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# Optimal minimum time control for a direct current motor using bang-bang control

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**Abstract.** The calculation of variations initially it was a technique that was born in the field of physics and its great contributions have been taken advantage of by the engineering sciences in problems that involve finding a minimum trajectory that minimizes time, energy among other variables of interest. The following document will show how to find an optimal control law in order to minimize the time to bring the system that in this case is a direct current motor from a known initial state to a desired final state, using the Bang-control technique. Bang In direct current motor, the variables to be controlled are the field current, the input voltage and the angular velocity developed by the motor. For all the above, the control technique is not only innovative but also useful and practical for energy systems.

## 1. Introduction

In control theory, a bang-bang controller (2-step or on-off controller), also known as hysteresis controller, is a feedback controller that changes abruptly between two states. These controllers can be performed in terms of any element that provides hysteresis. They are often used to control a plant that accepts a binary input, for example, an oven that is completely on or completely off. The most common residential thermostats are the bang-bang controllers. Heaviside's step function in its discrete form is an example of a bang-bang control signal. Due to the discontinuous control signal, the systems that include bang-bang controllers are variable structure systems, and the bang-bang controllers are therefore variable structure controllers [1,2].

In the problems of optimal control, it sometimes happens that a control is restricted to being between a lower and an upper limit. If the optimal control changes from one extreme to the other (that is, it is never strictly within the limits), that control is known as a bang-bang solution [3]. Bang-bang controls often arise in minimum time problems Figure 1. For example, if it is desired to stop a car in the shortest possible time in a certain position sufficiently in front of the car, the solution is to apply maximum acceleration to the single change point, and then apply the maximum braking to stop exactly. In the desired position [4].

A familiar everyday example is to bring the water to a boil in the shortest possible time, which is achieved by applying heat and then turning it off when the water comes to a boil. A domestic example of a closed circuit is the majority of thermostats, in which the heating element or the air conditioning



compressor is operating or not, depending on whether the measured temperature is above or below the set point [5,6]. Bang-bang solutions also arise when the Hamiltonian is linear in the control variable; the application of the minimum or maximum principle of Pontryagin [7]. will then lead to pushing the control to its upper or lower limit according to the sign of the coefficient of  $u(t)$  in the Hamiltonian [8].

In summary, bang-bang controls are actually optimal controls in some cases, although they are often also implemented due to their simplicity or convenience [9].

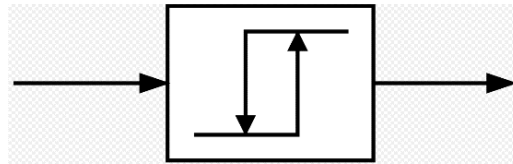


Figure 1. Symbol for a Bang-Bang control.

## 2. Content

*Definition 1 (Minimum time).* Given an  $x_0$  and  $x_1 \in \mathbb{R}^n$  such that  $\exists T, x_1 \in \mathcal{R}_\Omega(T, x_0)$ . Let  $t^*$  be the minimum value of  $t$  such that  $x_1 \in \mathcal{R}_\Omega(t, x_0)$ , That is, Equation (1).

$$t^* = \inf\{t > 0 \mid x_1 \in \mathcal{R}_\Omega(t, x_0)\}, \quad (1)$$

Is the minimum time well defined? The answer is yes, since the set that is taken very small is not empty, bounded below and closed (this comes from the continuity of the mapping  $t \rightarrow \mathcal{R}_\Omega(t, x_0)$ ). The above allows us to establish the following result [10].

*Theorem 1.* If there is  $T > 0$  and  $u \in L^\infty(0, T, \Omega)$  such that  $x(0) = x_0$  and  $x(T) = x_1$ , then there is a path  $x^*$  associated with the control  $u^*$  of optimal time which leads the system from  $x_0$  to  $x_1$ .

On the other hand, also by continuity  $t \rightarrow \mathcal{R}_\Omega(t, x_0)$ , it follows the Equation (2).

$$x_1 \in \partial \mathcal{R}_\Omega(t^*, x_0) = \mathcal{R}_\Omega(t^*, x_0) - \text{int}(\mathcal{R}_\Omega(t^*, x_0)). \quad (2)$$

Finally, when the convexity of  $\mathcal{R}_\Omega(t, x_0)$  is tested, the following is deduced.

*Corollary 1 (Bang-Bang principle).* Denote  $\text{conv}(\Omega)$  to the convex envelope of  $\Omega$ . So, we will have Equations (3) and Equation (4).

$$\mathcal{R}_\Omega(t, x_0) = \{x(t) \mid \in L^\infty(0, T; \Omega)\}, \quad (3)$$

$$\mathcal{R}_{\text{conv}(\Omega)}(t, x_0) = \{x(t) \mid \in L^\infty(0, T; \text{conv}(\Omega))\}. \quad (4)$$

As  $\text{conv}(\Omega) = \text{conv}(\partial\Omega)$  it concludes as shown in Equation (5).

$$\mathcal{R}_\Omega(t, x_0) = \mathcal{R}_{\partial\Omega}(t, x_0). \quad (5)$$

The above equality means that the achievable points are described only by controls that take values at the boundary of the  $\Omega$  set. For example, if  $\Omega = [a, -a]$  with  $a > 0$ , you will have Equation (6).

$$\mathcal{R}_\Omega(t, x_0) = \mathcal{R}_{\{-a, a\}}(t, x_0). \quad (6)$$

Implying that a point is attainable by a control or at values of  $\Omega = [a, -a]$  if and only if, it is attainable by a control that only takes values in  $\{-a, a\}$ . By definition a control  $u$  is said to be extreme if the

associated trajectory satisfies  $x(t) \in \partial\mathcal{R}_\Omega(t, x_0)$  for all  $t$ . In particular, all minimum time control is extreme [11,12].

### 3. Application

Consider the circuit of the direct current motor shown in the Figure 2, it is sought to bring the system from an initial state  $x(0) = x_0$  to a final state  $x(T) = x_f$ , restricted to the time needed to carry the system from initial state to final state be minimal [11,12].

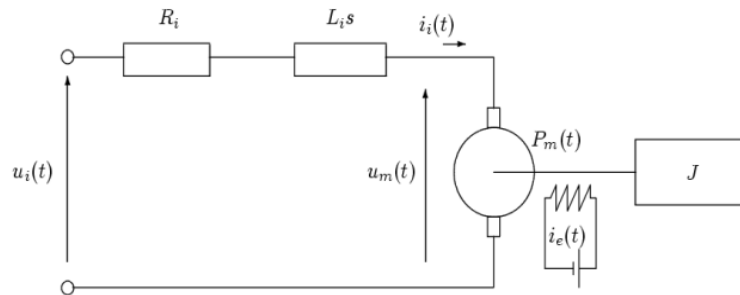


Figure 2. Direct current (DC) motor.

The state equations that describe the behavior of the system described above is given by Equation (7) [8].

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -ax_2(t) + u(t) \end{cases} \quad (7)$$

Being  $x_2$  the position of the motor,  $(x_2)'(t)$  the motor speed, and the time constant of the motor, the input  $u(t)$  refers to the switching function whose ends are +1 and -1.

The following is to find the group of functions that will be denoted as O+ and O-, by taking  $u(t) = 1$  or  $u(t) = -1$  the solutions for the differential equations described in Equation (7) are presented below the Equation (8).

$$\begin{cases} x_1(t) = -\frac{c_1}{a} e^{-at} \pm \frac{1}{a} t \pm \frac{1}{a^2} e^{-at} + c_2 \\ x_2(t) = c_1 e^{-at} \pm \frac{1}{a} t \pm \frac{1}{a} [1 - e^{-at}] \end{cases} \quad (8)$$

These equations define the families of curves which pass through the origin, assuming  $c_1 = c_2 = 0$ , since the system is invariant the time the following system is obtained Equation (9)

$$\begin{cases} x_1(t) = \pm \frac{1}{a} t \pm \frac{1}{a^2} e^{-at} \\ x_2(t) = \pm \frac{1}{a} t \pm \frac{1}{a} [1 - e^{-at}] \end{cases} \quad (9)$$

To determine O+ which corresponds to the input  $u(t) = 1$  solving the system again you get Equation (10).

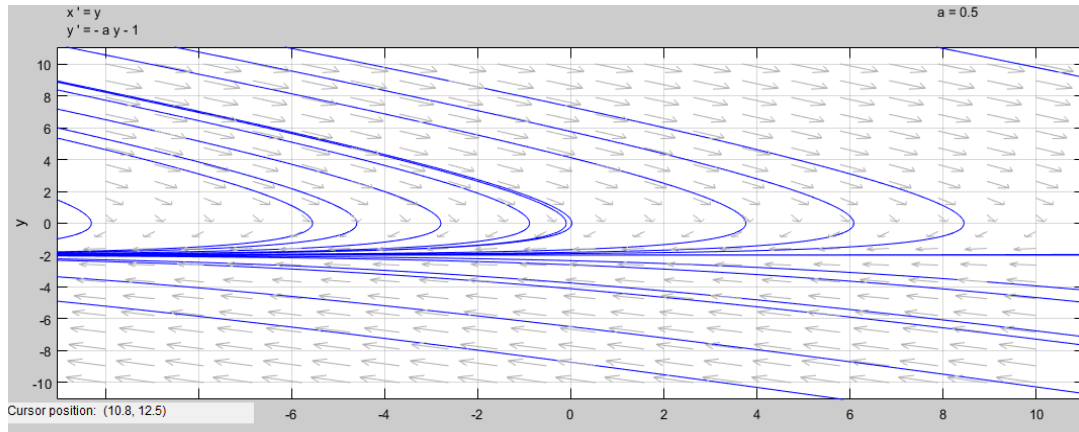
$$x_1(t) = -\frac{1}{a^2} \ln \left( -a \left[ x_2(t) - \frac{1}{a} \right] \right) - \frac{1}{a} x_2(t). \quad (10)$$

Similarly, for O- with  $u(t) = -1$ , you get: Equation (11).

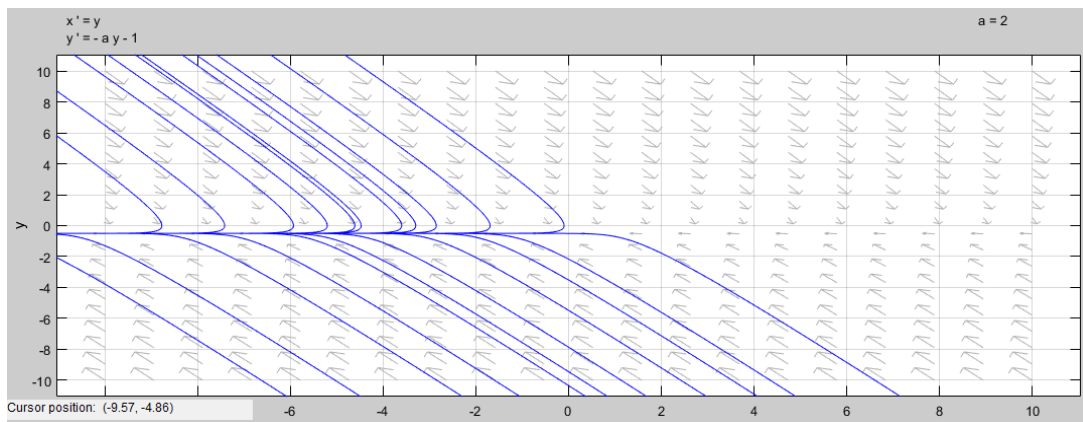
$$x_1(t) = \frac{1}{a^2} \ln \left( a \left[ x_2(t) + \frac{1}{a} \right] \right) - \frac{1}{a} x_2(t). \quad (11)$$

**4. Results**

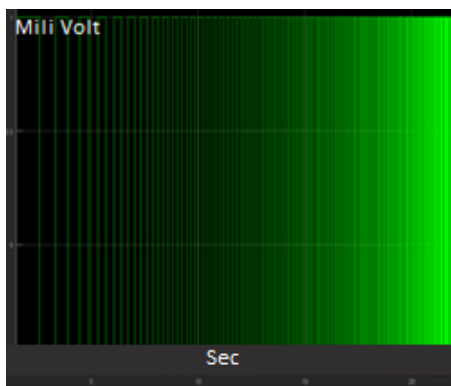
The following figures show the phase diagram for different values of the constant  $a$  and the input  $u(t)$ . Figure 3 and Figure 4 you can notice the differences when the value of the constant varies and the input signal remains constant, in them you can see that the phase diagram tends to different equilibrium points. Figure 5 shows the square commutation signal that increases its commutation time value over time to determine the response in the system; and in Figure 6 the system response quickly adjusts to the reference providing fast and stable control. Finally, Figure 7 shows the dynamic error which is high when the system is switched, but as the controller starts to operate the error decreases.



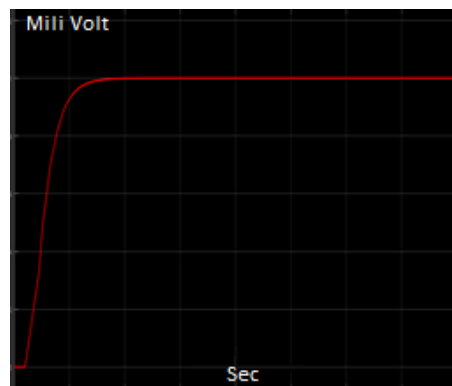
**Figure 3.** Phase diagram for  $a = 0.5$  and  $u(t) = -1$ .



**Figure 4.** Phase diagram for  $a = 2$  and  $u(t) = -1$ .



**Figure 5.** Controller switching signal.



**Figure 6.** System response.

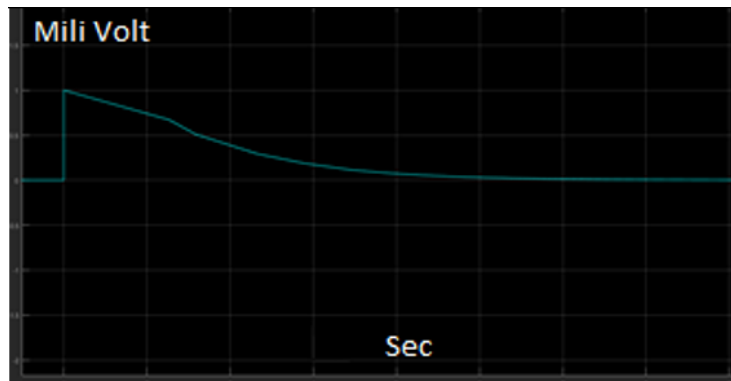


Figure 7. Error dynamics.

## 5. Conclusions

Functional validation has allowed us to observe that it is necessary to properly estimate the dead zone in DC motors in order to establish a more convenient simulation model for small-scale establishment points. The behavior of minimum time and zero impulse envelopes has been observed in the different tests presented, this demonstrates that the controller can be an appropriate solution in mobile arms or conveyor belts where fast responses and fine precision are required.

Switching control is included within the set of hybrid control techniques that combine continuous descriptions with transitions (discontinuities) generally of a discrete type. A high gain proportional control (in its on-off control form) can be seen as a form of switched control due to the discontinuous nature of the resulting signal. The effect of this type of control is a greater force applied to the actuator, which may eventually have high frequency micro-oscillations (chattering) with corresponding harmful effects. The Bang-Bang control is a particular kind of switched control that can be derived by the minimum time optimization problem. This article allowed us to show how a Bang-Bang control strategy, based on the same principle of switching between two admissible values, concentrates the energy of the control signal in a more efficient way, making the desired result at the output obtained with a lower consumption reflected in the minimization of the functional cost.

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