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Efficient Operative Cost Reduction in Distribution Grids Considering the Optimal Placement and Sizing of D-STATCOMs Using a Discrete-Continuous VSA

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Abstract: The problem of reactive power compensation in electric distribution networks is addressed in this research paper from the point of view of the combinatorial optimization using a new discrete-continuous version of the vortex search algorithm (DCVSA). To explore and exploit the solution space, a discrete-continuous codification of the solution vector is proposed, where the discrete part determines the nodes where the distribution static compensator (D-STATCOM) will be installed, and the continuous part of the codification determines the optimal sizes of the D-STATCOMs. The main advantage of such codification is that the mixed-integer nonlinear programming model (MINLP) that represents the problem of optimal placement and sizing of the D-STATCOMs in distribution networks only requires a classical power flow method to evaluate the objective function, which implies that it can be implemented in any programming language. The objective function is the total costs of the grid power losses and the annualized investment costs in D-STATCOMs. In addition, to include the impact of the daily load variations, the active and reactive power demand curves are included in the optimization model. Numerical results in two radial test feeders with 33 and 69 buses demonstrate that the proposed DCVSA can solve the MINLP model with best results when compared with the MINLP solvers available in the GAMS software. All the simulations are implemented in MATLAB software using its programming environment.

Keywords: discrete-continuous vortex search algorithm; radial distribution networks; distribution static compensators; annual operational costs minimization; reactive power compensation; daily active and reactive demand curves



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1. Introduction

Around the world, electric distribution networks are the channels through which electricity is supplied to millions of end-users at medium- and low-voltage levels [1]. These grids are constantly submitted to increments in the load consumption each year. Therefore, these networks must be prepared to receive new users and guarantee the quality, reliability and security of the public service [2,3]. Due to the operational voltages, i.e., typically between 10 kV and 15 kV for the Colombian medium-voltage distribution grids, these electrical networks experience higher energy losses as compared to the transmission and sub-transmission systems [4]. In the Colombian context the power system has energy losses of about 1.5% to 2.0% in terms of the total energy transmitted. In the medium-voltage grids energy losses can vary from 5% to 18%, and the lowest losses are found in networks of the grid operators who invested to improve the quality of their grids [5]. The difference between energy losses is principally caused by the radial topology which is typically adopted while building the distribution networks to minimize investment costs. This

topology is in contrast with the strong meshed structures that are found in power systems. The main problem with the radial topology of the electric distribution grids is the high percentages of energy losses caused by the electricity distribution tasks [6]. To reduce the energy losses in the distribution service, the Utilities can use various methodologies, such as (i) grid reconfiguration [7], (ii) optimal location of shunt capacitors [8] and (iii) optimal location of dispersed power generation plants [9]. These approaches can achieve different levels of reduction of energy losses. Of these methodologies, distributed generation is the best option for energy loss reduction. However, its initial installation costs can be very high as compared to the reconfiguration and shunt capacitor approaches. However, the main problem of the capacitor banks is that they inject only in fixed steps of reactive power (discrete nature of the capacitive compensation with shunt capacitors), while the daily active and reactive power demands along the electric distribution grids are highly variables and continuous [10]. Recognizing the power saving that can be achieved with reactive power compensation with shunt devices, in recent decades, compensators based on power electronics have gained taken importance in distribution levels due to their versatility and capacity to vary the reactive power injections as a function of the demand behavior. These devices are the distribution static compensators (D-STATCOMs) [11], which have, among others, such advantages as: (i) high reliability, (ii) low operative costs, and (iii) long useful life (typically 5 to 15 years).

The literature on this specific topic has reported several approaches regarding the use of D-STATCOMs for dynamic reactive power compensation at medium-voltage levels. Some of these approaches are presented below. The authors of [11] presented the complete of determining the optimal placement and sizing of D-STATCOMs using analytical and heuristic optimization methods. Also, they presented some objective classical functions used for improving the grid performance, such as voltage stability and power loss indices. In [6] the authors proposed a multi-objective particle swarm optimizer to locate and size D-STATCOMs considering the simultaneous reconfiguration of the electrical network. As objective functions are considered the minimizing the active power losses, the voltage stability index and the loadability factor of the distribution lines, the main characteristic of this approach is that the optimization process is made only during the peak load condition, which is not considered a proper scenario due to possible over-sizing of the compensator devices, since the active and reactive power consumptions are variable inputs. The authors in [12] presented a heuristic cost-based approach to evaluate the impact of the optimal placement and sizing of D-STATCOMs in isolated power systems for automatic voltage support. However, the model considers the unique nodal representation of the power system, which oversimplifies the real behavior of the power system. The authors of [7] presented a fuzzy multi-objective approach based on the ant colony optimization algorithm to solve the simultaneous reconfiguration and optimal allocation (sizing and siting) of photo-voltaic sources and D-STATCOMs in the distribution system. The goal of the authors was to minimize the grid losses, improve the voltage profiles and increase in the feeder load balancing. Numerical results were tested in the IEEE 33-bus test feeder. A heuristic method based on voltage and power losses indices was proposed in [13] to locate and size D-STATCOMs in radial electric distribution networks. Numerical validations of this heuristic approach were made in the IEEE 33-bus test feeder. However, the authors consider only the peak load condition in their analysis. In [14] the authors presented the implementation of a genetic algorithm to locate and size D-STATCOMs in distribution systems using the optimal power flow tool available in the DigSILENT software. The main advantage of this approach is that it is tested in a real distribution grid. However, the authors do not present any cost analysis. In [15] the authors analyze the problem of the optimal location of D-STATCOMs considering simultaneous grid reconfiguration to reduce the amount of active power losses. The solution of the problem is derived using the differential evolution algorithm. However, the authors only consider the peak load condition in their analysis. Additional works that have addressed the problem of the optimal siting and sizing of D-STATCOMs with meta-heuristic techniques are listed in Table 1.

Table 1. Meta-heuristic approaches applied to the optimal location and sizing of D-STATCOMs in electric distribution networks.

Optimization Technique	Refs.
Particle swarm optimization	[6,16,17]
Genetic algorithms	[14,18–20]
Cuckoo search algorithm	[21–24]
Immune algorithm	[25]
Harmony search algorithm	[26,27]
Imperialist competitive algorithm	[28]

Based on the aforementioned presentation, the main state-of-the-art contributions of this research are as follows: (i) the application of the new version of the vortex search algorithm with and discrete-continuous codification to solve the problem of the optimal siting and sizing of D-STATCOMs in electric distribution networks considering daily active and reactive power demand curves; and (ii) verification of the advantages, such as the possibility of managing the amount of reactive power injection in D-STATCOMs, that the dynamic reactive power compensation in electric distribution systems using D-STATCOMs has for achieving greater reductions in the annual energy losses. It is worth mentioning that to locate and size the D-STATCOMs in the electric distribution network is considered an objective function for the reduction of the annual operative costs of the energy losses. The conductors add to the investment costs associated with the installation of the D-STATCOMS, which is subjected to the classical power balance equations that generate a mixed-integer nonlinear programming problem (MINLP). This optimization model is solved in this research with the help of a master–slave optimization approach in which the master–slave relationship is based on DCVSA, which is entrusted with the optimal location and sizing of the D-STATCOMs. The slave stage (successive approximation power flow (SAPF) method) deals with the determination of the electrical variables to calculate the costs of the energy losses during the daily operation.

The remainder of this research has the following structure: Section 2 presents the exact MINLP formulation to present the problem of optimal siting and sizing of D-STATCOMs in electric distribution networks considering the extent of reduction of the annual operational costs caused by energy losses and the cost of additional investment in the dynamic reactive power compensators. Section 3 describes the proposed solution, i.e., master–slave optimization approach based on the hybridization of the DCVSA and the SAPF methods. Section 4 presents the main characteristics of the test feeders that correspond to IEEE 33- and IEEE 69-bus, respectively. Finally, Section 6 presents the conclusions derived from this work and some possible future works.

2. MINLP Formulation

The problem of the optimal integration of D-STATCOMs in distribution grids for reactive power compensation can be formulated with a MINLP model where the integer part of this model is related to the possible installation of the D-STATCOM in a small subset of distribution buses, while the continuous part is related to the optimal sizing of these devices. Here, we present the general MINLP using the rectangular representation of the power balance equations.

2.1. Objective Function Formulation

The objective function of the optimization problem corresponds to the minimization of the annual operative costs attributed to the grid energy losses added to the annualized investment costs in D-STATCOMs. The objective function and its components are presented in Equation (1).

$$\min A_{\text{cost}} = f_1 + f_2, \tag{1}$$

$$f_1 = C_{\text{kWh}} T \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}} \sum_{m \in \mathcal{N}} Y_{km} V_{kh} V_{mh} \cos(\delta_{kh} - \delta_{mh} - \theta_{km}) \Delta_h,$$

$$f_2 = T \left(\frac{k_1}{k_2} \right) \sum_{k \in \mathcal{N}} \left(\alpha \left(Q_k^{\text{D-STATCOM}} \right)^2 + \beta Q_k^{\text{D-STATCOM}} + \gamma \right) Q_k^{\text{D-STATCOM}},$$

where A_{cost} corresponds to the objective function value f_1 , which is the component of the objective function associated with the cost of the grid energy losses, f_2 is the component of the objective function related with the annualized investments costs in D-STATCOMs, C_{kWh} is the average energy costs; T is a constant associated with the planning horizon (i.e., 365 days), Y_{km} is the magnitude of the component of the nodal admittance matrix associated with nodes k and m , which have an angle θ_{km} , V_{kh} and V_{mh} are the magnitudes of the voltage variables associated with buses k and m and the period h , with angles δ_{kh} and δ_{mh} , respectively, Δ_h is the time associated with the power flow evaluation (for this research it is assumed to be 1 h), k_1 and k_2 are positive constants of annualization of the investment costs and the life-time of the D-STATCOM, respectively, α , β and γ are positive constants associated with variable costs of installation of the D-STATCOM with nominal reactive power generation capacity $Q_k^{\text{D-STATCOM}}$. Observe that \mathcal{H} and \mathcal{N} are the sets that contain all the periods of time and all the buses of the network, respectively.

2.2. Set of Constraints

The set of constraints associated with the problem of the optimal allocation and sizing D-STATCOMs in AC distribution networks are mainly associated with the power balance equations and the binary constraints associated with the installation of these devices. All the restrictions for this problem are listed below.

$$P_{kh}^s - P_{kh}^d = \sum_{k \in \mathcal{N}} \sum_{m \in \mathcal{N}} Y_{km} V_{kh} V_{mh} \cos(\delta_{kh} - \delta_{mh} - \theta_{km}), \forall \{k \in \mathcal{N} \ \& \ h \in \mathcal{H}\}, \tag{2}$$

$$Q_{kh}^s + Q_k^{\text{D-STATCOM}} - Q_{kh}^d = \sum_{k \in \mathcal{N}} \sum_{m \in \mathcal{N}} Y_{km} V_{kh} V_{mh} \sin(\delta_{kh} - \delta_{mh} - \theta_{km}), \forall \{k \in \mathcal{N} \ \& \ h \in \mathcal{H}\}, \tag{3}$$

$$V_{\min} \leq V_{kh} \leq V_{\max}, \forall \{k \in \mathcal{N} \ \& \ h \in \mathcal{H}\}, \tag{4}$$

$$z_k Q_{\min}^{\text{D-STATCOM}} \leq Q_k^{\text{D-STATCOM}} \leq z_k Q_{\max}^{\text{D-STATCOM}}, \forall \{k \in \mathcal{N}\}, \tag{5}$$

$$\sum_{k \in \mathcal{N}} z_k \leq N_{\text{available}}^{\text{D-STATCOM}}, \tag{6}$$

where P_{kh}^s and Q_{kh}^s are the active and reactive power injections of the generator connected at node i in the period h , and P_{kh}^d and Q_{kh}^d are the active and reactive power consumption in the constant power demand connected at bus i in the period of time h , V_{\min} and V_{\max} are the lower and upper voltage bounds allowed for all nodes of the network at each period time, $Q_{\min}^{\text{D-STATCOM}}$ and $Q_{\max}^{\text{D-STATCOM}}$ are the minimum and maximum limits of the distribution system compensators that can be installed along the distribution grid, z_k is the binary variable associated with the allocation ($z_k = 1$ or $z_k = 0$) of a D-STATCOM in the network, and $N_{\text{available}}^{\text{D-STATCOM}}$ is the maximum number of D-STATCOMs that may be installed in the electrical distribution network.

2.3. Model Interpretation

The interpretation of the mathematical model (1)–(6) associated with the optimal allocation and sizing of D-STATCOMS in distribution grids is as follows: Equation (1) defines the objective function of the optimization problem which is defined as function of the annual cost of grid losses in all the conductors of the networks (see the term f_1) and the annualized cost of investment in D-STATCOMs (see the term f_2). This objective function indicates whether installing reactive power compensators is an attractive solution

for reducing the annualized operating cost of the network caused the loss of energy in all the conductors in the distribution grids considering daily active and reactive power curves. Equation (2) define the active power balance at each node of the network for each period of time, while Equation (3) is a similar expression regarding the reactive power balance, for identifying the direct effect of the reactive power compensation with the D-STATCOM device installed at bus k on the voltage magnitudes and angles for each period of time h . It is worth mentioning that the injection of the reactive power in the D-STATCOM is considered constant throughout the planning period. Inequality constraint (4) defines the upper and lower regulation bounds of the magnitudes of the voltage in all nodes of the network. This is a typical imposition made by the regulatory authorities in the power sector. Inequality constraint (5) shows the integer nature of the allocation problem of D-STATCOMs in distribution networks because the binary variable z_k indicates whether or not this device is installed or not at node k . Inequality constraint (6) limits the number of D-STATCOMs that can be installed in the electrical distribution grid.

Remark 1. *The optimization model (1)–(6) demonstrates an MINLP structure due to the presence of binary variables associated with the allocation of the D-STATCOMs in the grid. Continuous variables relate with active and reactive power generation and voltage magnitudes and angles, while the nonlinear structure is defined based on the presence of trigonometric functions and products among voltage magnitudes in the active and reactive power balance constraints.*

Due to the complex MINLP structure of the optimization model (1)–(6) in this paper is proposed is a master–slave optimization methodology to solve this mathematical model by combining with the successive approximations of power flow (SAPF) and a meta-heuristic optimization algorithm known in the specialized literature as the VSA. All the details of this solution will be addressed in the next section.

3. Solution

To solve the MINLP model (1)–(6) for the optimal allocation and sizing of D-STATCOMs in electric distribution grids, here is proposed a master–slave optimization methodology using the VSA in the master stage and the SAPF in the slave stage. Please note that the master stage defines the nodes where the D-STATCOMs will be installed and their optimal sizes. For this process, this paper proposes discrete-continuous codification for the VSA. On the other hand, the slave stage is entrusted with the determination of the daily energy losses by solving the multi-period power flow problem. The master and slave stages are described in the following subsections.

3.1. Slave Stage

The slave stage in the problem of the optimal allocation and sizing of D-STATCOMs along distribution feeders can be considered to be the heart of this solution because this stage resolves the power balance equality constraints for each period h , which is necessary for determining the operative states of the network, i.e., the daily grid losses defined in the term f_1 of the objective function (1). To solve these equations, here we rewrite them using its complex domain equivalent [29].

$$\mathbf{S}_{sh}^* = \text{diag}(\mathbf{V}_{sh}^*)[\mathbb{Y}_{ss}\mathbf{V}_{sh} + \mathbb{Y}_{sd}\mathbf{V}_{dh}], \quad \forall \{h \in \mathcal{H}\}, \quad (7)$$

$$\mathbf{S}_{qh}^* - \mathbf{S}_{dh}^{s,*} = \text{diag}(\mathbf{V}_{dh}^*)[\mathbb{Y}_{ds}\mathbf{V}_{sh} + \mathbb{Y}_{dd}\mathbf{V}_{dh}], \quad \forall \{h \in \mathcal{H}\}, \quad (8)$$

where \mathbf{S}_{sh} is the complex vector that contains all the apparent power generation in the slack nodes at each period h , \mathbf{V}_{sh} is the vector that contains all the voltage variables in complex form for all the slack nodes at each period of time t , \mathbf{V}_{dh} is the vector that contains all the voltage variables in complex form for all the demand nodes at each period of time t , \mathbb{Y}_{ss} , \mathbb{Y}_{sd} , \mathbb{Y}_{ds} and \mathbb{Y}_{dd} are submatrices obtained from the nodal admittance matrix that relates slack and demand nodes, respectively, \mathbf{S}_{qh}^* is a complex vector that contains the variables

associated with the reactive power generation in the D-STATCOMs, i.e., $\mathbf{S}_{qh}^* = jQ_{gh}$, \mathbf{S}_{dh}^* is the vector that contains all the apparent power demands at each period h . It is worth mentioning that x^* corresponds to the conjugate value of the complex number x , and $\mathbf{diag}(x)$ represents the conversion of the vector x into a diagonal matrix. Please note that \mathbf{V}_{sh} are known variables since these correspond to the voltage values in all the slack nodes, while the \mathbf{V}_{dh} are unknown variables that must be determined for each period. In addition, if the reactive power injections in the D-STATCOMs are known (defined in the slave optimization stage), and the demand voltages can be iteratively obtained from (8) as follows:

$$\mathbf{V}_{dh}^{k+1} = \mathbb{Y}_{dd}^{-1} \left[\mathbf{diag}^{-1} \left(\mathbf{V}_{dh}^{k,*} \right) \left[\mathbf{S}_{qh}^* - \mathbf{S}_{dh}^{s,*} \right] - \mathbb{Y}_{ds} \mathbf{V}_{sh} \right], \quad \forall \{h \in \mathcal{H}\}, \quad (9)$$

where k is the iterative counter, such that for $k = 0$, plane-voltages are defined as $\mathbf{V}_{dh}^0 = \mathbf{1} \angle 0^\circ$ as the initial voltage condition using the per-unit representation. The recursive power flow formula defined by (9) is known as the successive approximation power flow method originally proposed in [29], where the iterations are made until the convergence criterion is met, which is defined as follows: $\max \left\{ \left\| \mathbf{V}_{dh}^{k+1} \right\| - \left\| \mathbf{V}_{dh}^k \right\| \right\} \leq \epsilon$, where epsilon is defined as the maximum convergence error.

Remark 2. In the SAPF method defined by the recursive Formula (4) its convergence can be ensured by applying the Banach fixed-point theorem as demonstrated in [29], based on the properties of the demand admittance matrix \mathbb{Y}_{dd} .

To determine the value of the objective function regarding the annual costs of the grid losses, the solution found by the SAPF method is used (suppose that the solution of the power flow is $\mathbf{U}_h = \left[\mathbf{V}_{sh} \ \mathbf{V}_{dh}^{k+1} \right]^T$, which allows transforming the component f_1 in the objective function (1) into

$$f_1 = C_{kWh} T \sum_{h \in \mathcal{H}} \text{real} \left\{ \mathbf{U}_h^T \mathbb{Y}^* \mathbf{U}_h \right\}. \quad (10)$$

3.2. Master Stage

The master stage in the problem of the optimal allocation and sizing of D-STATCOMs along distribution feeders can be considered to be the brain of this methodology because it explores and exploits the solution space. Here, we proposed a new version of the VSA that works with a hybrid codification that determines the nodes where the D-STATCOMs will be located (integer part) and their corresponding optimal sizes (continuous part), which constitutes a new meta-heuristic optimization approach that is an improvement over the discrete version recently proposed in [8] and named DCVSA.

The VSA is a recently developed meta-heuristic optimization methodology for solving nonlinear programming problems in the continuous domain by using the behavior of the stirred fluids that generates vortical demeanors in pipes [30]. The implementation of the VSA to solve optimization problem has the following advantages: (i) it corresponds to a trajectory algorithm that generates its neighborhood using a Gaussian distribution, which allows exploration of all regions of the solution space that contains potential solutions [31]; (ii) it exploits the solution space around the promising regions using a variable radius, which at each iteration reduces the size of the hyper-ellipse that contains all the solutions [8]; and (iii) at the end of the iterative procedure, all the solutions generated by the Gaussian distribution are placed at the same point of the optimal solution which increases the repeatability of the algorithm at each global evaluation of the optimization methodology [32].

3.2.1. Proposed Hybrid Discrete-Continuous Codification

To explore the solution space, the VSA uses a codification to organize the decision variables. Here we propose a combined codification that contains discrete and continuous numbers. For example, the proposed codification for the problem of the optimal placement and sizing of D-STATCOMs in electric distribution grids, the potential solution w_i^t takes the following structure:

$$w_i^t = [n, 2, \dots, k | Q_{\min}^{\text{D-STATCOM}}, Q_{\max}^{\text{D-STATCOM}}, \dots, q], \tag{11}$$

where i corresponds to the position of the potential solution and t the current iteration. Furthermore, k represents an arbitrary node, n as the last node number, and q an aleatory number between $Q_{\min}^{\text{D-STATCOM}} \leq q \leq Q_{\max}^{\text{D-STATCOM}}$. Please note that the codification vector has a dimension $1 \times 2N_{\text{available}}^{\text{D-STATCOM}}$, where the first part, i.e., the first $N_{\text{available}}^{\text{D-STATCOM}}$ positions contains the discrete variables associated with the nodes where the D-STATCOMs will be placed, and the remaining components of the vector contain their optimal sizes defined with continuous codification.

3.2.2. Generation of the Initial Solution

The set of potential solutions in the VSA is generated apart from the initial solution that corresponds to the initial center of the hyper-ellipse that covers the total solution space at the beginning of the process of iteration. For example, the selected center of the solution space is μ_0 , suppose an optimization is defined in the d -dimensional space where $d = 3$,

$$\mu_0 = \left\{ \frac{x_1^{\min} + x_1^{\max}}{2}, \frac{x_2^{\min} + x_2^{\max}}{2}, \frac{x_3^{\min} + x_3^{\max}}{2} \right\}, \tag{12}$$

where x_j^{\min} and x_j^{\max} represents the lower and upper bounds of the decision variables.

3.2.3. Generation of the Candidate Solutions

The generation of the set containing the candidate (potential) solutions $C_i^t(x) = w_i^t = \{x_1, x_2, \dots, x_d\}$, being i , the subindex associated with the i -th individual within the population is built using a Gaussian distribution as reported in [30], which takes the following structure:

$$w_i^t = p(\zeta_i^t, \mu_t, v) = \left((2\pi)^d |v| \right)^{1/2} e^{-\frac{1}{2} \frac{(\zeta_i^t - \mu_t)^T (\zeta_i^t - \mu_t)}{v}}. \tag{13}$$

Please note that in this distribution probability, $\zeta_i^t \in \mathbb{R}^{d \times 1}$ is a vector composed of aleatory numbers, $\mu_t \in \mathbb{R}^{d \times 1}$ is the current center of the hyper-ellipse in the iteration t , and $v \in \mathbb{R}^{d \times d}$ is known as the covariance matrix. Following the recommendation in [30], this matrix can be simplified using equal variances in the diagonal and null co-variances in its non-diagonal elements. These characteristics of the covariance matrix are formulated as follows (for the 3-dimensional example):

$$\sigma_0 = \frac{\max\{x_1^{\max}, x_2^{\max}, x_3^{\max}\} - \min\{x_1^{\min}, x_2^{\min}, x_3^{\min}\}}{2}, \tag{14}$$

being the covariance matrix defined as $v = \sigma_0 I_{d \times d}$, where $I_{d \times d}$ is a diagonal identity matrix. It is important to highlight that at the beginning of the iteration procedure the radius r_t with $t = 0$ as recommended by the authors of [30] must be assigned as σ_0 . In addition, the variable radius of the hyper-ellipse in the VSA plays an important role, since this governs the behavior of the vector of random variables ζ_i^t as $\zeta_i^t = r_t \text{rand}(d)$, where $\text{rand}(d)$ is an aleatory vector with d -dimension that contains numbers between 0 and 1 generated with a normal distribution.

3.2.4. Bounding the Candidate Solutions

When the Gaussian distribution (13) is applied to generate each candidate solution w_i^t there exists the real possibility that it lies outside its bounds, which implies that a procedure for revision is needed to revise every variable inside w_i^t to ensure that it meets fulfill its lower and upper bounds [8]. The rule to revise the upper and lower bounds is presented below.

$$w_i^t = \begin{cases} x^{\min} + (x^{\max} - x^{\min})\text{rand} & x^{\min} \leq x \leq x^{\max} \\ w_i^t & \text{otherwise} \end{cases} \quad (15)$$

where rand corresponds to a random number between 0 and 1 generated with a normal distribution structure.

Remark 3. *The main modification in the proposed DCVSA is that the first $N_{\text{available}}$ components of the potential solution w_i^t are rounded to guarantee that these have an integer structure, which is associated with the nodes where the D-STATCOMs will be installed.*

3.2.5. Selection of the New Center of the Hyper-Ellipse

The evaluative process of the VSA to explore promising regions of the solution space is defined by the location of the center of the hyper-ellipse, i.e., μ_{t+1} , which must be selected as the best solution in the current population. The best solution corresponds to the minimum value of the fitness function found after evaluating all individuals in $C_i^t(x)$, i.e., $\mu_{t+1} = w_{i,\text{best}}^t$. The following fitness function is employed to solve the problem of the optimal placement and sizing of D-STATCOMs in electric distribution grids. The fitness function corresponds with an adaptation of the original objective function to deal with possible lack of feasible solutions in the solution space.

$$A_f = A_{\text{cost}} + \beta \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}} \left[\begin{array}{l} \max\{0, V_{kh} - V_{\max}\} + \\ \max\{0, V_{\min} - V_{kh}\} \end{array} \right], \quad (16)$$

where β is known as the penalization factor that enlarges the effect of a voltage violation for each node at each period. Please note that $w_{i,\text{best}}^t = \min_{i,t} \{A_f(w_i^t)\}$.

Remark 4. *The remaining constraints in the optimization model (1)–(6) are guaranteed by the application of the SAPF in (9) and the generation of the variables in the VSA procedure. In addition, if the voltage regulation bounds are met, then the fitness function A_f takes the same value as the objective function.*

3.2.6. Reduction of the Hyper-Ellipse Radius

The radius in the VSA represents a variable that balances the exploration and exploitation of the solution space during the iterative procedure. The original VSA approach proposed in [30] recommends the implementation of the incomplete Gamma function to continuously reduce the radius of the hyper-ellipse; however, to simplify the implementation of this algorithm in any programming environment, an exponential adaptation is used in this research for the radius variation reported in [32] and as defined in (17).

$$r_{t+1} = \sigma_0 \left(1 - \frac{t}{t_{\max}}\right) e^{(-\gamma_1 \frac{t}{t_{\max}})}, \quad (17)$$

where γ_1 is an adjustable parameter that governs the speed of reduction of the radius. This parameter has been used heuristically in previous literature as 6 [8] because it emulates incomplete Gamma function with 95% of similarity. Please note that t_{\max} is the maximum number of iterations performed in the optimization process.

3.2.7. Stopping Conditions

The optimization process followed by the proposed DCVSA ends when one of the following criteria is met:

- ✓ If the maximum number of iteration, i.e., t_{\max} is attained, then, the optimal solution found by the DCVSA corresponds to the current center of the hyper-ellipse.
- ✓ If after τ_{\max} , consecutive iterations the center of the hyper-ellipse remains constant, then the optimal solution reached by the DCVSA is the current center of the hyper-ellipse.

3.2.8. Algorithmic Implementation of the DCVSA

The implementation of the master optimization stage based on the new discrete-continuous version of the VSA is presented in Algorithm 1, which follows the structure proposed in [8].

Algorithm 1: Schematic implementation of the DCVSA to optimal allocation and sizing of D-STATCOMs in electric distribution networks.

Data: Read the information of the AC distribution network to feed the optimization model (1)–(6).
 Select the initial radius of the hyper-ellipse r_0 and its initial center μ_0 ;
 Generate the potential solutions w_i^0 using (13);
 Revise the upper and lower bounds for each w_i^0 employing (15);
 Solve the SAPF problem using the recursive Formula (9) for each w_i^0 ;
 Determine the value of the fitness function for each w_i^0 , i.e., $A_f(w_i^0)$ using (16);
 Determine the best current solution, i.e., $w_{i,best}^0 = \min_{i,t} \{A_f(w_i^0)\}$;
for $t = 1 : t_{\max}$ **do**
 Update the center of the hyper-ellipse $\mu_t = w_{i,best}^{t-1}$;
 Determine the new radius of the hyper-ellipse r_t with (17);
 Generate the new potential solutions w_i^t using (13);
 Revise the upper and lower bounds for each w_i^t employing (15);
 Solve the SAPF problem using the recursive Formula (9) for each w_i^t ;
 Determine the value of the fitness function for each w_i^t , i.e., $A_f(w_i^t)$ using (16);
 Determine the best current solution, i.e., $w_{i,best}^t = \min_{i,t} \{A_f(w_i^t)\}$;
 if $\tau \geq \tau_{\max}$ **then**
 Reports as the solution of the problem μ_t ;
 break;
 end
end
Result: Return the optimal solution found

4. Electric Distribution Test Feeders

The evaluation of the proposed MINLP model and the DCVSA to solve the problem of the optimal location and sizing of D-STATCOMs in electric distribution grids is made in two classical and well-known distribution test feeders composed of 33 and 69 buses with radial structure, which will be denominated in this research as the IEEE 33-bus and the IEEE 69-bus, respectively. Both electrical distribution networks are operated with 12.66 kV at the substation node located at bus 1. The schematic connection between nodes in these test feeders are depicted in Figure 1.

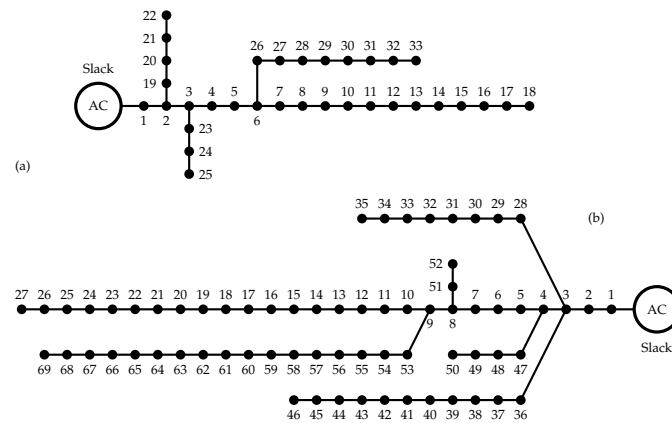


Figure 1. Nodal connections of the test feeder (a) IEEE 33-bus (b) IEEE 69-bus.

The IEEE 33-bus has a total active and reactive power demand of 3715 kW and 2300 kvar with 210.9876 kW of power losses at the peak load condition; while the IEEE 69-bus has a total active and reactive power demand of 3890.7 kW and 2693.6 kvar with 224.9520 kW of power losses in the same operative scenario. All information regarding branches and loads are reported in Tables 2 and 3 [8].

Table 2. Electrical parameters of the 33-node test feeder.

Node <i>i</i>	Node <i>j</i>	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node <i>i</i>	Node <i>j</i>	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0477	100	60	17	18	0.7320	0.5740	90	40
2	3	0.4930	0.2511	90	40	2	19	0.1640	0.1565	90	40
3	4	0.3660	0.1864	120	80	19	20	1.5042	1.3554	90	40
4	5	0.3811	0.1941	60	30	20	21	0.4095	0.4784	90	40
5	6	0.8190	0.7070	60	20	21	22	0.7089	0.9373	90	40
6	7	0.1872	0.6188	200	100	3	23	0.4512	0.3083	90	50
7	8	1.7114	1.2351	200	100	23	24	0.8980	0.7091	420	200
8	9	1.0300	0.7400	60	20	24	25	0.8960	0.7011	420	200
9	10	1.0400	0.7400	60	20	6	26	0.2030	0.1034	60	25
10	11	0.1966	0.0650	45	30	26	27	0.2842	0.1447	60	25
11	12	0.3744	0.1238	60	35	27	28	1.0590	0.9337	60	20
12	13	1.4680	1.1550	60	35	28	29	0.8042	0.7006	120	70
13	14	0.5416	0.7129	120	80	29	30	0.5075	0.2585	200	600
14	15	0.5910	0.5260	60	10	30	31	0.9744	0.9630	150	70
15	16	0.7463	0.5450	60	20	31	32	0.3105	0.3619	210	100
16	17	1.2890	1.7210	60	20	32	33	0.3410	0.5302	60	40

Table 3. Electrical parameters of the 69-node test feeder.

Node <i>i</i>	Node <i>j</i>	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node <i>i</i>	Node <i>j</i>	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0005	0.0012	0	0	3	36	0.0044	0.0108	26	18.55
2	3	0.0005	0.0012	0	0	36	37	0.0640	0.1565	26	18.55
3	4	0.0015	0.0036	0	0	37	38	0.1053	0.1230	0	0
4	5	0.0251	0.0294	0	0	38	39	0.0304	0.0355	24	17
5	6	0.3660	0.1864	2.6	2.2	39	40	0.0018	0.0021	24	17
6	7	0.3810	0.1941	40.4	30	40	41	0.7283	0.8509	1.2	1
7	8	0.0922	0.0470	75	54	41	42	0.3100	0.3623	0	0
8	9	0.0493	0.0251	30	22	42	43	0.0410	0.0475	6	4.3
9	10	0.8190	0.2707	28	19	43	44	0.0092	0.0116	0	0
10	11	0.1872	0.0619	145	104	44	45	0.1089	0.1373	39.22	26.3
11	12	0.7114	0.2351	145	104	45	46	0.0009	0.0012	39.22	26.3
12	13	1.0300	0.3400	8	5	4	47	0.0034	0.0084	0	0
13	14	1.0440	0.3450	8	5.5	47	48	0.0851	0.2083	79	56.4
14	15	1.0580	0.3496	0	0	48	49	0.2898	0.7091	384.7	274.5
15	16	0.1966	0.0650	45.5	30	49	50	0.0822	0.2011	384.7	274.5
16	17	0.3744	0.1238	60	35	8	51	0.0928	0.0473	40.5	28.3
17	18	0.0047	0.0016	60	35	51	52	0.3319	0.1114	3.6	2.7
18	19	0.3276	0.1083	0	0	9	53	0.1740	0.0886	4.35	3.5
19	20	0.2106	0.0690	1	0.6	53	54	0.2030	0.1034	26.4	19
20	21	0.3416	0.1129	114	81	54	55	0.2842	0.1447	24	17.2
21	22	0.0140	0.0046	5	3.5	55	56	0.2813	0.1433	0	0
22	23	0.1591	0.0526	0	0	56	57	1.5900	0.5337	0	0
23	24	0.3460	0.1145	28	20	57	58	0.7837	0.2630	0	0
24	25	0.7488	0.2475	0	0	58	59	0.3042	0.1006	100	72
25	26	0.3089	0.1021	14	10	59	60	0.3861	0.1172	0	0
26	27	0.1732	0.0572	14	10	60	61	0.5075	0.2585	1244	888
3	28	0.0044	0.0108	26	18.6	61	62	0.0974	0.0496	32	23
28	29	0.0640	0.1565	26	18.6	62	63	0.1450	0.0738	0	0
29	30	0.3978	0.1315	0	0	63	64	0.7105	0.3619	227	162
30	31	0.0702	0.0232	0	0	64	65	1.0410	0.5302	59	42
31	32	0.3510	0.1160	0	0	11	66	0.2012	0.0611	18	13
32	33	0.8390	0.2816	14	10	66	67	0.0047	0.0014	18	13
33	34	1.7080	0.5646	19.5	14	12	68	0.7394	0.2444	28	20
34	35	1.4740	0.4873	6	4	68	69	0.0047	0.0016	28	20

To evaluate the effect of the daily load variation in both test feeders the active and reactive power curves depicted in Figure 2 are considered.

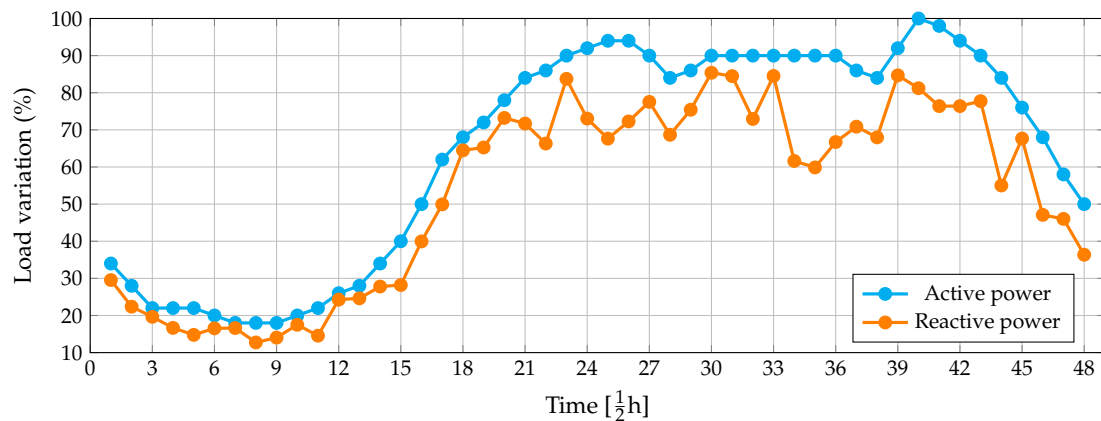


Figure 2. Typical behavior of the active and reactive power consumption in Colombia [2].

The information to construct the aforementioned daily active and reactive power curves is presented in Table 4 for the sake of reproducibility of the results that will be presented in the next section.

Table 4. Behavior of the daily active and reactive power demands.

Period	Act. (pu)	React. (pu)	Period	Act. (pu)	React. (pu)
1	0.1700	0.1477	25	0.4700	0.3382
2	0.1400	0.1119	26	0.4700	0.3614
3	0.1100	0.0982	27	0.4500	0.3877
4	0.1100	0.0833	28	0.4200	0.3434
5	0.1100	0.0739	29	0.4300	0.3771
6	0.1000	0.0827	30	0.4500	0.4269
7	0.0900	0.0831	31	0.4500	0.4224
8	0.0900	0.0637	32	0.4500	0.3647
9	0.0900	0.0702	33	0.4500	0.4226
10	0.1000	0.0875	34	0.4500	0.3081
11	0.1100	0.0728	35	0.4500	0.2994
12	0.1300	0.1214	36	0.4500	0.3336
13	0.1400	0.1231	37	0.4300	0.3543
14	0.1700	0.1390	38	0.4200	0.3399
15	0.2000	0.1410	39	0.4600	0.4234
16	0.2500	0.1998	40	0.5000	0.4061
17	0.3100	0.2497	41	0.4900	0.3820
18	0.3400	0.3224	42	0.4700	0.3820
19	0.3600	0.3263	43	0.4500	0.3887
20	0.3900	0.3661	44	0.4200	0.2751
21	0.4200	0.3585	45	0.3800	0.3383
22	0.4300	0.3316	46	0.3400	0.2355
23	0.4500	0.4187	47	0.2900	0.2301
24	0.4600	0.3652	48	0.2500	0.1818

To evaluate the objective function defined in (1) the parameters reported in Table 5 are considered. Some of these parameters have been taken from [6,33].

Table 5. Parametrization of the objective function.

Par.	Value	Unit	Par.	Value	Unit
C_{kWh}	0.1390	US\$/kWh	T	365	Days
Δ_h	0.50	h	α	0.30	US\$/MVar ³
β	−305.10	US\$/MVar ²	γ	127380	US\$/MVar
k_1	6/2190	1/Days	k_2	10	Years

It is worth mentioning that in the evaluation of the f_2 component, the variable $Q_k^{D-STATCOM}$ is defined in MVar [33].

5. Computational Implementation and Results

The solution of the MINLP model (1)–(6) for the optimal siting and sizing of D-STATCOMs in electric distribution networks using the DCVSA was implemented in the MATLAB version 2020b on a PC with an AMD Ryzen 7 3700 2.3-GHz processor and 16.0 GB RAM, running on a 64-bit version of Microsoft Windows 10 Single language. In addition, to compare the efficiency and robustness of the proposed optimization approach the MINLP model was also solved in the GAMS software with BONMIN and COUENNE solvers [9]. The parameterization of the DVSA was taken from [8] and it is presented in Table 6.

Table 6. Parametrization of the DCVSA.

Discrete-continuous vortex search algorithm	
Population size:10	Iterations' number: 1000
Population building: Gaussian Distribution	
SAPF method	
Iterations' number: 1000	Convergence error: 1×10^{-10}
Experimental tests in each test feeder	
Consecutive evaluations	100

5.1. IEEE 33-Bus

In Table 7 are listed the optimal solutions for the problem of the optimal placement and sizing of D-STATCOMs in electric distribution networks using the GAMS optimizers and the proposed optimization DCVSA approach.

Table 7. Optimal location and sizes of the D-STATCOMs in the IEEE 33-bus.

Approach	Location and Size Node (MVar)	A_{cost} (US \$/Year)
Caso base	—	112,740.90
COUENNE	{16(0.0109), 17(0.0224), 18(0.2065)}	107,589.50
BONMIN	{17(0.0339), 18(0.0227), 30(0.2395)}	102,447.29
DCVSA	{14(0.1599), 30(0.3591), 32(0.1072)}	98,497.90

Please note that the solutions reported in Table 7 show that: (i) the best solution is reported by the proposed DCVSA with the selection of the nodes 14, 30 and 32 to locate the D-STATCOMs with sizes of 0.1599 MVar, 0.3591 MVar, and 0.1072 MVar, respectively. These reduce the operating cost of the electric distribution grid by about US\$14243 per year of operation, i.e., 12.63% with compared to the base case; and (ii) the GAMS solvers are stuck in local optimal solutions. The BONMIN solver reduces the total annual grid cost by about 9.13%, while the COUENNE achieves a reduction of only 4.57%. These problems in the solutions reported by the GAMS solvers are clearly attributable to the nonlinear non-convex behavior of the objective function and the power balance constraints, which makes it impossible to combine classical branch and bound method, which, with interior points, finds the global optimum. This is possible only in mixed-integer convex problems.

To present the effect of the optimal location on the costs of the annual energy losses, we desegregate the investment and the operative costs associated with the installation of the D-STATCOMs reported in Table 7. In the Figure 3 are presented the desegregated costs for the IEEE 33-bus test system.

The behavior of the annual investment and operating costs presented in Figure 3 shows that the proposed DCVSA decides to make an inversion in D-STATCOMs that duplicates the inversion made by the GAMS solvers. However, this behavior of the DCVSA leads to considerable reduction in the cost of annual energy losses and helps to improve the grid behavior significantly as compared with the GAMS solvers. In addition, if we observe the cost of annual energy losses of the BONMIM solver, these are about US $\$98.67 \times 10^3$, which is itself higher than the summation of the costs reported with the proposed DCVSA which is about US $\$98.50 \times 10^3$ —This result proves that the proposed approach is more suitable for solving complex mixed-integer non-convex optimization problems than the GAMS package as shown in [9].

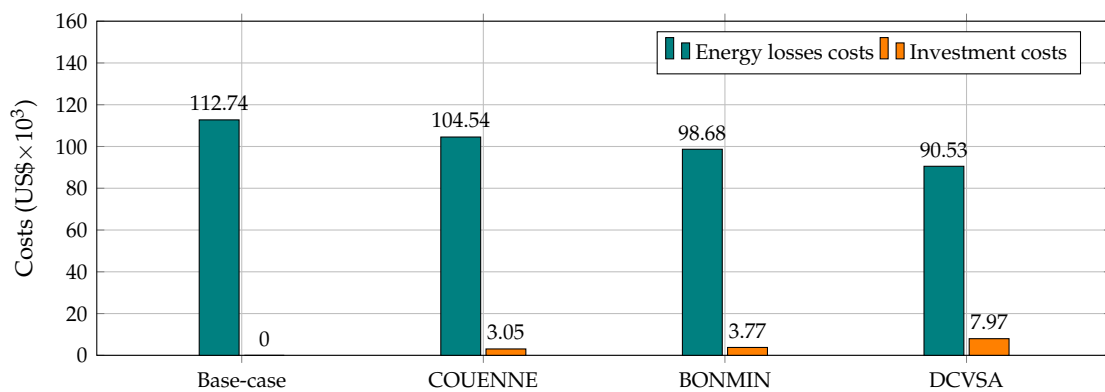


Figure 3. Distribution of the operative and investment costs for the comparative and proposed approach.

To demonstrate the efficiency of the proposed DCVSA we report the first six solutions found by this approach in Table 8.

Table 8. Lists of solutions found by the proposed DCVSA in the IEEE 33-bus.

Sol.	Location and Size Node (MVar)	A_{cost} (US \$/Year)	Rep.
1	{14(0.1599), 30(0.3591), 32(0.1072)}	98,497.90	36
2	{11(0.0659), 14(0.1148), 30(0.4578)}	98,564.29	22
3	{10(0.0642), 14(0.1175), 30(0.4574)}	98,565.03	10
4	{11(0.0787), 15(0.1019), 30(0.4578)}	98,567.91	8
5	{12(0.1110), 14(0.0666), 30(0.4591)}	98,569.08	2
6	{12(0.0804), 15(0.0972), 30(0.4591)}	98,570.12	4

Please note that the results in Table 8 demonstrate that: (i) after 100 consecutive evaluations, the proposed DCVSA is found to have a 36% efficiency at finding the same optimal solution, (ii) all the solutions in Table 8 are indeed better than the results achieved by the GAMS optimizers in reaching the final objective function value, and (iii) it is observed that the node 30 appears in all the solutions with sizes larger than 350 kvar, which implies that this is the most sensitive node for the placement of D-STATCOMs in the IEEE 33-bus test system.

It is also important to mention that about processing times the proposed approach takes about 76.85 s to solve the optimization problem with excellent numerical results in contrast with the 6.6 s taken by the BONMIN solver that is stuck in a local solution which is worse than all the solutions listed in Table 8.

5.2. IEEE 69-Bus

To present the effectiveness and robustness of the proposed DCVSA to solve the problem of the optimal siting and sizing of D-STATCOMs in the IEEE 69-bus test feeder, first six solutions reached by our proposed method are presented in Table 9. Please note that the base case for this test feeder has an annual cost of US \$119,715.63.

Regarding the solutions reported in Table 9 it is observed that: (i) in all the solutions the node 61 and 21 are recur, except in solution 2 where node 17 replaces node 21, which implies that these are the most suitable nodes to locate D-STATCOMs. On the basis of maximum reduction in the annual operating costs, node 61 with the D-STATCOM of the largest size appears in all the simulation cases; (ii) the solutions 3 to 6 are comparable because the differences among their annual costs are less than US \$0.4100 per year of operation and the fact that in all these node 61 node features as the location for a D-STATCOM with a nominal rate of 0.5741 MVar; and (iii) the proposed optimization approach for the IEEE 69-bus test system has an efficiency of the 49%, i.e., among 100 consecutive evaluations, 49 find global optimal solutions that cause the reduction of about 13.97% in the annual

operative cost as compared to the base case when D-STATCOMs are installed at nodes 61, 61 and 64 with nominal rates of 0.0839 MVar, 0.4601 MVar, and 0.1139 MVar, respectively.

Table 9. Lists of solutions found by the proposed DCVSA in the IEEE 69-bus.

Sol.	Location and Size Node (MVar)	A_{cost} (US \$/Year)	Rep.
1	{21(0.0839), 61(0.4601), 64(0.1139)}	102,990.80	49
2	{17(0.0862), 61(0.4597), 64(0.1139)}	103,022.77	1
3	{21(0.0695), 26(0.0143), 61(0.5741)}	103,101.25	3
4	{21(0.0704), 27(0.0134), 61(0.5741)}	103,101.31	1
5	{21(0.0687), 25(0.0152), 61(0.5741)}	103,101.52	1
6	{22(0.0695), 26(0.0143), 61(0.5741)}	103,101.66	4

Remark 5. Comparisons with the GAMS solvers are not presented since the BONMIN and the COUENNE failed to solve the MINLP model for the IEEE 69-bus test feeder. This further demonstrates the efficiency and robustness of the DCVSA for addressing this complex optimization problem from the meta-heuristic optimization point of view.

If we split the optimal solution yield by the proposed DCVSA in Table 9, then, the required inversion in D-STATCOMs is US \$8372.08 against the reduction in the costs of the energy losses of US \$25,096.92 seen in the base case. This clearly compensates for the required inversions in the D-STATCOMs, because it leads to a reduction of 13.97% in the annual operating costs for the grid operator, i.e., US \$16,724.84 per year of operation.

It is also important to mention the processing times the proposed approach takes. It is about 300.68 s to solve the optimization problem with excellent numerical results while the GAMS solvers diverge after 2187.96 s of exploration in the solution space.

5.3. Daily Operation of the D-STATCOMs

To verify the dynamic behavior of the D-STATCOMs in the reactive power compensation in electric distribution systems considering the daily active and reactive power demand curves presented in Figure 2, we present the D-STATCOMs' behavior in Figure 4a,b for the IEEE 33- and 69-buses test feeders, respectively.

From results in Figure 4 it may be noted that: (i) for the IEEE 33-bus test feeder all the D-STATCOMs work in their nominal capacities from the periods 16 to 47, while for the other periods, the amount of reactive power varies as a function of the total active and reactive power demand because, for these periods (see period 1 to period 16 in Figure 2), the total active and reactive power consumption is lower than the 50%. This implies that excessive injections of reactive power can worsen the grid performance in terms of energy losses; (ii) with the daily operation of the D-STATCOMs presented in Figure 4 the annual energy costs also reduced from US \$90,527.63 to US \$89,314.22, i.e., an additional reduction of about USD 1231.41 per year of operation; (iii) in the IEEE 69-bus test feeder the dynamic behavior of the D-STATCOMs shows that during the periods 1 to 17 and in the last, i.e., the 47th period, the amount of the reactive power injection was lower than the nominal rates of the D-STATCOMs. This was caused by the low demand in these periods of time (see Figure 2); and (iv) if the D-STATCOMs are operated throughout the day at their nominal values (see Table 9) the cost of annual energy losses is about US \$94,618.71. If, however, these are operated with the dynamic curves presented in Figure 4b, then, the cost of annual energy losses is about US \$93027.84, i.e., an additional reduction of US \$1590.87 per year of operation.

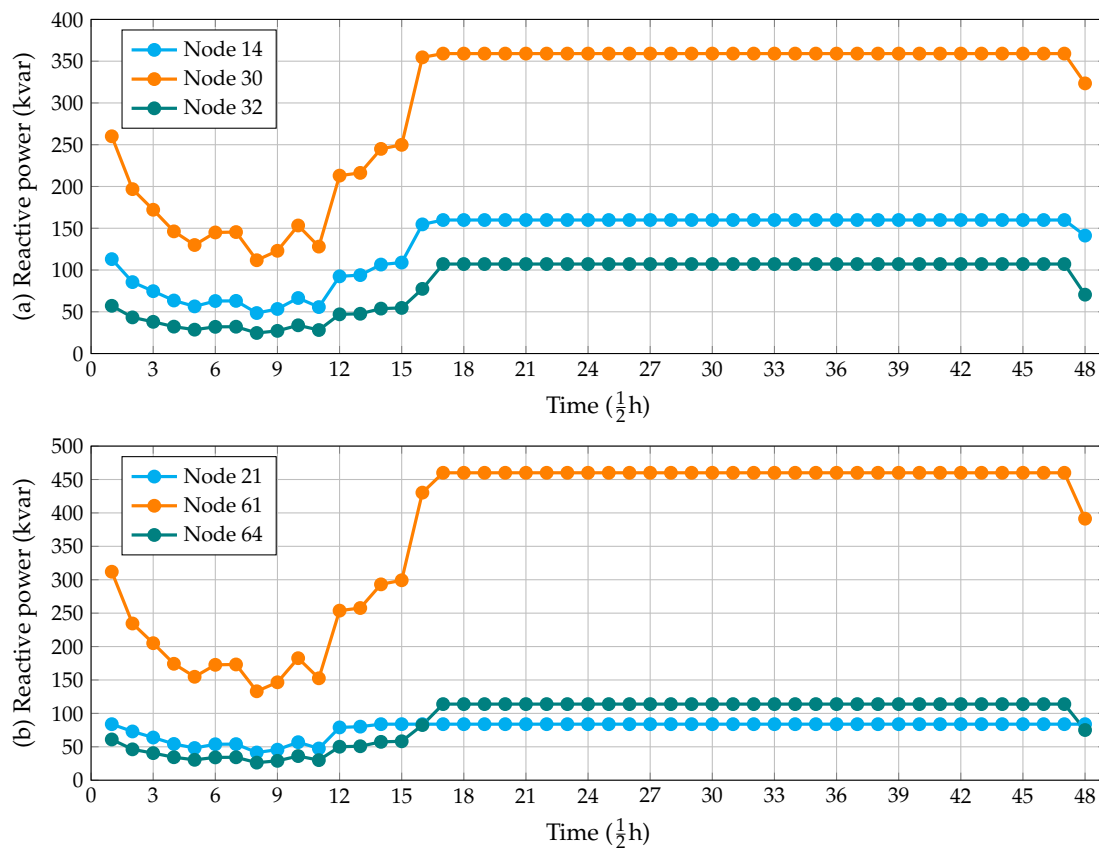


Figure 4. Dynamic reactive power behavior in a typical operation day for the D-STATCOMs in electric distribution networks: (a) IEEE 33-bus, and (b) IEEE 69-bus.

It is also important to mention that the dynamic behavior of the reactive power injections in Figure 4 in the IEEE 33- and IEEE 69-bus test feeder confirms the advantages of using D-STATCOMs instead fixed capacitor banks because the former can vary their reactive power injections as function of the demand demeanor, which is not possible when fixed capacitors are installed.

6. Conclusions and Future Works

The problem of the optimal siting and sizing of D-STATCOMs in electric distribution networks to minimize the annual operational costs of the grid caused by the energy losses during a typical day of operation was formulated as an MINLP model in research. The model proposed in this study incorporates of the a new version of the VSA that uses a discrete-continuous codification (DCVSA) and the SAPF method. The DCVSA is used for determining the optimal locations for and sizes of the D-STATCOMs and the SAPF method entrusted determines the voltage variables to calculate the annual energy losses of the grid and their cost. Numerical results in the IEEE 33- and IEEE 69-bus test feeders demonstrated that the proposed hybrid optimization approach showed a better numerical performance than the MINLP solvers BONMIN and COUENNE in the GAMS package. With the IEEE 33-bus test feeder, the DCVSA caused an annual reduction about 12.63%, while the BONMIN and the COUENNE solvers could bring about reductions of about 9.13% and 4.57%, respectively. With the IEEE 69-node test feeder, the reduction with DCVSA was about 13.97% and that with the GAMS solvers was variable.

The evaluation of the dynamic reactive power compensation using the D-STATCOMs sized and located by the proposed DCVSA approach showed that depending on the amount of active and reactive power demands throughout the day, the reactive power injection must be varied to achieve even greater improvement in the annual performance of the grid. Numerical results showed that for the IEEE 33-bus and IEEE 69-bus test feeders, additional

annual reductions of about US \$1231.41 and US \$1590.87 were achieved as compared to fixed reactive power injections.

As future works, the following research topics may explore: (i) the reformulation of the MINLP model into a mixed-integer convex model to find the global optimum with the application of the Branch and Bound and the interior point methods; and (ii) the simultaneous location and sizing of the renewable energy resources and D-STATCOMs to improve the grid performance during the planning horizon, considering annual load increments.

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Conflicts of Interest: The authors declare no conflict of interest.

References

1. Alam, M.S.; Arefifar, S.A. Energy Management in Power Distribution Systems: Review, Classification, Limitations and Challenges. *IEEE Access* **2019**, *7*, 92979–93001. [\[CrossRef\]](#)
2. Montoya, O.D.; Gil-González, W. Dynamic active and reactive power compensation in distribution networks with batteries: A day-ahead economic dispatch approach. *Comput. Electr. Eng.* **2020**, *85*, 106710. [\[CrossRef\]](#)
3. Montoya, O.; Gil-González, W.; Garces, A. Numerical methods for power flow analysis in DC networks: State of the art, methods and challenges. *Int. J. Electr. Power Energy Syst.* **2020**, *123*, 106299. [\[CrossRef\]](#)
4. Sadovskaia, K.; Bogdanov, D.; Honkapuro, S.; Breyer, C. Power transmission and distribution losses—A model based on available empirical data and future trends for all countries globally. *Int. J. Electr. Power Energy Syst.* **2019**, *107*, 98–109. [\[CrossRef\]](#)
5. Montoya, O.D.; Serra, F.M.; Angelo, C.H.D. On the Efficiency in Electrical Networks with AC and DC Operation Technologies: A Comparative Study at the Distribution Stage. *Electronics* **2020**, *9*, 1352. [\[CrossRef\]](#)
6. Marjani, S.R.; Talavat, V.; Galvani, S. Optimal allocation of D-STATCOM and reconfiguration in radial distribution network using MOPSO algorithm in TOPSIS framework. *Int. Trans. Electr. Energy Syst.* **2018**, *29*, e2723. [\[CrossRef\]](#)
7. Tolabi, H.B.; Ali, M.H.; Rizwan, M. Simultaneous Reconfiguration, Optimal Placement of DSTATCOM, and Photovoltaic Array in a Distribution System Based on Fuzzy-ACO Approach. *IEEE Trans. Sustain. Energy* **2015**, *6*, 210–218. [\[CrossRef\]](#)
8. Gil-González, W.; Montoya, O.D.; Rajagopalan, A.; Grisales-Noreña, L.F.; Hernández, J.C. Optimal Selection and Location of Fixed-Step Capacitor Banks in Distribution Networks Using a Discrete Version of the Vortex Search Algorithm. *Energies* **2020**, *13*, 4914. [\[CrossRef\]](#)
9. Montoya, O.D.; Molina-Cabrera, A.; Chamorro, H.R.; Alvarado-Barrios, L.; Rivas-Trujillo, E. A Hybrid Approach Based on SOCP and the Discrete Version of the SCA for Optimal Placement and Sizing DGs in AC Distribution Networks. *Electronics* **2020**, *10*, 26. [\[CrossRef\]](#)
10. Grisales-Noreña, L.; Montoya, O.D.; Ramos-Paja, C.A. An energy management system for optimal operation of BSS in DC distributed generation environments based on a parallel PSO algorithm. *J. Energy Storage* **2020**, *29*, 101488. [\[CrossRef\]](#)
11. Sirjani, R.; Jordehi, A.R. Optimal placement and sizing of distribution static compensator (D-STATCOM) in electric distribution networks: A review. *Renew. Sustain. Energy Rev.* **2017**, *77*, 688–694. [\[CrossRef\]](#)
12. Saxena, N.K.; Kumar, A. Cost based reactive power participation for voltage control in multi units based isolated hybrid power system. *J. Electr. Syst. Inf. Technol.* **2016**, *3*, 442–453. [\[CrossRef\]](#)
13. Gupta, A.R.; Kumar, A. Energy Savings Using D-STATCOM Placement in Radial Distribution System. *Procedia Comput. Sci.* **2015**, *70*, 558–564. [\[CrossRef\]](#)
14. Samimi, A.; Golkar, M.A. A Novel Method for Optimal Placement of STATCOM in Distribution Networks Using Sensitivity Analysis by DiGSILENT Software. In Proceedings of the 2011 Asia-Pacific Power and Energy Engineering Conference, Wuhan, China, 25–28 March 2011. [\[CrossRef\]](#)
15. Jazebi, S.; Hosseini, S.; Vahidi, B. DSTATCOM allocation in distribution networks considering reconfiguration using differential evolution algorithm. *Energy Convers. Manag.* **2011**, *52*, 2777–2783. [\[CrossRef\]](#)

16. Devi, S.; Geethanjali, M. Optimal location and sizing determination of Distributed Generation and DSTATCOM using Particle Swarm Optimization algorithm. *Int. J. Electr. Power Energy Syst.* **2014**, *62*, 562–570. [[CrossRef](#)]
17. Devi, S.; Geethanjali, M. Placement and Sizing of D-STATCOM Using Particle Swarm Optimization. In *Lecture Notes in Electrical Engineering*; Springer: Chennai, India, 2014; pp. 941–951. [[CrossRef](#)]
18. Bagherinasab, A.; Zadehbagheri, M.; Khalid, S.A.; Gandomkar, M.; Azli, N.A. Optimal Placement of D-STATCOM Using Hybrid Genetic and Ant Colony Algorithm to Losses Reduction. *Int. J. Appl. Power Eng.* **2013**, *2*. [[CrossRef](#)]
19. Singh, B.; Singh, S. GA-based optimization for integration of DGs, STATCOM and PHEVs in distribution systems. *Energy Rep.* **2019**, *5*, 84–103. [[CrossRef](#)]
20. Karami, H.; Zaker, B.; Vahidi, B.; Gharehpetian, G.B. Optimal Multi-objective Number, Locating, and Sizing of Distributed Generations and Distributed Static Compensators Considering Loadability using the Genetic Algorithm. *Electr. Power Compon. Syst.* **2016**, *44*, 2161–2171. [[CrossRef](#)]
21. Rukmani, D.K.; Thangaraj, Y.; Subramaniam, U.; Ramachandran, S.; Elavarasan, R.M.; Das, N.; Baringo, L.; Rasheed, M.I.A. A New Approach to Optimal Location and Sizing of DSTATCOM in Radial Distribution Networks Using Bio-Inspired Cuckoo Search Algorithm. *Energies* **2020**, *13*, 4615. [[CrossRef](#)]
22. Yuvaraj, T.; Ravi, K.; Devabalaji, K.R. Optimal Allocation of DG and DSTATCOM in Radial Distribution System Using Cuckoo Search Optimization Algorithm. *Model. Simul. Eng.* **2017**, *2017*, 2857926. [[CrossRef](#)]
23. Nguyen, K.P.; Fujita, G.; Dieu, V.N. Cuckoo Search Algorithm for Optimal Placement and Sizing of Static Var Compensator in Large-Scale Power Systems. *J. Artif. Intell. Soft Comput. Res.* **2016**, *6*, 59–68. [[CrossRef](#)]
24. Yuvaraj, T.; Ravi, K. Multi-objective simultaneous DG and DSTATCOM allocation in radial distribution networks using cuckoo searching algorithm. *Alex. Eng. J.* **2018**, *57*, 2729–2742. [[CrossRef](#)]
25. Taher, S.A.; Afsari, S.A. Optimal location and sizing of DSTATCOM in distribution systems by immune algorithm. *Int. J. Electr. Power Energy Syst.* **2014**, *60*, 34–44. [[CrossRef](#)]
26. Yuvaraj, T.; Devabalaji, K.; Ravi, K. Optimal Placement and Sizing of DSTATCOM Using Harmony Search Algorithm. *Energy Procedia* **2015**, *79*, 759–765. [[CrossRef](#)]
27. Zhang, T.; Xu, X.; Li, Z.; Abu-Siada, A.; Guo, Y. Optimum Location and Parameter Setting of STATCOM Based on Improved Differential Evolution Harmony Search Algorithm. *IEEE Access* **2020**, *8*, 87810–87819. [[CrossRef](#)]
28. Sedighzadeh, M.; Eisapour-Moarref, A. The Imperialist Competitive Algorithm for Optimal Multi-Objective Location and Sizing of DSTATCOM in Distribution Systems Considering Loads Uncertainty. *INAE Lett.* **2017**, *2*, 83–95. [[CrossRef](#)]
29. Montoya, O.D.; Gil-González, W. On the numerical analysis based on successive approximations for power flow problems in AC distribution systems. *Electr. Power Syst. Res.* **2020**, *187*, 106454. [[CrossRef](#)]
30. Doğan, B.; Ölmez, T. Vortex search algorithm for the analog active filter component selection problem. *AEU—Int. J. Electron. Commun.* **2015**, *69*, 1243–1253. [[CrossRef](#)]
31. Özkıs, A.; Babalık, A. A novel metaheuristic for multi-objective optimization problems: The multi-objective vortex search algorithm. *Inf. Sci.* **2017**, *402*, 124–148. [[CrossRef](#)]
32. Montoya, O.D.; Gil-Gonzalez, W.; Grisales-Norena, L.F. Vortex Search Algorithm for Optimal Power Flow Analysis in DC Resistive Networks With CPLs. *IEEE Trans. Circuits Syst. II Express Briefs* **2020**, *67*, 1439–1443. [[CrossRef](#)]
33. Sharma, A.K.; Saxena, A.; Tiwari, R. Optimal Placement of SVC Incorporating Installation Cost. *Int. J. Hybrid Inf. Technol.* **2016**, *9*, 289–302. [[CrossRef](#)]