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Hybrid GA-SOCP Approach for Placement and Sizing of Distributed Generators in DC Networks

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Abstract: This research addresses the problem of the optimal location and sizing distributed generators (DGs) in direct current (DC) distribution networks from the combinatorial optimization. It is proposed a master–slave optimization approach in order to solve the problems of placement and location of DGs, respectively. The master stage applies to the classical Chu & Beasley genetic algorithm (GA), while the slave stage resolves a second-order cone programming reformulation of the optimal power flow problem for DC grids. This master–slave approach generates a hybrid optimization approach, named GA-SOCP. The main advantage of optimal dimensioning of DGs via SOCP is that this method makes part of the exact mathematical optimization that guarantees the possibility of finding the global optimal solution due to the solution space’s convex structure, which is a clear improvement regarding classical metaheuristic optimization methodologies. Numerical comparisons with hybrid and exact optimization approaches reported in the literature demonstrate the proposed hybrid GA-SOCP approach’s effectiveness and robustness to achieve the global optimal solution. Two test feeders compose of 21 and 69 nodes that can locate three distributed generators are considered. All of the computational validations have been carried out in the MATLAB software and the CVX tool for convex optimization.

Keywords: direct current networks; optimal power flow analysis; metaheuristic optimization; master-slave optimization; genetic algorithms; second-order cone programming

1. Introduction

Recently, the study and analysis of direct current (DC) networks have increased, due to the development of power electronics, advances in renewable generation, and the advantages over alternative current (AC) networks [1,2]. Initially, the DC networks began to operate at high-level voltages in the power transmission for long-distance between systems, since it allows for reducing transmission power losses, enhancing grid stability systems, easily incorporating off-shore wind systems, and managing power flow both direction simply [3–5]. Consecutively, the integration of the DC networks has been widened to medium- and low-voltage level applications, generating concepts, such as the microgrids [6,7].

The main advantages of DC systems as compared to conventional AC systems can be summarized, as follows: (i) fewer power converter devices, (ii) no requirement of synchronizing generators, (iii) better voltage profiles, (iv) less power losses, and (v) do not require frequency and reactive power analysis and controls [8–10].

Two kinds of analysis are used to study DC networks. The first analysis is dynamic, being performed under the time domain, and integrates concepts as primary and secondary controllers [11,12]. The second analysis is static, which computes all of the state variables under stationary conditions. This analysis is known as tertiary control [13], which addresses power flow analysis [2,14], optimal power flow [15], economic dispatch strategies [16], and voltage stability analysis [17,18]. Furthermore, these analyses are integrated with other problems, such as the placement and sizing of multiple distributed generators (DGs) [19], as well as placement and operation of battery energy storage systems in DC networks [20].

In this study, we focus on the problem of the optimal placement and sizing of DGs in DC networks. Although, in the specialized literature, there are few approaches in this area. In [21], an analytical methodology for sizing energy storage systems in DC networks was proposed. This methodology evaluated the profiles of currents that were injected by energy storage systems in order to minimize power losses. In [22], a feasibility study for the integration of photovoltaic systems in a standalone building was carried out. This study considered strong weather scenarios for off-grid systems and variable electricity prices for the grid-connected system. In [23], a probabilistic optimization methodology for estimating uncertainties in loads, plug-in electric vehicles, and renewable generation in DC distribution networks was analyzed. In [19], a hybrid strategy for optimal location and dimensioning of DGs in DC grids was provided, which mixed a hyperplanes search method (HSM) with semidefinite programming (SDP) model. The HSM relaxed the binary variables, while the SDP computed the optimal power flow. In [24], a similar strategy using a sequential quadratic optimal power flow was presented. Both of the techniques ensure the optimal global solution only for a relaxed problem, where the complexity lies in the number of candidate solutions to recover the binary variables. In [25], a tutorial in the general algebraic modeling system (GAMS) for the optimal placement and dimensioning of DGs in DC grids were presented. In [26], the optimal placement and sizing of photovoltaic generators in DC networks were carried out, when considering an environmental-economic dispatch as an objective function. However, the approach presented in [26] was solved in GAMS, which does not guarantee the optimal global solution, since the problem is nonlinear non-convex mixed-integer. In [27], three metaheuristic methods for optimal location and dimensioning of DGs in DC systems were studied. A classical genetic algorithm was employed in the locations stage, which assigns the location of the DGs. While the black-hole, continuous genetic algorithm, and particle swarm metaheuristic methods were used to dimension the DGs. The main problem with these methods is that they have multiple tuning parameters that significantly affect their performance. Additionally, they do not ensure the optimal global solution to the problem. In [28], a methodology for optimal sizing of multiple devices, such as wind turbines, photovoltaic sources, and batteries in hybrid AC/DC grids was provided, which considered the life cycle cost and the life span of the renewable energy resources in the objective function.

After the revision mentioned above of the state-of-the-art, this research's main contributions can be summarized, as follows:

- ✓ The reformulation of the problem of the optimal placement and sizing of DGs in DC networks that allows transforming the original mixed-integer nonlinear programming (MINLP) model into a mixed-integer second-order cone programming (MI-SOCP) one.
- ✓ The solution of the proposed MI-SOCP model with a hybrid master-slave optimization methodology, where the master stage is guided by a genetic algorithm to determine the best locations for the DGs, while the slave stage allows for solving the optimal power flow problem by using the SOCP formulation via interior point methods to determine the best sizes of the DGs.
- ✓ The computational results demonstrate that the hybrid GA-SOCP approach has an efficiency higher than 70% to find the global optimal solution in both test feeders analyzed. In addition, the proposed approach presents better solutions than commercial solvers available in GAMS and classical metaheuristics such as genetic algorithms and particle swarm optimizer, and derived approaches

It is worth mentioning that the master–slave hybrid GA-SOCP approach has not previously been proposed in specialized literature for addressing the problem of the optimal placement and sizing of DGs in electrical networks, even if these are under AC or DC paradigms of operation. This situation was identified as a gap in the scientific literature that this research tries to fill.

The remainder of this document is organized, as follows: Section 2 presents the original MINLP formulation of the problem of the optimal placement and dimensioning of DGs in DC networks by highlighting its nonlinearities and non-convexities. Section 3 presents the proposed hybrid optimization method, which is composed of a conic reformulation problem of the optimal power flow problem in DC networks and the classical GA. Section 4 exposes the main characteristics of the test feeders, which are composed of 21 and 69 nodes with radial structures, where the possibility of installing three DGs is considered. Section 5 presents all of the computational validations of the proposed hybrid GA-SOCP approach and their comparisons with nonlinear solvers available in GAMS and master–slave optimization approaches that are based on genetic algorithms and particle swarm optimization methods and its variants. Section 6 shows the concluding remarks derived from this work and some possible future researches in the area of distribution system optimization.

2. MINLP Model

The problem of the optimal placement and sizing of DGs in DC networks corresponds to an MINLP model since: (i) the problem of sizing of these power sources is modeled with binary variables, and (ii) the problem of the location of these DGs is modeled with continuous variables (power generation and voltage profiles). The optimization objective in this MINLP formulation corresponds to the minimization of the grid power losses produced by the current circulation in all the networks' branches. The mathematical structure of this objective function is presented in Equation (1).

$$\min p_{\text{loss}} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} G_{ij} v_i v_j, \tag{1}$$

where p_{loss} is the value of the objective function regarding power losses in all of the branches of the network, v_i and v_j are the voltage values at nodes i and j , respectively, and G_{ij} is the conductance value taken from the admittance nodal matrix that defines the physical interconnection among all the nodes of the DC network. Observe that \mathcal{N} is the set that contains all of the nodes of the network.

Regarding model constraints, these are related with operational conditions of the DC distribution networks. These constraints are presented below:

$$p_i^s + p_i^{gd} - p_i^d = \sum_{j \in \mathcal{N}} G_{ij} v_i v_j, \forall i \in \mathcal{N} \tag{2}$$

$$p_{ij} = g_{ij} v_i (v_i - v_j), \forall (i, j) \in \mathcal{L} \tag{3}$$

$$\sum_{i \in \mathcal{N}} p_i^{gd} \leq \alpha \sum_{i \in \mathcal{N}} p_i^d, \tag{4}$$

$$\sum_{i \in \mathcal{N}} y_i \leq N_{dg}^{ava}, \tag{5}$$

$$y_i p_i^{gd, \min} \leq p_i^{gd} \leq y_i p_i^{gd, \max} \forall i \in \mathcal{N} \tag{6}$$

$$p_{ij}^{\min} \leq p_{ij} \leq p_{ij}^{\max} \forall (i, j) \in \mathcal{L} \tag{7}$$

$$v^{\min} \leq v_i \leq v^{\max} \forall i \in \mathcal{N} \tag{8}$$

$$y_i \in \{0, 1\} \forall i \in \mathcal{N} \tag{9}$$

where p_i^s , p_i^{gd} , p_i^d , and p_{ij} represent the power generation in the voltage controlled source (slack nodes), power generation in distributed sources, power consumption in demand nodes, and power flow in distribution lines, respectively. α is a parameter that defines the percentage of power penetration of renewable generation as function of the total demand; y_i is the binary variable associate with the

location ($y_i = 1$) or no ($y_i = 0$) of a distributed generator at node i ; N_{dg}^{ava} is the maximum number of distributed generators available for location in the DC network, $p^{gd,min}$ and $p^{gd,max}$ are the minimum and maximum power generation capabilities allowed for each distributed generator connected at node i . p_{ij}^{min} and p_{ij}^{max} are the lower and upper bounds for the power flow in line that connects nodes i and j , respectively; and, v^{min} and v^{max} are the minimum and maximum voltage bounds in all nodes of the DC distribution network, respectively.

The complete interpretation of the MINLP model defined from (1) to (9) is as follows: Equation (1) defines the objective function of the problem regarding the minimization of the total grid power losses in the whole DC network; Equation (2) is the set of power balance equations applied at each node of the network; Equation (3) defines the amount of power that flows at each line of the DC grid. Expression (4) determines the maximum penetration of the distributed generation in the whole DC distribution network; Inequality constraint (5) bounds the number of generators available for location in the DC grid; inequalities (6), (7), and (8) present the upper and lower bounds for distributed generators, power flow in branches, and voltage variables, respectively. Finally, Expression (9) shows the binary nature of the decision variable regarding the location of distributed generators.

Remark 1. *In the problem of the optimal placement and sizing of distributed generators in DC distribution networks is complex due to two main reasons: (i) the binary nature of the location problem makes necessary integer programming methods to define the best possible location of the distributed generators (master optimization problem); and (ii) the optimal sizing of the distributed generators for each possible location provided by the master stage is a nonlinear non-convex optimization problem due to the product between voltage variables in Expressions (1) to (3).*

To solve the MINLP model (1)–(9), this research proposes a hybrid optimization approach based on genetic algorithms to address the problem of the optimal location of distributed generators and the second-order cone programming model to solve the problem of the optimal sizing of these devices. The main advantage of using SCOP instead of metaheuristics for optimal power flow analysis is that for each possible combination of distributed generator locations, the solution is the global optima [29], which is not possible with any particle swarm derived method that corresponds to the most classical approach for solving this kind of problems in literature [27].

3. Proposed Hybrid GA-SOCP Optimization Approach

We proposed a hybrid GA-SOCP approach, which works by using a master–slave optimization strategy, where the master stage guides the discrete part of the problem (i.e., defines the location of the DGs), and the slave stage deals with their optimal sizing, in order to address the problem of the optimal location of distributed generators in DC networks in this research.

3.1. Slave Stage: Second-Order Cone Programming

The SOCP is a branch of the convex optimization that allows transforming some nonlinear non-convex optimization problems into conic convex equivalents that guarantee the global optimum of the equivalent model [30]. In the case of the optimal power flow problem, the SOCP equivalent model has the same global optimum that the original nonlinear model, which is the main advantage of this technique in power system analysis [29]. Here, we present the SOCP version of the optimal power flow problem that addresses distributed generators' optimal sizing. For this purpose, let us suppose that all of the binary variables regarding the distributed generators' location, i.e., y_i has been fixed by the master stage, which will be presented in this study. In this sense, the SOCP deals with the Equality constraints (2) and (3), which are the only continuous nonlinear non-convex constraints.

The convexification of the optimal power flow problem is made using the following auxiliary variable:

$$z_{ij} = v_i v_j, \tag{10}$$

note that, from (10), we know that $z_{ii} = v_i^2$ and $z_{jj} = v_j$, which implies that if we elevate both sides of (10) at the square, then we have:

$$z_{ij}^2 = z_{ii} z_{jj}. \tag{11}$$

Now, in order to obtain an equivalent representation of (11), we use the hyperbolic relation of the product between two variables as follows:

$$\begin{aligned} z_{ij}^2 &= z_{ii} z_{jj}, \\ &= \frac{1}{4} (z_{ii} + z_{jj})^2 - \frac{1}{4} (z_{ii} - z_{jj})^2, \\ (2z_{ij})^2 + (z_{ii} - z_{jj})^2 &= (z_{ii} + z_{jj})^2, \\ \left\| \begin{matrix} 2z_{ij} \\ z_{ii} - z_{jj} \end{matrix} \right\| &= z_{ii} + z_{jj}. \end{aligned} \tag{12}$$

Note that Equation (12) is the conic equivalent of (11), since, to reach this result, only algebraic manipulations have been made; in addition, it is still non-convex due to the equality symbol; nevertheless, it can be relaxed with a lower-equal symbol as recommended in [13,31,32], which becomes (12) into a conic convex constraint, as presented below:

$$\left\| \begin{matrix} 2z_{ij} \\ z_{ii} - z_{jj} \end{matrix} \right\| \leq z_{ii} + z_{jj}. \tag{13}$$

Remark 2. To obtain a SOCP equivalent of the optimal power flow problem, we can recur to substitute (11) into Expressions (1) to (3), which produce equivalent mixed-integer second-order cone programming (i.e., MISOCP) model to address the problem of the optimal placement and sizing of distributed generators in DC distribution networks.

The complete MISOCP model is presented below:

$$\min p_{\text{loss}} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} G_{ij} z_{ij}, \tag{14}$$

$$p_i^s + p_i^{gd} - p_i^d = \sum_{j \in \mathcal{N}} G_{ij} z_{ij}, \forall i \in \mathcal{N} \tag{15}$$

$$p_{ij} = g_{ij} (z_{ii} - z_{ij}), \forall (i, j) \in \mathcal{L} \tag{16}$$

$$\left\| \begin{matrix} 2z_{ij} \\ z_{ii} - z_{jj} \end{matrix} \right\| \leq z_{ii} + z_{jj}, \forall (i, j) \in \mathcal{N} \tag{17}$$

$$\text{Expressions (4) to (9)}, \tag{18}$$

Remark 3. Observe that the slave optimization stage solves the proposed MI-SOCP model once all the binary variables have been assigned on the master stage with the main advantage that the SOCP structure of the power flow model guarantees the global optimum finding [30].

3.2. Master Stage: Genetic Algorithm

The genetic algorithm is one of the most classical optimization methodologies from the family of combinatorial nature-inspired optimization methods [33]. The GA algorithm deals with complex optimization problems that involve integer (binary) variables, using three evolution rules: selection, recombination, and mutation [34]. Here, we select the GA optimizer to deal with the problem of the location of the distributed sources in DC networks using an integer combination that allows for minimizing the infeasibilities caused by classical binary codifications as recommended in [34]. The complete optimization structure of the proposed GA is presented below.

3.2.1. Initial Population

The most of nature-inspired optimization algorithms work with a list of potential solutions named “population”. This list has the complete information regarding the solution of the problem under study. Here, we consider an integer population to determine the location of the DGs in the Dc network. This population W^t (being t the iterative counter) takes the following form:

$$W^t = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1a} \\ w_{21} & w_{22} & \cdots & w_{2a} \\ \vdots & \vdots & \ddots & \vdots \\ w_{b1} & w_{b2} & \cdots & w_{ba} \end{bmatrix}$$

where a is the number of variables of interest and b is the number of the individuals in the population.

Note that the component w_{km} in the initial population matrix is calculated, as follows:

$$w_{km} = \text{round}(1 + \text{rand}(\cdot)(n - 1)), \forall k = 1, 2, \dots, b, \forall m = 1, 2, \dots, n. \tag{19}$$

where $\text{round}(\cdot)$ determines the integer part of the argument and $\text{rand}(\cdot)$ is a random number generated with a normal Gaussian distribution in the interval $[0, 1]$. It is worth remembering that n is the number of nodes and it corresponds to the cardinality of the set \mathcal{N} , i.e., $n = |\mathcal{N}|$.

Remark 4. *An essential stage during the creation of the initial population is that it is mandatory to guarantee that each potential individual W_i^t is different from each other $W_j^t; \forall j \neq i$. This condition is known in genetic algorithms terminology as the diversity verification.*

3.2.2. Fitness Function Evaluation

It is needed to define the fitness function in order to evaluate the initial population. The fitness function in metaheuristic optimization is the “adaptation” of the objective function to deal with possible infeasibilities in the solution space [35,36]. However, in the proposed MI-SOCP model, we guarantee that each individual is feasible, since the SOCP model is correctly constrained by the optimization model (14)–(18). In this sense, our proposed methodology’s fitness function is equal to the objective function that is defined in Equation (14).

3.2.3. Selection

The selection (tournament) is the procedure where a subset of parents is chosen from the initial population to obtain a set of potential solutions (offsprings). Here, we select the most straightforward selection method that consists of selecting two individuals (a different one to each other) from the population, named as the parents.

3.2.4. Recombination

The recombination is a heuristic procedure where the genetic information of the parents is crossed to generate offsprings. To present this procedure, let us consider two parents that are provided by the selection stage as follows:

$$W_i^t = [1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0] \tag{20}$$

$$W_j^t = [0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1]. \tag{21}$$

Note that, in this example, the each parent has 10 genes ($n = 10$). To make the recombination it is generate a random number between 1 and $n - 1$. Consider that this number is 6. With this number is make the recombination procedure by adding the first 6 genes of the individual W_i^t with the last 4 genes of W_j^t ; in addition, the first 6 genes of the individual W_j^t are added to the last 4 genes of W_i^t . The resulting offsprings (Os_i^t and Os_j^t) are presented below.

$$Os_i^t = [1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1], \tag{22}$$

$$Os_j^t = [0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0]. \tag{23}$$

3.2.5. Mutation

The mutation is a procedure in which a subset of genes is changed. This procedure modifies the initial information of the parents after the recombination procedure. To make this procedure, we use the mutation with a unique point for each offspring, which implies that two random number between 1 and $n - 1$ is generated. Suppose that, for Os_i^t , the random number is 3 and for Os_j^t this number is 9, which produces the following mutations in both offsprings.

$$Os_i^t = [1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1], \tag{24}$$

$$Os_j^t = [0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 0]. \tag{25}$$

Note that both of the offsprings are different from their parents, since both have new information added by the mutation procedure.

3.2.6. Fitness Function Evaluations and Replacement of the Population

Once the offsprings have been created by the selection, recombination, and mutation stages, these are evaluated in the fitness function, i.e., optimization model (14)–(18). Let us suppose that the fitness function of the offspring Os_i^t is $z(Os_i^t)$ and the fitness function of the offspring Os_j^t is $z(Os_j^t)$. Subsequently, the potential offspring that will be part of the population is selected as the minimum of both fitness function values: (i) select Os_i^t if $z(Os_i^t) \geq z(Os_j^t)$, or (ii) select Os_j^t otherwise.

In order to decide whether the winner offspring is added to the population, two aspects must be verified: (i) is the fitness function of the winner offspring better than the worst individual in the population? If the answer is positive, then, (ii) is the winner offspring different from each of the individuals in the population? If the answer is also positive, then the winner offspring replace the worst individual in the population.

Note that, if one of the previous questions has a negative response, then a new iteration is started without modification in the population.

3.2.7. Stopping Criteria

In order to finalize the searching procedure of the proposed genetic algorithm, the following aspects are considered:

- ✓ If the maximum number of iteration t_{\max} is reached, the optimization procedure ends by reporting the best solution obtained in the current population.
- ✓ If during τ_{\max} consecutive iterations, the best objective function has not improved, then the optimization procedure ends by reporting the best solution obtained in the current population.

Remark 5. *The genetic algorithm that is presented in the subsections mentioned above is known in the specialized literature as the Chu & Beasley genetic algorithm [34]. It does not replace all the population individuals in the same iteration, as is made by the classical genetic algorithm.*

3.3. Pseudocode of the Proposed Hybrid Optimization Algorithm

Algorithm 1 summarized the main aspects of the proposed hybrid GA-SOCP method for optimal placement and sizing of distributed generators in DC distribution networks.

Algorithm 1: Proposed master-slave optimization approach for the optimal location and sizing of distributed generators (DGs) in DC distribution networks

- 1: **Inputs:**
 - 2: Read the data of the DC network;
 - 3: Define the number of generators available, i.e., N_{dg}^{ava} ;
 - 4: Define the percentage of penetration of distributed generation i.e., α ;
 - 5: Generate the initial population W^0 ;
 - 6: Evaluate the fitness function for each individual in the population (see the slave stage);
 - 7: Make $t = 0$;
 - 8: **while** $t \leq t_{\max}$ **do**
 - 9: Select two parents from the population;
 - 10: Make recombinations for generating two offspring individuals;
 - 11: Apply the mutation operator for each offspring population;
 - 12: Evaluate the fitness function of the offspring individuals, i.e., slave stage (see optimization model (14)–(18));
 - 13: Select the offspring with the minimum fitness function;
 - 14: Replace the worst individual in the population with the selected offspring if the offspring has the best objective function and fulfills the diversity criterion;
 - 15: **if** $\tau \geq \tau_{\max}$ **then**
 - 16: Select as the solution of the problem, the individual with the best objective function in the current population W^t ;
 - 17: Return the optimal solution concerning the location and sizing of DGs (see the slave stage);
 - 18: **break**;
 - 19: **end if**
 - 20: $t = t + 1$;
 - 21: **end while**
 - 22: **Output:**
 - 23: The best solution is found for the MINLP model;
-

4. Test Systems

In this section, the main characteristics of the 21- and 69-node test feeders are presented. These grids are mainly used in power system analysis for DC networks, including voltage stability, power flow analysis, and distributed generators' optimal location [37]. Both of the test feeders are described below.

4.1. 21-Node Test Feeder

The 21-node test feeder is a DC distribution network that is composed of 21 nodes and 20 branches that generates a radial configuration where the voltage-controlled source is connected at node 1. This source supports a voltage profile of 1.00 pu [19]. The complete information regarding constant power consumption and branches are presented in Figure 1 and Table 1, respectively [19].

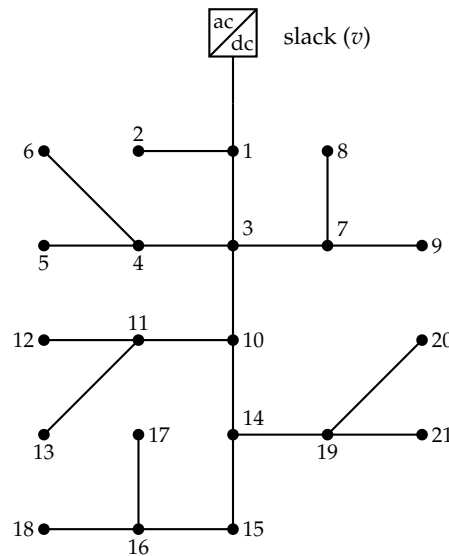


Figure 1. Electrical connection between nodes in the 21-node test feeder.

Table 1. Parametric information of the 21-node test system.

Node <i>i</i>	Node <i>j</i>	R_{ij} [p.u]	P_j [p.u]	Node <i>i</i>	Node <i>j</i>	R_{ij} [p.u]	P_j [p.u]
1	2	0.0053	0.70	11	12	0.0079	0.68
1	3	0.0054	0.00	11	13	0.0078	0.10
3	4	0.0054	0.36	10	14	0.0083	0.00
4	5	0.0063	0.04	14	15	0.0065	0.22
4	6	0.0051	0.36	15	16	0.0064	0.23
3	7	0.0037	0.00	16	17	0.0074	0.43
7	8	0.0079	0.32	16	18	0.0081	0.34
7	9	0.0072	0.80	14	19	0.0078	0.09
3	10	0.0053	0.00	19	20	0.0084	0.21
10	11	0.0038	0.45	19	21	0.0082	0.21

Note that all of the values in Table 1 are calculated when considering 1 kV and 100 kW as the voltage and power bases, respectively.

4.2. 69-Node Test Feeder

This test feeder is a classical AC distribution network that is employed in the literature, such as shunt devices’ optimal location for power losses minimization [37]. Notwithstanding, in this study, we used its DC adaptation reported in [37]. Figure 2 depicts the 69-node test feeder configuration and Table 2 reports the branch parameters and load information. Furthermore, for simulation purposes, 12.66 kV and 100 kVA are used as the test system’s voltage and power bases.

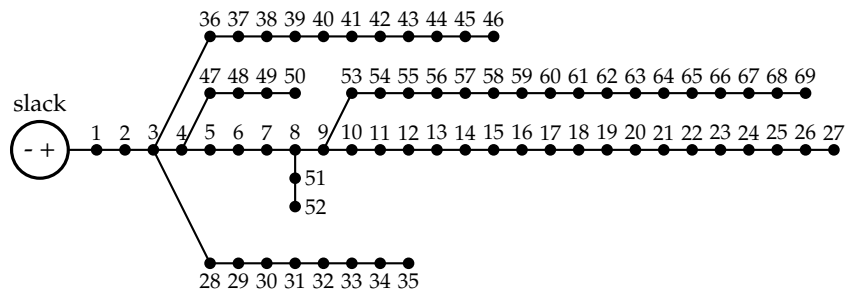


Figure 2. Schematic connection among nodes for the 69-node test feeder.

Table 2. Parametric information of the 69-node test system.

Node <i>i</i>	Node <i>j</i>	R_{ij} [Ω]	P_j [kW]	Node <i>i</i>	Node <i>j</i>	R_{ij} [Ω]	P_j [kW]
1	2	0.0005	0	3	36	0.0044	26
2	3	0.0005	0	36	37	0.0640	26
3	4	0.0015	0	37	38	0.1053	0
4	5	0.0215	0	38	39	0.0304	24
5	6	0.3660	2.6	39	40	0.0018	24
6	7	0.3810	40.4	40	41	0.7283	102
7	8	0.0922	75	41	42	0.3100	0
8	9	0.0493	30	42	43	0.0410	6
9	10	0.8190	28	43	44	0.0092	0
10	11	0.1872	145	44	45	0.1089	39.22
11	12	0.7114	145	45	46	0.0009	39.22
12	13	1.0300	8	4	47	0.0034	0
13	14	1.0440	8	47	48	0.0851	79
14	15	1.0580	0	48	49	0.2898	384.7
15	16	0.1966	45	49	50	0.0822	384.7
16	17	0.3744	60	8	51	0.0928	40.5
17	18	0.0047	60	51	52	0.3319	3.6
18	19	0.3276	0	9	53	0.1740	4.35
19	20	0.2106	1	53	54	0.2030	26.4
20	21	0.3416	114	54	55	0.2842	24
21	22	0.0140	5	55	56	0.2813	0
22	23	0.1591	0	56	57	1.5900	0
23	24	0.3463	28	57	58	0.7837	0
24	25	0.7488	0	58	59	0.3042	100
25	26	0.3089	14	59	60	0.3861	0
26	27	0.1732	14	60	61	0.5075	1244
3	28	0.0044	26	61	62	0.0974	32
28	29	0.0640	26	62	63	0.1450	0
29	30	0.3978	0	63	64	0.7105	227
30	31	0.0702	0	64	65	1.0410	59
31	32	0.3510	0	65	66	0.2012	18
32	33	0.8390	10	66	67	0.0047	18
33	34	1.7080	14	67	68	0.7394	28
34	35	1.4740	4	68	69	0.0047	28

5. Computational Implementation

The proposed hybrid optimization model, i.e., the GA-SOCP method, has been implemented in the MATLAB software version 2019b on a desktop computer with an INTEL(R) Core(TM) i7-7700 2.8-GHz processor and 16.0 GB of RAM running on a 64-bit version of Microsoft Windows 10 Home. To validate our proposed MI-SOCP formulation solved with the proposed hybrid GA-SOCP approach, we employed different MINLP solvers that were available in the GAMS software (i.e., SBB and CONOPT solvers), and some metaheuristic approaches reported in [37]. In [37], the genetic algorithm has been combined with three continuous methods, as follows: black-hole optimizer (GA-BHO),

particle swarm optimization (GA-PSO), and continuous genetic algorithm (GA-CGA). In addition, the following simulating conditions are considered:

- ✓ Three distributed generators are available for installation in the DC test feeder; wherein, the case of the 21-node test system, their maximum individual allowed size is 1.5 pu and, in the case of the 69-node test feeder, this bound is 12 pu.
- ✓ The maximum power penetration in the 21-node test feeder is 40% and, in the case of the 69-node test feeder, the maximum penetration is 40% regarding the total power demand of each one of the DC networks.

Remark 6. All the genetic algorithms employed as the master stage in the proposed and comparison methods have adjusted using a trial and error procedure, which have been produced as parameters: 10 individuals in the population, 100 iterations, recombination, and mutation rates 50%. In addition, to evaluate statistically speaking these methods, 100 consecutive iterations were made for each method.

5.1. 21-Node Test Feeder

Table 3 reports the numerical comparisons between the proposed hybrid optimization method and the literature approaches for the 21-node test feeder. Note that this problem’s optimal solution corresponds to nodes 9, 12, and 16, with power injections of 0.8350 pu, 1.0258 pu, and 1.4632 pu, respectively.

Table 3. Comparison of different approaches in the 21-node test feeder.

Method	Nodes	σ [p.u.]	μ [p.u.]	min [p.u.]	t_{ave} [s]
GA-BHO	{9,12,16}	2.2761×10^{-03}	0.0368	0.0318	111.9440
GA-CGA	{9,12,16}	1.3537×10^{-03}	0.0329	0.0311	33.1974
GA-PSO	{9,12,16}	1.8437×10^{-03}	0.0319	0.0306	58.0934
MIQP	{9,12,16}	---	0.0306	0.0306	8.9688
SSB	{9,12,16}	---	0.0306	0.0306	5.3653
GA-SOCP	{9,12,16}	1.1050×10^{-03}	0.0309	0.0306	52.146

From Table 3, we can observe that:

- ✓ All of the comparison methods, as well as the proposed approach, identifies the best optimal location in nodes 9, 12, and 16. However, due to the aleatory nature of the continuous metaheuristics (i.e., BHO, PSO, and CGA), both of them are stuck local optimums, and only the GA-PSO approach can deal with the global optimal solution.
- ✓ The mixed-integer quadratic programming (MIQP) model and the SBB solver (for the exact MINLP model) reach the global optimal solution with the lowest computational effort, which is attributable to the efficiency of the GAMS interface.
- ✓ The standard deviation of the proposed GA-SOCP programming is the lowest when compared with metaheuristics; this situation occurs because the proposed approach works with an exact optimization method based on second-order cone programming that has a unique solution for each combination of DGs provided by the GA in the master stage.

Figure 3 presents the histogram after the 100 consecutive evaluations of the proposed methodology for optimal placement and sizing distributed generators in DC distribution networks in order to present the efficiency of the proposed GA-SOCP approach.

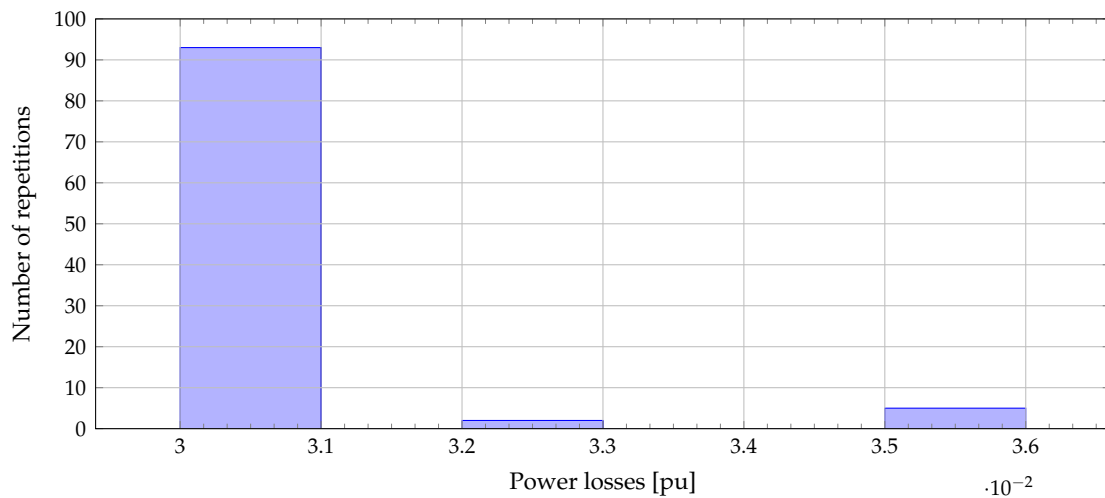


Figure 3. Behavior of the proposed hybrid GA-PSO after 100 consecutive evaluations in the 21-node test feeder.

Note that the histogram presented in Figure 3 shows that more than 90 solutions (93 exactly) reached by the proposed hybrid GA-SOCP method has the capability of finding the global optimal solution (i.e., 0.0306 pu) regarding power losses, which confirms the efficiency and robustness of the studied methodology for optimal placement and sizing DGs in DC distribution networks.

Remark 7. We can ensure that the solution finds by the proposed GA-SOCP approach is the optimal one, since all the possibilities (i.e., 1140 options) for the location of DGs in the 21-node test feeder have been evaluated comprehensively via nested loops, which have confirmed that nodes 9, 12 and 16 are the best possible location for these DGs reaching a final power losses of 0.0306 pu. It is worth mentioning that this exhaustive evaluation takes about 800 s.

Figure 4, which presents the percentage of improvement reached by the comparative methods in contrast to the proposed hybrid optimization approach, shows the total grid improvement about the power losses minimization.

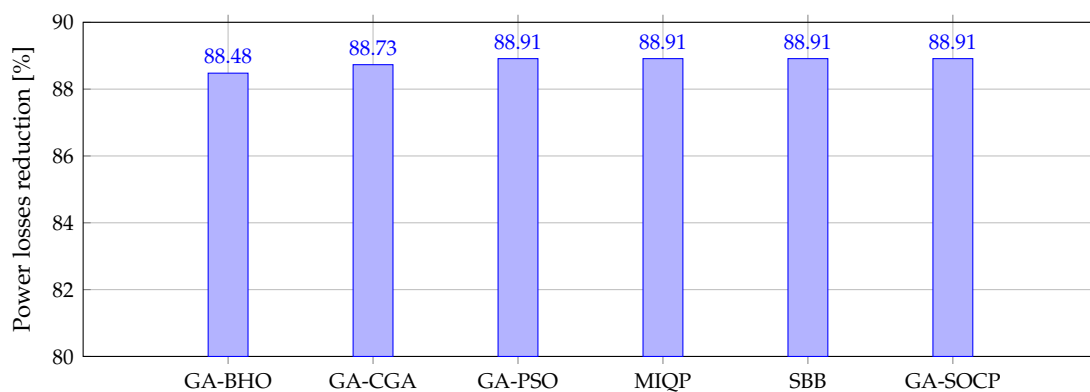


Figure 4. Percentage of improvement regarding power losses minimization in the 21-node test feeder.

Figure 4 shows the positive impact that has the inclusion of distributed generation in DC distribution networks, since all of the comparative methods and the proposed hybrid approach present improvements that are higher than 88% with a total penetration of 60% in a distributed generation regarding total power consumption. In addition, these results confirm the global capabilities of the proposed hybrid GA-SOCP method for dealing with the problem of the optimal location and sizing DGs in DC feeders.

5.2. 69-Node Test Feeder

Table 4 presents the numerical results of the proposed and comparative methods. These results allow observing that:

- ✓ All of the comparative methods are stuck in locally optimal solutions, including the MIQP approach and the MINLP model solved in GAMS through the SBB solver since these finds final power losses about 0.1660 pu and 0.1574 pu, respectively. However, the best optimal solution is reached by the GA-SOCP method with 0.1573 pu.
- ✓ Some of the metaheuristic approaches (i.e., GA-CGA and GA-PSO) find the best positions for installing the DGs being these nodes 21, 61, and 64; nevertheless, due to the random procedures in the slave stages, these find local solutions.
- ✓ Even if standard deviations of the GA-PSO and GA-CGA are lower than the proposed GA-SOCP, these values imply that these methods are always trapped in the local solutions, and they do not have the capacity of scape from these solutions to explore other promissory regions of the solution space.

Table 4. Comparison of different approaches in the 69-node test feeder.

Method	Nodes	σ [p.u.]	μ [p.u.]	min [p.u.]	t_{ave} [s]
GA-BHO	{23,61,67}	2.5207×10^{-03}	0.1633	0.1593	713.7193
GA-CGA	{21,61,64}	3.4801×10^{-04}	0.1648	0.1603	218.0169
GA-PSO	{21,61,64}	5.4023×10^{-04}	0.1689	0.1588	378.4731
MIQP	{22,61,64}	---	0.1660	0.1660	24.1868
SBB	{22,61,65}	---	0.1574	0.1574	12.4270
GA-SOCP	{21,61,64}	3.1295×10^{-03}	0.1591	0.1573	66.9100

Note that the best optimal solution reported by the GA-SOCP proposed approach connects DGs in nodes 21, 61, and 64 with generation capabilities of about 1.4140 pu, 10.2630 pu, and 3.8803, respectively. Once again, we can confirm that this solution is the global optimum, since, after the exhaustive evaluation of the solution space (i.e., 50116 possible locations) via nested loops, the same set of nodes and power generations have been identified as the best global solution. It is important to highlight that this exhaustive evaluation takes about 7000 s to revise the whole solution space, which implies that 66.91 s reported by the proposed GA-SOCP approach demonstrates its efficiency and robustness global finding capabilities.

Figure 5 presents the histogram with the optimization behavior of the proposed GA-SOCP method for locating and sizing DGs in DC distribution networks.

From Figure 5, it can be noted that more than 70% of the solutions that were found by the proposed GA-SOCP method find the global optimal solution, and 24% of the additional solutions are better than the mean solutions reported in Table 4 (see μ column). These results confirm the proposed GA-SOCP optimization approach’s efficiency and robustness for solving the optimal placement and sizing of DGs in DC distribution networks with global optimum finding capabilities.

Figure 6 presents the percentage of power losses improvement reached by the comparative and the proposed optimization approaches.

Note that the percentage of reduction of power losses reported in plot 6 confirms the global optimization capabilities of the proposed GA-SOCP approach, since it is the only one methodology that reports the highest percentage of reduction with 89.78%, only followed by the SBB methodology with 89.77% of reduction.

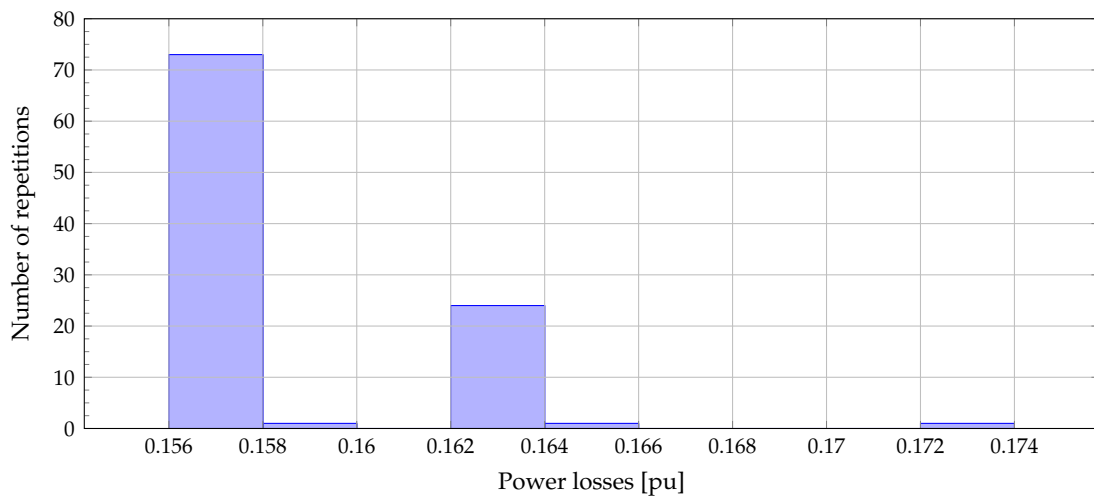


Figure 5. Behavior of the proposed hybrid GA-PSO after 100 consecutive evaluations in the 69-node test feeder.

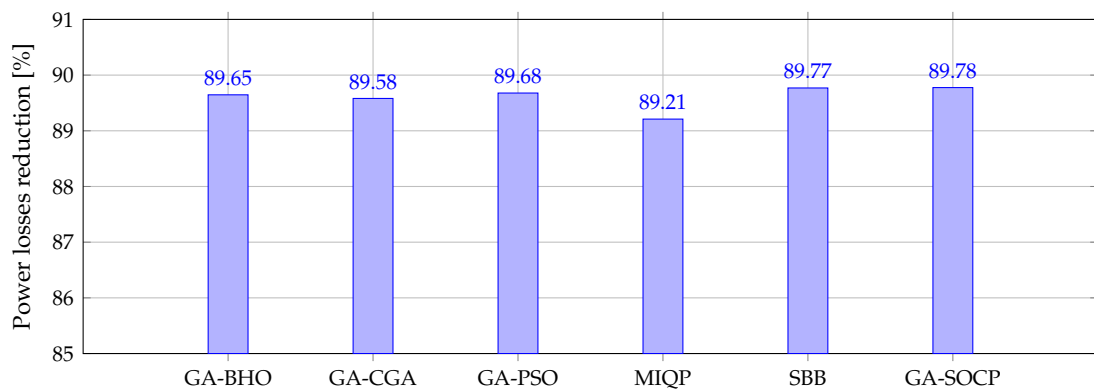


Figure 6. Percentage of improvement regarding power losses minimization in the 69-node test feeder.

5.3. General Commentaries

After applying the comparative and the proposed optimization methodology for optimal locating and sizing DGs in DC distribution networks, we can affirm that:

- ✓ The processing times in both test systems have been lower than 70 s, which implies that for planning purposes, our approach is perfectly applicable since, with a fraction of the time taken by the exhaustive search method, this can deal with the global optimal solution. In addition, the processing times of the comparative metaheuristic methods increase rapidly as a function of the test feeder size (see the last columns of Tables 3 and 4) with the main disadvantage that due to the random behavior of the slave stages, i.e., BHO, CGA, and PSO, it is not possible to find the optimal solution of the studied optimization problem.
- ✓ The methodologies implemented in GAMS, i.e., the MIQP and the MINLP model, both solved with the SBB tool available in GAMS, have speedy processing times. However, the linearization of the power flow equations in the MIQP approach and the nonlinear and non-convex structure of the exact MINLP model make that these get stuck in optimal locations as the solution space grows.
- ✓ The main advantages of the proposed hybrid GA-SOCP master-slave optimization method are: (i) the possibility of using a compact codification with integer numbers (nodes) that allows for guaranteeing feasibility in all the stages of the genetic algorithm procedures, and (ii) the guarantee of the global optimal solution for each combination of distributed generators thanks to the second-order cone structure of the optimal power flow problem in the slave stage.

6. Conclusions and Future Works

A hybrid master-slave optimization approach for optimal placement and sizing distributed generators in DC distribution networks while using a combination of the classical Chu & Beasley genetic algorithm in the master stage and a second-order cone programming model in the slave stage, i.e., GA-SOCP. This algorithm has an important advantage regarding classical metaheuristics, since solving the optimal power flow problem for optimal dimensioning the distributed generators provided by the master stage with a SCOP model guarantees the existence and uniqueness of the optimal solution, which is not possible with combination methods, such as BHO, CGA, and PSO, where random procedures affect the optimization process. Numerical comparisons with classical metaheuristics and exact methodologies demonstrate that, for the studied test feeders, the proposed GA-SOCP approach deals with the global optimal solution. Simultaneously, it is not the case of the comparative methods in the 69-node test feeder scenario. The effectiveness of the proposed GA-SOCP method to find the optimal solution after 100 consecutive evaluations is about 73% for the 21-node test feeder, and 93% in the case of the 69-node test system, which implies the high efficiency of the studied hybrid optimization approach to deal with MINLP problems.

Regarding future works, it will be possible to make the following researches: (i) extend the proposed optimization methodology to the optimal location and sizing of distributed generators and capacitor banks in AC distribution networks, (ii) to apply the proposed GA-SOCP approach to the problem of the optimal placement and sizing battery energy storage systems in AC and DC networks, and (iii) to develop an economic dispatch approach to optimal locating renewable generation at isolated distribution networks when considering variations in the load curve and the primary energy resources.

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Abbreviations

The following abbreviations are used in this manuscript:

Acronyms	
AC	Alternating Current
DG	Distributed Generator
GA	Chu & Beasley genetic algorithm
HSM	Hyperplanes search method
NLP	Nonlinear Programming
CVX	Matlab Software for Disciplined Convex Programming
GAMS	General Algebraic Modeling System
MI	Mixed-integer
MINLP	Mixed-integer nonlinear programming
SOCP	Second-order cone programming
SDP	Semidefinite programming

Sets and subscripts

\mathcal{L}	Set of lines
\mathcal{N}	Set of nodes
d	Demand
gd	Distributed generator
s	Slack node
i or j	Node

Parameters

G	Conductance matrix
G_{ij}	Component of the conductance matrix that relates nodes i and j
N_{dg}^{ava}	Maximum number of distributed generators available
g_{km}	Conductance between nodes k and m
p^d	Power demanded by loads
p^s	Power generated by generators
p^{gd}	Power generated by distributed source
$\text{round}(\cdot)$	Integer part of the argument
$\text{rand}(\cdot)$	Random number
Os_i^t	Offsprings of population i in period t

Variables

v_i	Voltage profile at node i
z_{ij}	cross-product of voltage i with voltage j
w_{km}	Population position k and m
y_i	binary variable associate with the location at node i
W	Population matrix

Control variables

p^s	Power generated by conventional generator
p^{gd}	Power generated by renewable energies
p_{km}	Power flow between nodes k and m
y_i	Binary variable for location of DG
p^s	Power generated vector by conventional generator
p^{gd}	Power generated vector by renewable energies

Limits

N_{gd}^{\max}	Maximum number of DG
v^{\min}, v^{\max}	Minimum and maximum voltage profile
V^{\min}, V^{\max}	Minimum and maximum voltage profile squared
$p^{s,\min}, p^{s,\max}$	Minimum and maximum by conventional generator
$p^{gd,\min}, p^{gd,\max}$	Minimum and maximum by generated source
$p_{km}^{\min}, p_{km}^{\max}$	Minimum and maximum power flow between nodes k and m

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