related to the objectives of a sustainable chain that helps to reduce the carbon footprint [42] and additionally integrates concepts of reverse and green logistics for the use of raw materials.

The proposed model can be expanded in future research, including the production safety inventory. The model can incorporate the features of uncertainty so that the design is completely stochastic based on different policies of collaboration and integration in the chain. It is possible to combine the collaborative approach, grouping the actors, and mixing the type of collaboration (information, capacity, inventories, among others). A model can be proposed that includes Shapley Value, where agents are part of the chain who collaborate and form coalitions in such a way that costs are minimized, and better profits are obtained.

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Compliance with ethical standards

Conflict of interest The authors do hereby declare that there is no conflict of interests of other works regarding the publication of this paper. This article does not contain any studies with human participants or animals performed by any of the authors and materials of third party are not used in this article.

Appendix A: Parameters values of production and transportation capacities, inventory levels, and defective items of suppliers

$Io^{S}_{s,m}$ Initial inventory level at supplier for the raw material (ton)

s	Gypsum	Slag	Clinker	Sacks
1	20	135	561	20
2	30	100	528	12

$CapT^{S}_{s,p,m}$ Transport capacity for the raw material from the supplier to the production plant (ton)

s	p	Gypsum	Slag	Clinker	Sacks
1	1	140,000	135,000	173,500	112,880
1	2	140,000	135,000	173,500	112,880
2	1	138,000	134,000	172,000	111,019
2	2	138,000	134,000	172,000	111,019

$Imax^{S}_{s}$ Maximum inventory capacity at the supplier (ton)

s	
1	380,186
2	370,759

$Fmax^{S}_{s,m}$ Maximum production capacity at the supplier for the raw material (ton)

s	Gypsum	Slag	Clinker	Sacks
1	50,000	55,000	53,500	22,880
2	48,000	44,000	82,000	21,019

$\alpha^{S}_{s,m}$ Expected percentage of defective items at supplier for the raw material

s	Gypsum	Slag	Clinker	Sacks
1	0.016	0.013	0.011	0.018
2	0.015	0.017	0.015	0.017

$\beta_{s,m}^{S}$ Screening rate of defective items at supplierfor the raw material				
s	Gypsum	Slag	Clinker	Sacks
1	0.98	0.99	0.99	0.97
2	0.98	0.99	0.99	0.96

$TT_{s,p}^{S}$ Transportation time from supplier to production plant (hours)				
s	Gypsum	Slag	Clinker	Sacks
1	0.98	0.99	0.99	0.97
2	0.98	0.99	0.99	0.96

Appendix B. Parameters values of production and transportation capacities, inventory levels, and defective items of production plants

$Io_{p,j}^{P}$ Initial inventory level at production plant for the finished product (sacks)		
p	A	B
1	275	427
2	239	226

$Imax_{p}^{P}$ Maximum inventory capacity at the production plant (sacks)	
p	
1	89,580
2	89,200

$CapT_{p,d,j}^{P}$ Transport capacity for the finished product from production plant to distribution center (ton)			
P	*d*	A	B
1	1	10,026	10,926
1	2	19,026	10,926
2	1	10,910	10,910
2	2	10,910	10,910

$CapPR^{P}_{p,q,t}$ Maximum production capacity at production plant working in regular time on the production resource in period time (Sacks)

p	q	1	2	3	4	5	6
1	Q1	10,547	10,562	10,512	10,684	10,563	50,522
1	Q2	10,573	10,559	10,540	10,681	10,547	10,554
2	Q1	10,543	10,563	10,565	10,688	10,591	10,592
2	Q2	10,537	10,554	10,597	10,669	10,576	10,524

$CapPE^{P}_{p,q,t}$ Maximum production capacity at production plant working in extra time on the production resource in period time (Sacks)

p	q	1	2	3	4	5	6
1	Q1	547	562	512	684	563	522
1	Q2	573	559	540	681	547	554
2	Q1	543	563	565	688	591	592
2	Q2	537	554	597	669	576	524

$CapTI^{P}_{p}$ Maximum capacity of transport input in the plant (ton)

p	
1	49,580
2	49,200

$CapTO^{P}_{p}$ Maximum capacity of transport output in the plant (sacks)

p	
1	48,643
2	42,249

$\mu^{P}_{p,j}$ Screening rate of defective items at production plant for the finished product

p	A	B
1	0.99	0.98
2	0.97	0.99

$\alpha^{P}_{p,j}$ Expected percentage of defective items at production plant for the finished product

p	A	B
1	0.012	0.011
2	0.010	0.009

$\beta^{P}_{p.m}$ Screening rate of defective items at production plant for the raw material

p	Gypsum	Slag	Clinker	Sacks
1	0.99	0.97	0.99	0.98
2	0.98	0.98	0.97	0.97

$MA^{P}_{m,j}$ Combination of raw material necessary to produce the finished product (sack)

p	Gypsum	Slag	Clinker	Sacks
1	0.0025	0.0095	0.038	0.002
2	0.0025	0.007	0.0405	0.002

$Sub^{P}_{p,j}$ Expected percentage of quantities to manufacture the finished product for subcontracting at production plant

p	A	B
1	0.1	0.1
2	0.1	0.1

$TT^{P}_{p,d}$ Transportation time from the production plant to the distribution center (Hours)

p	1	2
1	0	0
2	0	0

Appendix C. Parameters values of production and transportation capacities, inventory levels, and defective items of distribution centers

$Io^{D}_{d,j}$ Initial inventory level at distribution center for the finished product (sacks)

d	1	2
1	1,250	700
2	1,000	820

$Imax_d^D$ Maximum inventory capacity at distribution center (sacks)	
d	
1	5,800
2	6,200

$CapTO_d^D$ Maximum capacity of transport output in the distribution center (sacks)	
d	
1	15,800
2	16,200

$\delta_{d,j}^D$ Screening-rate of defective items at distribution center for finished product		
d	A	B
1	0.99	0.97
2	0.98	0.99

$TT_{d,r}^D$ Transportation time from the distribution center to the retailer (sacks)		
d	1	2
1	0	0
2	0	0

Appendix D. Parameters values of production and transportation capacities, inventory levels, and defective items of retailers

$Io_{r,j}^R$ Initial inventory level at retailer for the finished product (tons)		
d	A	B
1	200	250
2	220	240

$Imax_r^R$ Maximum inventory capacity at retailer (sacks)	
r	
1	15,800
2	16,200

$CapTO_r^R$ Maximum capacity of transport input in the retailer (sacks)	
r	
1	15,800
2	16,200

Appendix E. Optimal values of decision variables

$Q^S_{s,m,t,e}$ Quantity of raw material m to purchase from the supplier s in period t (tons)

s	m	e	1	2	3	4	5
1	Gypsum	High	75	83	57	73	80
1	Gypsum	Medium	75	83	57	73	80
1	Gypsum	Low	56	83	57	62	80
1	Slag	High	197	253	213	244	271
1	Slag	Medium	197	253	213	244	271
1	Slag	Low	139	257	214	203	271
1	Clinker	High	896	1,323	857	1,135	1,243
1	Clinker	Medium	896	1,323	857	1,135	1,243
1	Clinker	Low	595	1,318	856	972	1,243
1	Sacks	High	56	67	45	58	64
1	Sacks	Medium	56	67	45	58	64
1	Sacks	Low	41	67	45	50	64
2	Gypsum	High	64	83	57	73	80
2	Gypsum	Medium	64	83	57	73	80
2	Gypsum	Low	45	83	57	62	80
2	Slag	High	233	254	214	215	272
2	Slag	Medium	233	254	214	215	272
2	Slag	Low	175	258	215	204	272
2	Clinker	High	933	1,328	861	1,139	1,248
2	Clinker	Medium	933	1,328	861	1,139	1,248
2	Clinker	Low	631	1,324	859	976	1,248
2	Sacks	High	64	67	45	58	64
2	Sacks	Medium	64	67	45	58	64
2	Sacks	Low	49	67	45	50	64

$QT^{S}_{s.p.m.t.e}$ Quantity of raw material m to transport from the supplier s to the plant p in period t (Tons)								
s	p	m	e	1	2	3	4	5
1	1	Gypsum	High	53	26	28	57	23
1	1	Gypsum	Medium	53	26	28	57	23
1	1	Gypsum	Low	53	26	28	57	23
1	1	Slag	High	201	93	105	199	87
1	1	Slag	Medium	201	93	105	199	87
1	1	Slag	Low	201	98	105	188	87
1	1	Clinker	High	803	408	421	881	349
1	1	Clinker	Medium	803	408	421	881	349
1	1	Clinker	Low	803	404	421	891	349
1	1	Sacks	High	42	21	22	45	18
1	1	Sacks	Medium	42	21	22	45	18
1	1	Sacks	Low	42	21	22	45	18
1	2	Gypsum	High	40.69	55.59	27.99	14.90	55.84
1	2	Gypsum	Medium	40.69	55.59	27.99	14.90	55.84
1	2	Gypsum	Low	22.00	55.59	27.99	4.30	55.84
1	2	Slag	High	128.33	156.18	104.83	41.72	180.39
1	2	Slag	Medium	128.33	156.18	104.83	41.72	180.39
1	2	Slag	Low	71.39	156.18	106.34	12.03	180.39
1	2	Clinker	High	644.83	899.94	426.89	241.38	880.48
1	2	Clinker	Medium	644.83	899.94	426.89	241.38	880.48
1	2	Clinker	Low	346.59	899.94	425.37	69.58	880.48
1	2	Sacks	High	32.55	44.47	22.39	11.92	44.67
1	2	Sacks	Medium	32.55	44.47	22.39	11.92	44.67
1	2	Sacks	Low	17.60	44.47	22.39	3.44	44.67
2	1	Gypsum	High	52.80	26.41	27.70	56.83	22.97
2	1	Gypsum	Medium	52.80	26.41	27.70	56.83	22.97
2	1	Gypsum	Low	52.80	26.41	27.70	56.83	22.97
2	1	Slag	High	200.64	93.31	105.26	199.02	87.28
2	1	Slag	Medium	200.64	93.31	105.26	199.02	87.28
2	1	Slag	Low	200.64	97.86	105.26	188.39	87.28
2	1	Clinker	High	802.56	408.38	421.04	880.66	349.10
2	1	Clinker	Medium	802.56	408.38	421.04	880.66	349.10
2	1	Clinker	Low	802.56	403.83	421.04	891.29	349.10
2	1	Sacks	High	42.24	21.12	22.16	45.46	18.37
2	1	Sacks	Medium	42.24	21.12	22.16	45.46	18.37
2	1	Sacks	Low	42.24	21.12	22.16	45.46	18.37
2	2	Gypsum	High	41	56	28	15	56
2	2	Gypsum	Medium	41	56	28	15	56
2	2	Gypsum	Low	22	56	28	4	56

$QT^{S}_{s.p.m.t.e}$ Quantity of raw material m to transport from the supplier s to the plant p in period t (Tons)

s	p	m	e	1	2	3	4	5
2	2	Slag	High	128	156	105	42	180
2	2	Slag	Medium	128	156	105	42	180
2	2	Slag	Low	71	156	106	12	180
2	2	Clinker	High	645	900	427	241	880
2	2	Clinker	Medium	645	900	427	241	880
2	2	Clinker	Low	347	900	425	70	880
2	2	Sacks	High	33	44	22	12	45
2	2	Sacks	Medium	33	44	22	12	45
2	2	Sacks	Low	18	44	22	3	45

$QP^{P}_{p.q.j.t.e}$ Quantity of product j to be produced in plant p on production resource q in period t (sacks)

p	q	j	e	1	2	3	4	5
1	Q1	A	High	10,547	7,751		4,600	
1	Q1	A	Medium	10,547	7,751		4,600	
1	Q1	A	Low	10,547	9,571		349,437	
1	Q1	B	High		2,811		6768	
1	Q1	B	Medium		2,811		6768	
1	Q1	B	Low		991		11,019	
1	Q2	A	High	10,573		11,080	11,362	9,187
1	Q2	A	Medium	10,573		11,080	11,362	9,187
1	Q2	A	Low	10,573		11,080	11,362	9,187
1	Q2	B	High					
1	Q2	B	Medium					
1	Q2	B	Low					
2	Q1	A	High					
2	Q1	A	Medium					
2	Q1	A	Low					
2	Q1	B	High	10,522	11,126			11,182
2	Q1	B	Medium	10,522	11,126			11,182
2	Q1	B	Low	4,881	11,126			11,182
2	Q2	A	High	5,755	216	10,589		9,620
2	Q2	A	Medium	5,755	216	10,589		9,620
2	Q2	A	Low	3,919	216	11,194		9,620
2	Q2	B	High		10,892	605	5,960	1,532
2	Q2	B	Medium		10,892	605	5,960	1,532
2	Q2	B	Low		10,892		1718	1,532

$QPR^{P}_{p.q.j.t.e}$ Production quantity of the finished product j at the plant p working in regular time on the production resource q in period t (sacks)

p	q	j	e	1	2	3	4	5
1	Q1	A	High	10,547	7.751		3,916	
1	Q1	A	Medium	10,547	7.751		4,600	
1	Q1	A	Low	10,547	9,571			
1	Q1	B	High		2,811		6768	
1	Q1	B	Medium		2,811		6084	
1	Q1	B	Low		991		10,684	
1	Q2	A	High	10,573		10,540	10,681	8,640
1	Q2	A	Medium	10,573		10,540	10,681	8,640
1	Q2	A	Low	10,573		10,540	10,681	8,640
1	Q2	B	High					
1	Q2	B	Medium					
1	Q2	B	Low					
2	Q1	A	High					
2	Q1	A	Medium					
2	Q1	A	Low					
2	Q1	B	High	10,522	10,563			10,591
2	Q1	B	Medium	10,522	10,563			10,591
2	Q1	B	Low	4,881	10,563			10,591
2	Q2	A	High	5,755		10,589		9,620
2	Q2	A	Medium	5,755		10,589		9,620
2	Q2	A	Low	3,919	216	10,597		9620
2	Q2	B	High		10,554	8,449	5,291	957
2	Q2	B	Medium		10,554	8,449	5,291	957
2	Q2	B	Low		10,338		1,718	957

$QPE^{P}_{p,q,j,t,e}$ Production quantity of the finished product j at the plant p working in extra time on the production resource q in period t (sacks)

p	q	j	e	1	2	3	4	5
1	Q1	A	High				684	
1	Q1	A	Medium					
1	Q1	A	Low				350	
1	Q1	B	High					
1	Q1	B	Medium				684	
1	Q1	B	Low				335	
1	Q2	A	High			540	681	547
1	Q2	A	Medium			540	681	547
1	Q2	A	Low			540	681	547
1	Q2	B	High					
1	Q2	B	Medium					
1	Q2	B	Low					
2	Q1	A	High					
2	Q1	A	Medium					
2	Q1	A	Low					
2	Q1	B	High		563			591
2	Q1	B	Medium		563			591
2	Q1	B	Low		563			591
2	Q2	A	High		215.83			
2	Q2	A	Medium		215.83			
2	Q2	A	Low			597		
2	Q2	B	High		338	597	669	576
2	Q2	B	Medium		338	597		576
2	Q2	B	Low		554			576

$QT^{P}_{p,d,j,t,e}$ Quantity of product j to transport from the plant p to the distribution center d in period t (Sacks)									
p	d	j	e	1	2	3	4	5	6
1	1	A	High	10,026	4,472	10,026	7,974	9,000	9,000
1	1	A	Medium	10,026	4,472	10,026	7,974	9,000	9,000
1	1	A	Low	10,026	6,290	10,026	7,802	9,000	9,000
1	1	B	High	427	1,090		6,200		1,090
1	1	B	Medium	427	1,090		6,200		1,090
1	1	B	Low	427	980		6,200		1,090
1	2	A	High	11,116	3,186	921	7,796	77	9,600
1	2	A	Medium	11,116	3,186	921	7,796	77	9,600
1	2	A	Low	11,116	3,186	921	3,769	77	9,600
1	2	B	High		1,690		494		1,690
1	2	B	Medium		1,690		494		1,690
1	2	B	Low				4,698		1,690
2	1	A	High	2,253					
2	1	A	Medium	2,253					
2	1	A	Low	434		172			
2	1	B	High	5,073	10,910	400		6,200	10,910
2	1	B	Medium	5,073	10,910	400		6,200	10,910
2	1	B	Low	3,743	10,910			6,200	10,910
2	2	A	High	3,684	214	10,483		9,523	
2	2	A	Medium	3,684	214	10,483		9,523	
2	2	A	Low	3,684	214	10,910		9,523	
2	2	B	High	5,580	10,910	200	5,906	6,400	10,910
2	2	B	Medium	5,580	10,910	200	5,906	6,400	10,910
2	2	B	Low	1,320	10,910		1,703	6,400	10,910

$QT^{D}_{d,r,j,t,e}$ Quantity of product j to transport from the distribution center d to the retailer r in period t (Sacks)

d	r	j	e		2	3	4	5	6
1	1	A	High	3,400	3,400	3,400	3,400	3,400	3,400
1	1	A	Medium	3,400	3,400	3,400	3,400	3,400	3,400
1	1	A	Low	3,400	3,400	3,400	3,400	3,400	3,400
1	1	B	High	3,400	3,400	3,400	3,400	3,400	3,400
1	1	B	Medium	3,400	3,400	3,400	3,400	3,400	3,400
1	1	B	Low	2,070	3,290	3,000	3,400	3,400	3,400
1	2	A	High	5,600	5,600	5,600	5,600	5,600	5,600
1	2	A	Medium	5,600	5,600	5,600	5,600	5,600	5,600
1	2	A	Low	5,600	5,600	5,600	5,600	5,600	5,600
1	2	B	High	2,800	2,800	2,800	2,800	2,800	2,800
1	2	B	Medium	2,800	2,800	2,800	2,800	2,800	2,800
1	2	B	Low	2,800	2,800	2,800	2,800	2,800	2,800
2	1	A	High	4,400	4,400	4,400	4,400	4,400	4,400
2	1	A	Medium	4,400	4,400	4,400	4,400	4,400	4,400
2	1	A	Low	4,400	4,380	4,400	800	4,400	4,400
2	1	B	High	3,800	3,800	3,800	3,800	3,800	3,800
2	1	B	Medium	3,800	3,800	3,800	3,800	3,800	3,800
2	1	B	Low		3,310	2,400	3,800	3,800	3,800
2	2	A	High	5,200	5,200	5,200	5,200	5,200	5,200
2	2	A	Medium	5,200	5,200	5,200	5,200	5,200	5,200
2	2	A	Low	5,200	5,200	5,200	5,200	5,200	5,200
2	2	B	High	2,600	2,600	2,600	2,600	2,600	2,600
2	2	B	Medium	2,600	2,600	2,600	2,600	2,600	2,600
2	2	B	Low	2,140	2,600	2,600	2,600	2,600	2,600

$QT^{R}_{r,c,j,t,e}$ Quantity of product j to transport from the retailer r to the client c in period t (sacks)

r	c	j	e	1	2	3	4	5	6
1	1	A	High	8,000	200				
1	1	A	Medium	10,740	1,000				2,740
1	1	A	Low	12,980		2,200			7,400
1	1	B	High				3,100	2,600	6,800
1	1	B	Medium		1,740		2,600	1,600	4,060
1	1	B	Low	820	6,600	400	3,600	7,400	1,800
1	2	A	High		7,600	7,800	7,900	8,200	9,000
1	2	A	Medium		4,060	7,400	8,200	7,200	8,000
1	2	A	Low		2,400	6,000	4,200	3,200	5000
1	2	B	High	7,450	7,200	7,200	4,800	5,000	
1	2	B	Medium	5,060	9,000	8,400	3,000	4,000	1,000
1	2	B	Low	1,500		3,200	2,000	2,000	800
2	1	A	High	10,560		8,600	7,700	8,000	9,800
2	1	A	Medium	6,260		10,200	10,000	8,000	11,260
2	1	A	Low	2,020	800	10,800	8,000	6,000	5,600
2	1	B	High	4,090	2,600	1,800	5,400	5,400	4,200
2	1	B	Medium	2,240	5,860	4,800	5,400	5,400	4,940
2	1	B	Low	5,180	3,400	5,400	5,400	2,600	7,200
2	2	A	High		10,800	2,200	3,100	2,800	1,000
2	2	A	Medium	4,760	10,340	600	800	2,800	
2	2	A	Low	9,000	10,000		2,800	4,800	
2	2	B	High	1,550	2,800	3,600			1,200
2	2	B	Medium	2,940		600			
2	2	B	Low		2,000				

$I^{P-}_{p,j,t,e}$ Shortage inventory level of the finished product j at plant p in period t (Sacks)

p	j	e	1	2	3	4	5	6
1	A	High						18,600
1	A	Medium						18,600
1	A	Low						18,600
1	B	High						2,780
1	B	Medium						2,780
1	B	Low						2,780
2	A	High						
2	A	Medium						
2	A	Low						
2	B	High						21,820
2	B	Medium						21,820
2	B	Low						21,820

$I^{P-}_{p,j,t,e}$ Shortage inventory level of the finished product j at plant p in period t (Sacks)

p	j	e	1	2	3	4	5	6
1	A	High						18,600
1	A	Medium						18,600
1	A	Low						18,600
1	B	High						2,780
1	B	Medium						2,780
1	B	Low						2,780
2	A	High						
2	A	Medium						
2	A	Low						
2	B	High						21,820
2	B	Medium						21,820
2	B	Low						21,820

$I^{D+}_{d,j,t,e}$ Excess Inventory level of the finished product j at the distribution center d in period t (sacks)

d	j	e	1	2	3	4	5	6
1	A	High	4,528		1,026			
1	A	Medium	4,528		1,026			
1	A	Low	2,710		1,198			
1	B	High		5,800				5,800
1	B	Medium		5,800				5,800
1	B	Low		5,800				5,800
2	A	High	6,200		1,804			
2	A	Medium	6,200		1,804			
2	A	Low	6,200		2,231			
2	B	High		6,200				6,200
2	B	Medium		6,200				6,200
2	B	Low		5,000				6,200

$I^{R+}_{r,j,t,e}$ Excess inventory level of the finished product j at retailer r in period t (Sacks)

r	*j*	*e*	1	2	3	4	5	6
1	A	High						
1	A	Medium			400		600	
1	A	Low		400			4,600	
1	B	High						
1	B	Medium	2390				850	2,990
1	B	Low			2,390	3,400	1,200	5,800
2	A	High	460	460	460	460	460	460
2	A	Medium		460	460	460	460	
2	A	Low						5,200
2	B	High						
2	B	Medium	460					460
2	B	Low					2,800	1,000

$I^{R-}_{r,j,t,e}$ Shortage inventory level of the finished product j at the retailer r in period t (Sacks)

r	*j*	*e*	1	2	3	4	5	6
1	A	High				100	500	1,700
1	A	Medium	2,740					2,340
1	A	Low	4,980					
1	B	High				700	1,100	700
1	B	Medium		1,150	2350	750		
1	B	Low						
2	A	High						
2	A	Medium						
2	A	Low						
2	B	High						
2	B	Medium						
2	B	Low						

References

1. Angerhofer, B.J., Angelides, M.C.: A model and a performance measurement system for collaborative supply chains. Decis. Support Syst. **42**(1), 283–301 (2006)
2. Arns, M., Fischer, M., Kemper, P., Tepper, C.: Supply chain modelling and its analytical evaluation. J. Oper. Res. Soc. **53**(8), 885–894 (2002)
3. Aviv, Y.: The effect of collaborative forecasting on supply chain performance. Manag. Sci. **47**(10), 1326–1343 (2001)
4. Aviv, Y.: Gaining benefits from joint forecasting and replenishment processes: the case of auto-correlated demand. Manuf. Serv. Oper. Manag. **4**(1), 55–74 (2002)
5. Aviv, Y.: On the benefits of collaborative forecasting partnerships between retailers and manufacturers. Manag. Sci. **53**(5), 777–794 (2007)

6. Barratt, M.: Understanding the meaning of collaboration in the supply chain. Supply Chain Manag. Int. J. **9**(1), 30–42 (2004)
7. Binder, M., Clegg, B.: Enterprise management: a new frontier for organisations. Int. J. Prod. Econ. **106**(2), 409–430 (2007)
8. Cárdenas, M., Reina, M.: La minería en Colombia: impacto socioeconómico y fiscal (2008)
9. Chan, F.T., Zhang, T.: The impact of collaborative transportation management on supply chain performance: a simulation approach. Expert Syst. Appl. **38**(3), 2319–2329 (2011)
10. Chedid, J.A., Vidal, G.H.: Análisis del Problema de Planificación de la Producción en Cadenas de Suministro Colaborativas: Una Revisión de la Literatura en el Enfoque de Teoría de Juegos (2012)
11. Chen, C.-L., Lee, W.-C.: Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices. Comput. Chem. Eng. **28**(6–7), 1131–1144 (2004)
12. Chen, T.-H., Chen, J.-M.: Optimizing supply chain collaboration based on joint replenishment and channel coordination. Transp. Res. Part E Logist. Transp. Rev. **41**(4), 261–285 (2005)
13. Cheung, C.F., Cheung, C., Kwok, S.: A knowledge-based customization system for supply chain integration. Expert Syst. Appl. **39**(4), 3906–3924 (2012)
14. Chopra, S., Meindl, P.: Supply Chain Management. Strategy, Planning & Operation, pp. 265–275. Springer, Berlin (2007)
15. de Kruijff, J.T., Hurkens, C.A., de Kok, T.G.: Integer programming models for mid-term production planning for high-tech low-volume supply chains. Eur. J. Oper. Res. **269**(3), 984–997 (2018)
16. Diabat, A., Al-Salem, M.: An integrated supply chain problem with environmental considerations. Int. J. Prod. Econ. **164**, 330–338 (2015)
17. Dobos, I., Pintér, M.: Cooperation in supply chains: a cooperative game theoretic analysis. Working paper. Corvinus University of Budapest (2010)
18. Doukidis, G.I., Chang, T.H., Fu, H.P., Lee, W.I., Lin, Y., Hsueh, H.C.: A study of an augmented CPFR model for the 3C retail industry. Supply Chain Manag. Int. J. (2007)
19. Ellram, L.M.: Supply-chain management: the industrial organisation perspective. Int. J. Phys. Distrib. Logist. Manag. **21**(1), 13–22 (1991)
20. Entezaminia, A., Heydari, M., Rahmani, D.: A multi-objective model for multi-product multi-site aggregate production planning in a green supply chain: considering collection and recycling centers. J. Manuf. Syst. **40**, 63–75 (2016)
21. Esmaeili, M., Aryanezhad, M.-B., Zeephongsekul, P.: A game theory approach in seller–buyer supply chain. Eur. J. Oper. Res. **195**(2), 442–448 (2009)
22. Fu, Y., Piplani, R.: Supply-side collaboration and its value in supply chains. Eur. J. Oper. Res. **152**(1), 281–288 (2004)
23. Golicic, S.L., Foggin, J.H., Mentzer, J.T.: Relationship magnitude and its role in interorganizational relationship structure. J. Bus. Logist. **24**(1), 57–75 (2003)
24. Han, J.-H., Lee, J.-Y., Kim, Y.-D.: Production planning in a two-level supply chain for production-time-dependent products with dynamic demands. Comput. Ind. Eng. **135**, 1–9 (2019)
25. Hennet, J.-C., Arda, Y.: Supply chain coordination: a game-theory approach. Eng. Appl. Artif. Intell. **21**(3), 399–405 (2008)
26. Holweg, M., Disney, S., Holmström, J., Småros, J.: Supply chain collaboration: making sense of the strategy continuum. Eur. Manag. J. **23**(2), 170–181 (2005)
27. Huang, Y., Huang, G.Q., Newman, S.T.: Coordinating pricing and inventory decisions in a multi-level supply chain: a game-theoretic approach. Transp. Res. Part E Logist. Transp. Rev. **47**(2), 115–129 (2011)
28. Huang, Z., Li, S.X.: Co-op advertising models in manufacturer–retailer supply chains: a game theory approach. Eur. J. Oper. Res. **135**(3), 527–544 (2001)
29. Huiskonen, J.: Maintenance spare parts logistics: special characteristics and strategic choices. Int. J. Prod. Econ. **71**(1–3), 125–133 (2001)
30. Jena, S.K., Sarmah, S.: Price competition and co-operation in a duopoly closed-loop supply chain. Int. J. Prod. Econ. **156**, 346–360 (2014)
31. John, T.,DeWitt, W., Keebler, J.S., Min, S., Nix, N., Smith, C., Zacharia, Z.: Defining supply chain management. J. Bus. Logist. (2001)
32. Kim, B., Oh, H.: The impact of decision-making sharing between supplier and manufacturer on their collaboration performance. Supply Chain Manag. Int. J. (2005)
33. Kwon, O., Im, G.P., Lee, K.C.: MACE-SCM: a multi-agent and case-based reasoning collaboration mechanism for supply chain management under supply and demand uncertainties. Expert Syst. Appl. **33**(3), 690–705 (2007)
34. Leng, M., Parlar, M.: Allocation of cost savings in a three-level supply chain with demand information sharing: a cooperative-game approach. Oper. Res. **57**(1), 200–213 (2009)

35. Leng, M., Parlar, M.: Game-theoretic analyses of decentralized assembly supply chains: non-cooperative equilibria vs. coordination with cost-sharing contracts. Eur. J. Oper. Res. **204**(1), 96–104 (2010)
36. Li, J., Wang, S., Cheng, T.E.: Competition and cooperation in a single-retailer two-supplier supply chain with supply disruption. Int. J. Prod. Econ. **124**(1), 137–150 (2010)
37. Long, Q.: A multi-methodological collaborative simulation for inter-organizational supply chain networks. Knowl.-Based Syst. **96**, 84–95 (2016)
38. Manthou, V., Vlachopoulou, M., Folinas, D.: Virtual e-Chain (VeC) model for supply chain collaboration. Int. J. Prod. Econ. **87**(3), 241–250 (2004)
39. Mena O'meara, N. A. (2010). Planificación Maestra de operaciones en la Gestión de la Cadena de Suministro en contexto de INcertidumbre en el Sector Cerámico. Propuesta de Modelado y Resolución Basada en Redes Neuronales Artificiales (ANN).
40. Mentzer, J.T., Foggin, J.H., Golicic, S.L.: Collaboration: the enablers, impediments, and benefits. Supply Chain Manag. Rev. **4**(4), 52–58 (2000)
41. Mishra, A.A., Shah, R.: In union lies strength: collaborative competence in new product development and its performance effects. J. Oper. Manag. **27**(4), 324–338 (2009)
42. Mishra, U., Wu, J.-Z., Sarkar, B.: A sustainable production-inventory model for a controllable carbon emissions rate under shortages. J. Clean. Prod. **256**, 120268 (2020)
43. Mohammaditabar, D., Ghodsypour, S.H., Hafezalkotob, A.: A game theoretic analysis in capacity-constrained supplier-selection and cooperation by considering the total supply chain inventory costs. Int. J. Prod. Econ. **181**, 87–97 (2016)
44. Nagarajan, M., Sošić, G.: Game-theoretic analysis of cooperation among supply chain agents: review and extensions. Eur. J. Oper. Res. **187**(3), 719–745 (2008)
45. Navarro, K.S., Chedid, J.A., Florez, W.F., Mateus, H.O., Cárdenas-Barrón, L.E., Sana, S.S.: A collaborative EPQ inventory model for a three-echelon supply chain with multiple products considering the effect of marketing effort on demand. J. Ind.Manag. Optim. **16**, 272–283 (2019)
46. Nyaga, G.N., Whipple, J.M., Lynch, D.F.: Examining supply chain relationships: do buyer and supplier perspectives on collaborative relationships differ? J. Oper. Manag. **28**(2), 101–114 (2010)
47. Ospina-Mateus, H., Montero-Perez, J., Acevedo-Chedid, J., Salas-Navarro, K., Morales-Londoño, N.: A mathematical model for the optimization of the non-metallic mining supply chain in the mining district of Calamarí-Sucre (Colombia). Applied Computer Sciences in Engineering, Cham, Springer International Publishing (2020)
48. Pimentel, B.S., Mateus, G.R., Almeida, F.A.: Mathematical models for optimizing the global mining supply chain. In: Intelligent Systems in Operations: Methods, Models and Applications in the Supply Chain, IGI global, pp 133–163 (2010)
49. Rafiei, H., Safaei, F., Rabbani, M.: Integrated production-distribution planning problem in a competition-based four-echelon supply chain. Comput. Ind. Eng. **119**, 85–99 (2018)
50. Raghunathan, S.: Information sharing in a supply chain: a note on its value when demand is nonstationary. Manag. Sci. **47**(4), 605–610 (2001)
51. Ramanathan, U.: Aligning supply chain collaboration using analytic hierarchy process. Omega **41**(2), 431–440 (2013)
52. Ramanathan, U., Gunasekaran, A.: Supply chain collaboration: impact of success in long-term partnerships. Int. J. Prod. Econ. **147**, 252–259 (2014)
53. Ren, Z.J., Cohen, M.A., Ho, T.H., Terwiesch, C.: Information sharing in a long-term supply chain relationship: the role of customer review strategy. Oper. Res. **58**(1), 81–93 (2010)
54. Renna, P., Argoneto, P.: Capacity sharing in a network of independent factories: a cooperative game theory approach. Robot. Comput. Integr. Manuf. **27**(2), 405–417 (2011)
55. Rosenthal, E.C.: A game-theoretic approach to transfer pricing in a vertically integrated supply chain. Int. J. Prod. Econ. **115**(2), 542–552 (2008)
56. Sahay, B.: Supply chain collaboration: the key to value creation. Work study (2003)
57. Sana, S.S., Chedid, J.A., Navarro, K.S.: A three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items. Appl. Math. Comput. **229**, 139–150 (2014)
58. Sarkar, B., Majumder, A., Sarkar, M., Dey, B.K., Roy, G.: Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction. J. Ind. Manag. Optim. **13**(2), 1085 (2017)
59. Sarkar, B., Omair, M., Kim, N.: A cooperative advertising collaboration policy in supply chain management under uncertain conditions. Appl Soft Comput **88**, 105948 (2020)
60. Seifert, D.: Collaborative planning, forecasting, and replenishment: how to create a supply chain advantage, AMACOM Div American Mgmt Assn (2003)
61. SeyedEsfahani, M.M., Biazaran, M., Gharakhani, M.: A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer–retailer supply chains. Eur. J. Oper. Res. **211**(2), 263–273 (2011)

62. Sheu, C., Rebecca Yen, H., Chae, B.: Determinants of supplier-retailer collaboration: evidence from an international study. Int. J. Oper. Prod. Manag. **26**(1), 24–49 (2006)
63. Shirodkar, S., Kempf, K.: Supply chain collaboration through shared capacity models. Interfaces **36**(5), 420–432 (2006)
64. Simatupang, T.M., Sridharan, R.: The collaborative supply chain. Int. J. Logist. Manag. **13**(1), 15–30 (2002)
65. Simatupang, T.M., Sridharan, R.: Benchmarking supply chain collaboration. Benchmark. Int. J. (2004)
66. Sinha, A.K., Aditya, H., Tiwari, M., Chan, F.T.: Agent oriented petroleum supply chain coordination: co-evolutionary particle swarm optimization based approach. Expert Syst. Appl. **38**(5), 6132–6145 (2011)
67. Stank, T.P., Keller, S.B., Daugherty, P.J.: Supply chain collaboration and logistical service performance. J. Bus. Logist. **22**(1), 29–48 (2001)
68. Vonderembse, M.A., Uppal, M., Huang, S.H., Dismukes, J.P.: Designing supply chains: Towards theory development. Int. J. Prod. Econ. **100**(2), 223–238 (2006)
69. Wang, H., Guo, M., Efstathiou, J.: A game-theoretical cooperative mechanism design for a two-echelon decentralized supply chain. Eur. J. Oper. Res. **157**(2), 372–388 (2004)
70. Xiao, T., Qi, X.: Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers. Omega **36**(5), 741–753 (2008)
71. Xie, J., Neyret, A.: Co-op advertising and pricing models in manufacturer–retailer supply chains. Comput. Ind. Eng. **56**(4), 1375–1385 (2009)
72. Yaghin, R.G.: Integrated multi-site aggregate production-pricing planning in a two-echelon supply chain with multiple demand classes. Appl. Math. Model. **53**, 276–295 (2018)
73. Zhang, X., Huang, G.Q.: Game-theoretic approach to simultaneous configuration of platform products and supply chains with one manufacturing firm and multiple cooperative suppliers. Int. J. Prod. Econ. **124**(1), 121–136 (2010)
74. Zhao, Y., Wang, S., Cheng, T.E., Yang, X., Huang, Z.: Coordination of supply chains by option contracts: a cooperative game theory approach. Eur. J. Oper. Res. **207**(2), 668–675 (2010)