

1 Highlights

2 **Scale-dependent coupling between galactic cosmic rays and trace gases revealed by multi-** 3 **fractal analysis**

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- 5 • MFDFA/MFDCCA reveal scale-dependent GCR–gas coupling
- 6 • CH₄ shows the widest multifractal spectrum; O₃ is intermediate
- 7 • Cosmic rays leave a subtle, scale-selective fingerprint on trace gases

8 Scale-dependent coupling between galactic cosmic rays and trace
9 gases revealed by multifractal analysis

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11 **Abstract**

Galactic cosmic rays (GCR) modulate atmospheric ionisation and may influence reactive greenhouse gases, yet linear correlations have proved inconclusive. We analyse 74 328 hourly observations (2016–2024) of pressure-corrected neutron-monitor counts and co-located CH₄ and O₃ mixing ratios from the high-alpine Jungfraujoch station using Multifractal Detrended Fluctuation Analysis (MFDFA) and its bivariate extension (MFDCCA). Cosmic rays exhibit a narrow, quasi-monofractal singularity spectrum ($\Delta\alpha_{\text{full}} = 0.43$), consistent with heliospheric modulation as a single dominant driver, whereas O₃ and CH₄ display progressively broader spectra ($\Delta\alpha = 0.78$ and 0.84). Cross-Hurst exponents exceed unity for small-to-moderate fluctuations, indicating super-persistent joint variability on 1–7 d (O₃) and 7–30 d (CH₄) horizons—time-scales compatible with HO_x/NO_x chemistry triggered by GCR ionisation. Quadratic fits to the cross-singularity spectra yield half-maximum widths of 0.39 (CH₄–GCR) and < 0.33 (O₃–GCR), quantifying a broader amplitude hierarchy for methane. Extreme gas anomalies, by contrast, show weak cross persistence, implicating dynamical intrusions rather than ionisation. Multifractal metrics thus expose a scale-selective GCR imprint masked in Pearson and Spearman statistics and suggest that incorporating GCR flux as a multiscale covariate could improve sub-monthly O₃/CH₄ predictions. The approach provides a transferable framework for disentangling cosmic-ray forcing from chemical and dynamical controls in other trace-gas records.

12 *Keywords:*

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14 galactic cosmic rays, methane, ozone, multifractal analysis, atmospheric ionisation, space
15 weather, trace-gas variability

16 **1. Introduction**

17 Galactic cosmic rays (CRs)—relativistic nuclei and electrons accelerated in supernova rem-
18 nants, pulsar magnetospheres and other high-energy astrophysical sites—impinge continuously
19 on Earth’s atmosphere. Their hadronic cascades convert a small fraction of the atmospheric
20 column into a dense swarm of secondary particles (muons, neutrons, pions, . . .) whose sea-level

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21 footprint responds both to heliospheric modulation and to local thermodynamic conditions (Don-
22 nini, 2021; Riádigos et al., 2022; Utomo, 2017). Seasonal and synoptic variations in tropospheric
23 temperature alter meson decay and attenuation lengths, imprinting a measurable temperature ef-
24 fect on muon and neutron monitors (Jeong and Oh, 2020; Maghrabi et al., 2015, 2023). Super-
25 imposed on this meteorological forcing, the 11- and 22-year solar cycles modulate CR fluxes
26 with lags of a few to ~ 14 months that depend on particle rigidity and solar magnetic polarity
27 (Tomassetti et al., 2022; Gong et al., 2025).

28 Beyond their utility as geophysical probes, CRs initiate ion-molecule chemistry that can per-
29 turb key oxidative and radiative tracers. Ion pairs produced in the middle atmosphere trigger
30 fast reactions—up to ten orders of magnitude faster than neutral-neutral pathways—that en-
31 hance HO_x and NO_x abundances (Catone et al., 2023; Satta et al., 2024). The ensuing catalytic
32 cycles destroy ozone and elevate hydroxyl radicals, thereby shortening the lifetime of methane
33 (Naik et al., 2013; Brune et al., 2020). Global-chemistry simulations demonstrate that persistent
34 GCR backgrounds reduce lower-stratospheric O_3 by $\gtrsim 10\%$ and can draw down CH_4 by tens
35 of percent under extreme flux scenarios (Matthes et al., 2017; Tabataba-Vakili et al., 2016). Yet
36 observational verification remains elusive because both CR and gas records exhibit strong trends,
37 long-range persistence, and multiple characteristic time scales.

38 Ionisation rates peak near the Pfozter maximum at $\sim 15\text{--}18$ km, where Galactic CRs deposit
39 $\mathcal{O}(10^4) \text{ eV g}^{-1} \text{ s}^{-1}$ into the air column (Meng et al., 1997; Schnadt et al., 2002; Jackman et al.,
40 2007, 2005). Each ion pair yields, on average, 1.2 reactive OH radicals and ~ 1.6 HO_2 radicals
41 via hydration of the primary ion cluster $\text{H}_2\text{O}^+(\text{H}_2\text{O})_n$ and subsequent proton transfer reactions
42 (Seinfeld and Pandis, 2016). In parallel, impact excitation of N_2 produces $\text{N}(^2\text{D})$ atoms that react
43 with O_2 to form NO, seeding the NO_x cycle (Calisto et al., 2011). The coupled $\text{HO}_x\text{--NO}_x$ system
44 drives two feedback loops: (a) NO catalytically destroys O_3 through the classic $\text{NO} + \text{O}_3 \rightarrow \text{NO}_2$
45 $+ \text{O}_2 / \text{NO}_2 + h\nu \rightarrow \text{NO} + \text{O}$ pathway; and (b) elevated OH shortens the CH_4 lifetime via CH_4
46 $+ \text{OH} \rightarrow \text{CH}_3 + \text{H}_2\text{O}$, thereby lowering CH_4 concentrations and further altering odd-hydrogen
47 budgets (Prather and Hsu, 2010; Naik et al., 2013). Model studies show that a 10% increase
48 in ionisation can raise the zonal-mean OH column by 2–4%, translating into a 1–2% decrease
49 in CH_4 and a concomitant ~ 1 DU (Dobson Unit) ozone loss in the lower stratosphere (Jackman
50 et al., 2016; Scheucher et al., 2018; Grenfell et al., 2007, 2012).

51 Altitude and latitude modulate the sign of these responses. In cold, low- NO_x air masses
52 above Antarctica, CR-induced NO_x can actually enhance O_3 by suppressing halogen-catalysed
53 loss, whereas within the chlorine-rich Arctic vortex the same mechanism amplifies O_3 depletion
54 (Calisto et al., 2011). Tropospheric CH_4 responds on multi-week time scales set by vertical
55 mixing and OH regeneration, whereas stratospheric O_3 can react within days as NO descends
56 from the mesosphere after solar proton events (Maliniemi et al., 2021; Konopka et al., 2007;
57 Portmann et al., 2012). These chemically distinct latency windows— $\sim 1\text{--}7$ days for O_3 versus ~ 7
58 30 days for CH_4 —provide physically motivated benchmarks against which to interpret the scale-
59 resolved MFDCCA spectra derived below. High-altitude stations provide the cleanest natural
60 laboratories for such tests. Jungfraujoch (3580 m.a.s.l.) offers continuous, in-situ monitoring of
61 CR counts alongside near-background CH_4 and O_3 concentrations. Because the site sits above
62 much of the planetary boundary layer, it minimises confounding anthropogenic variability while
63 retaining sensitivity to ionisation changes in the lower stratosphere and upper free troposphere.

64 Ionisation does more than seed HO_x/NO_x chemistry; it also changes the microphysical bud-
65 get of cloud condensation nuclei (CCN). Charged molecular clusters containing H_2SO_4 , NH_3
66 and organics grow more efficiently than neutral clusters, enhancing new-particle formation rates
67 by up to an order of magnitude under clean-marine conditions (Kirkby et al., 2011; Pierce, 2017).

68 Observational studies during the 1980s–1990s reported a positive correlation between low-cloud
69 cover and galactic CR flux (Svensmark and Friis-Christensen, 1997; Friis-Christensen and Svens-
70 mark, 1997; Marsh and Svensmark, 2000), although the signal weakened in the 2000s and re-
71 mains debated (Miyahara et al., 2023; Harrison et al., 2015). Even if the radiative impact is small,
72 the CR–CCN link is relevant here because increases in CCN modulate aqueous-phase production
73 of HO₂ and alter photolysis rates within cloudy columns (Carslaw et al., 2002). Hence cloud-
74 mediated feedbacks provide an additional, albeit indirect, avenue through which CR variability
75 can influence the budgets of O₃ and CH₄—reinforcing the motivation to search for multifractal
76 fingerprints of cosmic rays in these gases.

77 Recent analyses using satellite and reanalysis products report additional evidence and mixed
78 signals on CR–cloud couplings. For the Eastern Mediterranean, Christodoulakis et al. (2019) ex-
79 amined satellite cloud parameters versus CR proxies and discussed regional dependencies, while
80 Pierce (2017) synthesised CLOUD results on ion-mediated nucleation and implications for cloud
81 microphysics. Extending to broader temporal context, Varotsos et al. (2024) investigated the time
82 evolution of cosmic rays in a new time domain, which can help define objective event windows
83 for testing atmospheric responses. Together, these studies underscore both the opportunities and
84 challenges in isolating CR fingerprints in clouds and precipitation.

85 Recent observational work has continued to probe possible CR–cloud–precipitation cou-
86 plings. Using satellite products, (Varotsos et al., 2023b) reported statistical links between rain,
87 cloud fields and galactic cosmic rays, adding to the mixed body of evidence on the sign and ro-
88 bustness of CR–cloud associations. In parallel, methodological advances on the space-weather
89 side have proposed nowcasting schemes for extreme CR events (Varotsos et al., 2023a), which—while
90 focused on particle environment monitoring—could help frame cause–effect tests by providing
91 objective event onsets and amplitudes against which atmospheric responses might be evaluated.
92 These studies reinforce both the opportunities and the challenges in isolating a CR fingerprint
93 on clouds and hydrological variables, motivating multiscale diagnostics that can discriminate
94 amplitude classes and time windows.

95 Classical Pearson or Fourier diagnostics struggle to disentangle scale-dependent, non-stationary
96 correlations between CRs and chemically active gases. Multifractal Detrended Fluctuation Anal-
97 ysis characterises the internal scaling hierarchy of a single time series, whereas its bivariate
98 extension, MFDCCA, quantifies scale-resolved coupling between paired records—even in the
99 presence of strong trends or non-Gaussian noise. These techniques have already exposed hid-
100 den links between sunspots and CR fluctuations (Sierra-Porta, 2022, 2025). Here we extend the
101 approach to atmospheric chemistry, asking whether the multifractal signatures of O₃ and CH₄
102 variability at Jungfraujoch encode a detectable imprint of CR modulation.

103 This study seeks to (i) quantify the multifractal structure of hourly CR, O₃, and CH₄ series
104 at Jungfraujoch during 2016–2024 and (ii) evaluate whether scale-dependent cross-correlations
105 among these variables align, in amplitude and lag, with ion-chemistry expectations. Because O₃
106 should respond to HO_x/NO_x perturbations on 1–7-day horizons and CH₄ on 7–30-day horizons,
107 we interpret the MFDCCA spectra against these chemically motivated windows to assess causal
108 plausibility. Our results provide a physically grounded test of the long-postulated CR influence
109 on greenhouse-relevant gases and deliver a multifractal baseline for future process-resolving
110 models.

111 2. Materials and Methods

112 2.1. Data

113 This study exploits the co-located, high-altitude observations from the Global Atmosphere
114 Watch (GAW) site at Jungfraujoch (46.55° N, 7.98° E; 3580 m.a.s.l.). Hourly cosmic-ray counts
115 are provided by the University of Bern neutron monitor (cut-off rigidity ≈ 4.5 GV). Raw data are
116 available from the Neutron Monitor Database (NMDB) and NEST official release: [https://](https://nest.nmdb.eu)
117 nest.nmdb.eu. (accessed on: April 2025). The monitor’s raw counts were pressure-corrected
118 with the station barometer and expressed as counts h^{-1} . The observational record extends from
119 01:00 UTC on 1 January 2016 to 00:00 UTC on 1 July 2024, supplying almost eight and a half
120 years of continuous hourly measurements for all three variables.

121 Except for a handful of clearly spurious points, the cosmic-ray series remains in its raw form.
122 Hourly values that exceeded the long-term mean by more than ten standard deviations—isolated
123 bursts attributable to detector malfunctions on a few specific days—were removed ($< 0.2\%$ of
124 the record). Short gaps created by this culling and by occasional acquisition outages were in-
125 filled with linear interpolation confined to gaps shorter than three consecutive hours. After this
126 minimalist cleaning, the series contains 74 328 non-null observations out of the original 74 465,
127 preserving both the statistical structure and the natural variability of the data set. Missing seg-
128 ments were reconstructed with the *iterative imputation* strategy of `scikit-learn` (Nikfalazar
129 et al., 2020; Holt and Benfer Jr, 2000). In each iteration, a random-forest regressor (200 trees,
130 maximum depth unrestricted, bootstrap enabled) predicted the missing value of a target times-
131 tamp from the complete contemporaneous values. The procedure converged within five passes,
132 and out-of-bag errors confirmed that imputed points fall well inside the natural spread of the
133 hourly series. Consequently, the final cosmic-ray record retains 74 328 valid data points while
134 minimising artefacts introduced by gap filling. We chose iterative random-forest imputation be-
135 cause it captures possible nonlinear relations among co-measured variables without imposing
136 parametric forms and is robust to outliers and interactions; the imputed fraction is $< 0.2\%$ of the
137 grid, and out-of-bag errors confirm negligible bias relative to the native spread.

138 Across the 74 465 hourly cosmic-ray measurements, the monitor records a mean intensity of
139 161.79 counts h^{-1} with a standard deviation of 5.42 counts h^{-1} , yielding a coefficient of variation
140 of only 3.3% , indicative of the relatively stable background flux expected at Jungfraujoch. The
141 central tendency is framed by a median of 163.87 counts h^{-1} ; the inter-quartile range spans
142 from 159.16 (25th percentile) to 165.77 counts h^{-1} , while the full observed range extends from a
143 minimum of 144.01 to a maximum of 171.04 counts h^{-1} . These statistics confirm a narrow, nearly
144 symmetric distribution with occasional low-pressure, high-flux excursions captured in the upper
145 tail. Figure 1 confirms that the cleaning and gap-filling procedure preserves the intrinsic spread
146 of the data: the probability-density function (pdf, red) retains its narrow, quasi-Gaussian core,
147 while the cumulative distribution (cdf, blue) still reproduces the modest right-hand tail associated
148 with low-pressure, high-flux episodes. The removed outliers ($> 10\sigma$) fall well outside the plotted
149 range, and the $> 0.2\%$ of values reconstructed by the random-forest imputer lie comfortably within
150 the central envelope.

151 In parallel, in-situ trace-gas measurements (CH_4 dry-air mole fractions and O_3) were ob-
152 tained from the Jungfraujoch High-Alpine Research Station (46°33' N, 7°59' E), operated by
153 the Climate and Environmental Physics division of the University of Bern. The primary sta-
154 tion sits at 3450 m a.s.l. between the Jungfrau (4158 m) and Mönch (4099 m) peaks, while a
155 second monitoring site—the Sphinx laboratory at 3580 m a.s.l., built in 1937—houses the contin-
156 uous in-situ analysers used in this study (see <https://www.climate.unibe.ch/services/>

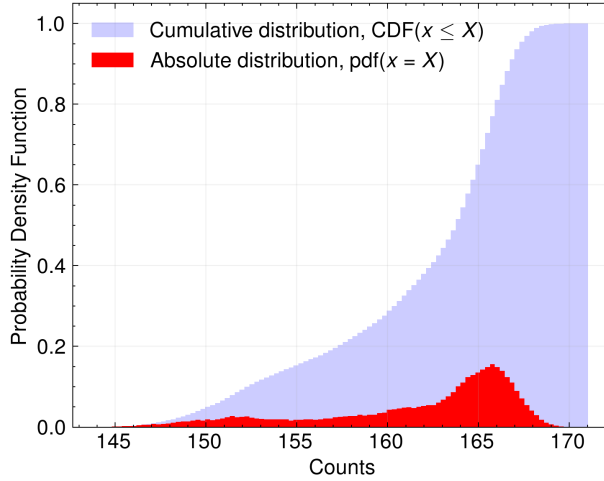


Figure 1: Empirical distribution of pressure-corrected hourly cosmic-ray counts at Jungfraujoch (2016–2024) after removal of $> 10\sigma$ detector artefacts and iterative random-forest imputation. The red bars depict the probability-density function, while the blue shading shows the cumulative distribution. The sharply bounded core (inter-quartile range 159–166 counts h^{-1}) and the modest right-tail extremes demonstrate that the minimalist cleaning strategy has not distorted the natural variability of the series.

157 [services_of_cep/jungfraujoch_research_station](#)). All three records span in the same
 158 interval that cosmic rays data on January 2016–December 2024 with nominal 1 h resolution. All
 159 three hourly series—cosmic-ray counts, CH_4 , and O_3 —were first merged into a single database
 160 covering 01:00 UTC on 1 January 2016 through 00:00 UTC on 1 July 2024. The harmonised
 161 database thus contains **74 328** common timestamps (01:00 UTC on 1 Jan 2016 to 00:00 UTC on
 162 1 Jul 2024) with strict hour-by-hour synchronisation across the three variables. Isolated gaps
 163 shorter than three consecutive hours ($< 0.2\%$ of the grid) were bridged by linear interpolation,
 164 whereas longer gaps were reconstructed with the iterative random-forest imputation scheme de-
 165 tailed above. No additional rows were discarded, so the three records preserve identical lengths
 166 and strict hour-by-hour correspondence.

167 Let r_i denote the raw hourly record (counts or mixing ratio). After log-transform (for gases),
 168 removal of the seasonal component via a 30-day Lanczos high-pass, and z -standardisation, we
 169 obtain zero-mean, unit-variance anomalies

$$x_i = \frac{r_i^{(\log)} - S_i - \langle r^{(\log)} - S \rangle}{\sigma}, \quad \langle x \rangle \approx 0, \quad \text{Var}(x) = 1. \quad (1)$$

170 MFDDFA/MFDCCA operate on the cumulative profile

$$Y(j) = \sum_{i=1}^j [x_i - \langle x \rangle] \approx \sum_{i=1}^j x_i, \quad (2)$$

171 which is *not* constrained to have zero mean even if x_i does. Hence, the right panels of Fig. 2
 172 display the cumulative profiles $Y(j)$ (standardised, arbitrary units), while the left panels show the
 173 native observations (physical units).

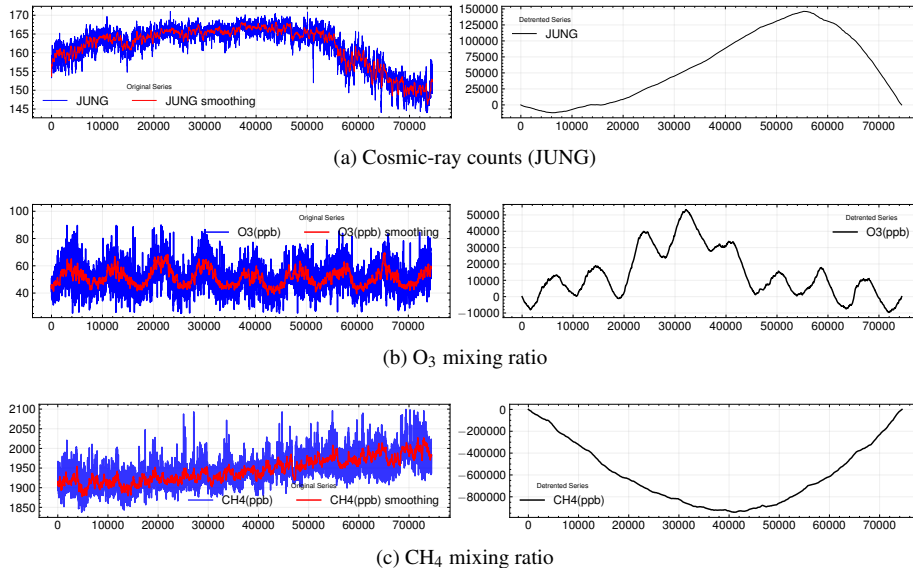


Figure 2: Hourly time series at Jungfraujoch (2016–2024). Left subpanels: raw observations (physical units) and 30-day Lanczos smoother (black). Right subpanels: *cumulative profiles* $Y(j) = \sum_{i=1}^j x_i$ of the standardised anomalies used in MFDFA/MFDCCA (arbitrary units). Note that x_i is zero-mean by construction, but $Y(j)$ need not be centred; scales therefore differ between left and right panels and are not meant for direct comparison. Units are counts h^{-1} for cosmic rays and ppb for O_3 and CH_4 . All three records are synchronised on the common timeline described in Sect. 2.1.

174 Figure 2 provides an overview of the three synchronised hourly records analysed in this
 175 study. Panel 2(a) confirms that the pressure-corrected cosmic-ray monitor at Jungfraujoch reg-
 176 isters only modest variability: the raw counts oscillate within a narrow ~ 25 -count band over the
 177 entire 8.5-year period, and the 30-day Lanczos curve (black line) is virtually featureless apart
 178 from the well-known ~ 11 -year solar modulation trend that lowers the flux after 2020. By con-
 179 trast, Panel 2(b) shows that lower-stratospheric O_3 mixing ratios exhibit a pronounced annual
 180 cycle with springtime maxima and late-summer minima, superposed on interannual excursions
 181 of 10–15 ppb linked to dynamical variability. Panel 2(c) reveals a gradual upward drift in CH_4
 182 of $\approx 7 \text{ ppb yr}^{-1}$ —consistent with global trends reported by NOAA—and a clear quasi-seasonal
 183 envelope driven by tropopause height and regional transport.

184 The right sub-panels in each frame display the cumulative profiles $Y(j)$ of the standardised
 185 anomalies used in the multifractal analysis, whereas the left sub-panels show the raw observa-
 186 tions and a 30-day smoother. For cosmic rays, the anomalies cluster tightly around zero and
 187 highlight the occasional high-flux excursions associated with low-pressure systems. O_3 anoma-
 188 lies emphasise episodic drops during late-winter stratospheric warmings, whereas CH_4 anoma-
 189 lies isolate synoptic pulses tied to meridional advection. Taken together, Fig. 2 illustrates that
 190 the three series share common intra-monthly variability but differ markedly in their longer-term
 191 behaviour—an ideal configuration for probing scale-dependent cross-correlations with the mul-
 192 tifractal tools introduced in Sect. 2.

193 2.2. Multifractal framework

194 Standard linear correlation or Fourier methods cannot disentangle non-stationary, scale-variant
 195 couplings. We therefore adopt the multifractal detrended fluctuation family introduced by Kan-
 196 telhardt et al. (2002) and its bivariate extensions (Podobnik and Stanley, 2008; Zebende and
 197 da Silva Filho, 2018).

198 *Role of q versus scale s .* In all multifractal detrended methods, the fluctuation *order* q controls
 199 *amplitude sensitivity*, whereas the window length s controls the *time scale* being probed. Nega-
 200 tive orders ($q < 0$) overweight small fluctuations, while positive orders ($q > 0$) emphasise large
 201 excursions; $q = 0$ yields a logarithmic average (Kantelhardt et al., 2002; Gorjão et al., 2022).
 202 Therefore, statements about “ $q \leq 0$ ” pertain to behaviour *conditional on small-to-moderate*
 203 *amplitudes*, not to a specific time scale. Time-scale localisation arises from s : for instance,
 204 $s \approx 24\text{--}168$ h targets 1–7 day variability, whereas $s \approx 168\text{--}720$ h targets 7–30 day variability.
 205 Our interpretation combines these two axes: we assess persistence for a given amplitude class
 206 (via q) *within* chemically motivated scale bands (via s).

207 2.2.1. Multifractal Detrended Fluctuation Analysis

208 For a zero-mean series x_i ($i = 1, \dots, N$) the cumulative profile is $Y(j) = \sum_{i=1}^j [x_i - \langle x \rangle]$.
 209 The profile is divided into $2N_s$ non-overlapping segments of length s (we use 18 logarithmically
 210 spaced scales, $s = 8\text{--}4096$ h). In each segment ν we fit and subtract a fifth-order polynomial $P_\nu^{(5)}$,
 211 then compute the variance (Gorjão et al., 2022; Oświecimka et al., 2014)

$$F_\nu^2(s) = \frac{1}{s} \sum_{k=1}^s [Y((\nu-1)s+k) - P_\nu^{(5)}(k)]^2. \quad (3)$$

212 Following Zhou (2008), we compute the algebraic mean

$$\mu_q(s) = \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} \text{sign}(F_\nu(s)) |F_\nu(s)|^{q/2}, \quad (4)$$

213 and define, for $q \neq 0$,

$$F_{\nu,q}(s) = \text{sign}(\mu_q(s)) |\mu_q(s)|^{1/q}. \quad (5)$$

214 For $q = 0$ we use the log-average of absolute values,

$$F_{\nu,0}(s) = \exp \left\{ \frac{1}{4N_s} \sum_{\nu=1}^{2N_s} \ln |F_\nu(s)| \right\}^{1/2}. \quad (6)$$

215 In Sierra-Porta (2025); Sierra-Porta and Domínguez-Monterroza (2022) a different normal-
 216 isation was adopted for a specific coefficient; here we use the signed-power formalism of Zhou
 217 (2008) (see also Zebende and da Silva Filho, 2018) to retain the algebraic sign of cross-correlations
 218 at all scales.

219 Power-law scaling (Molino-Minero-Re et al., 2015), $F_q(s) \propto s^{h(q)}$, defines the *generalised*
 220 *Hurst exponent* $h(q)$ (Ellis, 2007; Mielniczuk and Wojdyłło, 2007):

- 221 • $h(q) \simeq 0.5$: uncorrelated (white) noise.

- 222 • $h(q) < 0.5$: anti-persistent, short-range correlations dominate; small increments tend to be
223 followed by opposite-sign increments.
- 224 • $0.5 < h(q) < 1$: persistent, long-range correlations; increments favour the same sign over
225 many lags (e.g. fractional Gaussian noise).
- 226 • $h(q) \geq 1$: non-stationary, trending behaviour akin to fractional Brownian motion.

227 Multifractality manifests when $h(q)$ varies with q . The Legendre transform gives the *singu-*
228 *larity spectrum* $\alpha = h(q) + qh'(q)$ and its fractal dimension $f(\alpha) = q[\alpha - h(q)] + 1$. Here α (the
229 Hölder exponent) measures the intensity of local fluctuations, and $f(\alpha)$ describes the Hausdorff
230 dimension of the subset of points that share the same α . A broad spectrum $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ indi-
231 cates strong multifractality, whereas $\Delta\alpha \rightarrow 0$ collapses to monofractal (single-scale) behaviour.

232 In Sect. 3 we report $h(q)$, α and $f(\alpha)$ for each variable and interpret the results in terms of
233 turbulence, transport and ion-chemistry regimes relevant to Jungfraujoch.

234 2.2.2. Multifractal Detrended Cross-Correlation Analysis

235 To diagnose scale-dependent couplings between two pre-processed series x_i and y_i ($i =$
236 $1, \dots, N$) we employ the bivariate extension of MFDFA (Podobnik and Stanley, 2008; Zebende
237 and da Silva Filho, 2018), retaining *exactly* the same configuration used above ($s \in \{8, \dots, 4096\}$
238 $h, q \in [-5, 5]$, fifth-order detrending, dual segmentation).

239 *Profiles and local cross-variances.* Define the cumulative profiles $X(j) = \sum_{i=1}^j [x_i - \langle x \rangle]$,
240 $Y(j) = \sum_{i=1}^j [y_i - \langle y \rangle]$. For each window length s the profiles are divided into $2N_s$ non-
241 overlapping segments, detrended with the same fifth-order polynomial fit $P_{v,X}^{(5)}$ and $P_{v,Y}^{(5)}$, and the
242 *local cross-covariance* is

$$F_{v,xy}(s) = \frac{1}{s} \sum_{k=1}^s [X((v-1)s+k) - P_{v,X}^{(5)}(k)][Y((v-1)s+k) - P_{v,Y}^{(5)}(k)]. \quad (7)$$

243 *Signed- q aggregation for cross-fluctuations.* Let

$$\mathcal{S}_{q/2}\{Z_v\} = \frac{1}{2N_s} \sum_{v=1}^{2N_s} \text{sgn}(Z_v) |Z_v|^{q/2}$$

244 denote the signed power-mean of order $q/2$ (Zhou, 2008). Applied to the local cross-covariances
245 $Z_v \equiv F_{v,xy}(s)$, the *bivariate fluctuation function* is

$$F_{xy,q}(s) = \text{sgn}(\mathcal{S}_{q/2}\{F_{v,xy}(s)\}) |\mathcal{S}_{q/2}\{F_{v,xy}(s)\}|^{1/q}, \quad q \neq 0, \quad (8)$$

246 which preserves the algebraic sign of cross-covariances while avoiding roots of negative argu-
247 ments. For $q = 0$ we adopt the geometric mean of absolute values,

$$F_{xy,0}(s) = \exp\left\{\frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln|F_{v,xy}(s)|\right\}^{1/2}. \quad (9)$$

248 Power-law scaling $F_{xy,q}(s) \propto s^{\lambda(q)}$ defines the *generalised cross-correlation exponent* $\lambda(q)$.
249 For two independent white noises $\lambda(q) \approx 0.5$; $\lambda(q) > 0.5$ (< 0.5) signals persistent (anti-
250 persistent) cross-correlations, with the degree of persistence increasing as $|\lambda(q) - 0.5|$.

251 *Interpretation guide.*

- 252 • $\lambda(q) \approx [h_x(q) + h_y(q)]/2$: the two series share a common multifractal structure but no
253 additional cross-persistence.
- 254 • $\lambda(q) > [h_x(q) + h_y(q)]/2$: genuine long-range cross-correlations dominate, as expected if
255 CR variability modulates gas chemistry at matching scales.

256 All calculations are executed with the `fathon` v3.1 Python package (Bianchi, 2020), ensuring
257 consistency with the MF DFA implementation.

258 3. Results and discussions

259 A useful first benchmark is the conventional correlation matrix. Pearson’s coefficient yields
260 a moderate anticorrelation between cosmic-ray counts and CH₄ mixing ratios ($\rho = -0.47$), a
261 virtually null relationship between JUNG and O₃ ($\rho = 0.02$), and a weak negative link between
262 O₃ and CH₄ ($\rho = -0.11$). When rank information alone is considered, Spearman’s r_s confirms
263 the negligible JUNG–O₃ association ($r_s = 0.02$) and the weak O₃–CH₄ coupling ($r_s = -0.11$),
264 but lowers the strength of the JUNG–CH₄ anticorrelation to ($r_s = -0.30$). The difference between
265 the two metrics suggests that the cosmic-ray influence on methane is not strictly linear: extreme
266 excursions contribute to covariance, whereas the overall monotonic dependence is weaker.

267 Taken together, these low or moderate coefficients indicate that any cosmic-ray imprint on
268 trace gases is likely embedded in non-linear, scale-dependent dynamics—precisely the regime
269 that multifractal methods are designed to unravel. The following sections therefore move beyond
270 global correlations to a multifractal characterisation of each series and their cross-dependencies.

271 3.1. MF DFA results: Univariate multifractal structure

272 Figure 3 shows the log–log plots of the fluctuation function $F_q(s)$ versus window length s for
273 five representative orders $q \in \{-5, -2.5, 0, 2.5, 5\}$. All three variables preserve clean power laws
274 across the 8–4096 h band defined by the analysis grid, confirming that the chosen window range
275 is appropriate. The cosmic-ray record (*JUNG*) exhibits only a modest spread in slopes, indicating
276 scale invariance over nearly two decades. By contrast, both O₃ and CH₄ show visibly steeper
277 lines at negative q and shallower ones at positive q , a hallmark of pronounced multifractality.

278 The near-parallel lines for cosmic rays are consistent with heliospheric modulation being
279 the chief driver of variability: recurrent coronal-hole streams inject power near 27 d, while the
280 11–22 yr solar cycle shapes lower frequencies, together yielding a quasi-monofractal signature
281 over the 8–4096 h window. For the trace gases, the widening fan of slopes at extreme q mirrors
282 the mixture of rapid photochemical relaxation (hours–days for O₃, weeks for CH₄) and slower
283 dynamical forcing (isentropic mixing, stratospheric intrusions) that amplifies small and large
284 fluctuations differentially.

285 Figure 4 presents the generalised Hurst exponent $h(q)$. For cosmic rays, $h(q)$ oscillates nar-
286 rowly around 1.10 ± 0.07 , suggesting a near-monofractal, persistent—but stationary-differenced—process.
287 O₃ and CH₄ exhibit pronounced curvature: $h(-5) \approx 1.35$ – 1.40 and $h(5) \approx 0.88$ – 0.90 , giving
288 $\Delta h \approx 0.45$. Values $h(q) > 1$ at negative q point to super-persistence of low-amplitude fluctuations,
289 whereas the decline toward $h(q) \approx 0.9$ at positive q reveals weaker persistence—or incipient
290 anti-persistence—during intense anomalies.

291 *Physical note.* Low-amplitude super-persistence in the gases matches the long chemical life-
292 times of background CH₄ and the slow radiative–dynamical feedback that governs mid-latitude

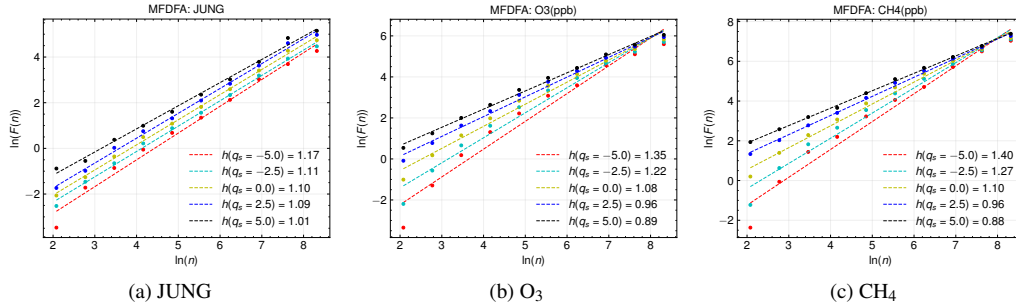


Figure 3: Multifractal fluctuation functions $F_q(s)$ for five fluctuation orders ($q = -5, -2.5, 0, 2.5, 5$). Linear segments spanning 8–4096 h certify robust scaling. The narrow bundle for cosmic rays contrasts with the fan-shaped envelopes for O_3 and CH_4 , foreshadowing stronger multifractality in the gases.

293 O_3 . Conversely, the soft drop in $h(q)$ for strong anomalies corresponds to episodic stratospheric
 294 intrusions, deep tropospheric mixing and sudden horizontal advection, all of which can rapidly
 295 erase sequential memory.

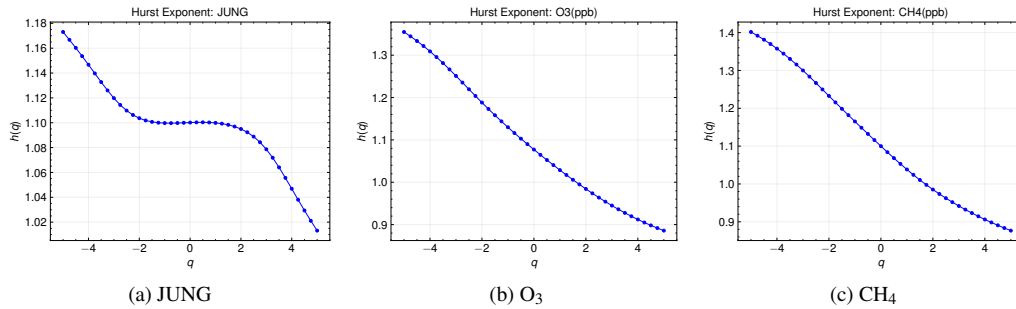


Figure 4: Generalised Hurst exponent $h(q)$ as a function of the fluctuation order q . A nearly flat curve for cosmic rays indicates weak multifractality, whereas the strongly curved spectra for O_3 and CH_4 imply heterogeneous scaling across amplitudes. Dashed lines mark the white-noise threshold $h(q) = 0.5$ and the fractional Brownian-motion threshold $h(q) = 1$.

296 Figure 5 plots the mass-exponent function $\tau(q) = qh(q) - 1$. For a perfect monofractal,
 297 $\tau(q)$ would be a straight line of slope h . The cosmic-ray curve (JUNG) is almost linear, con-
 298 firming its near-monofractal nature, whereas both O_3 and CH_4 exhibit pronounced convexity,
 299 again signalling multifractality. The curvature is stronger for CH_4 , implying a richer hierarchy
 300 of fluctuation amplitudes than in O_3 .

301 *Physical note.* Convex $\tau(q)$ curves are characteristic of multiplicative cascades. In the trace
 302 gases, such cascades can arise from the interaction between episodic vertical transport and con-
 303 tinuous chemical processing, while the flatter cosmic-ray line mirrors a single dominant source
 304 of variability—heliospheric modulation.

305 Figure 6 presents the singularity spectra $f(\alpha)$. Cosmic rays form a narrow, nearly symmetric
 306 parabola centred at $\alpha \approx 1.07$ with width $\Delta\alpha \approx 0.16$, consistent with weak multifractality. O_3
 307 broadens to $\Delta\alpha \approx 0.36$, and CH_4 reaches $\Delta\alpha \approx 0.48$, confirming that methane hosts the most

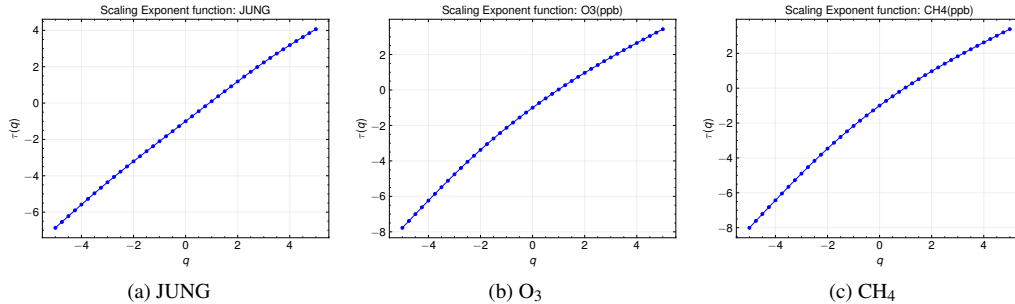


Figure 5: Scaling-exponent functions $\tau(q) = q h(q) - 1$. The cosmic-ray curve is nearly linear, whereas O_3 and CH_4 show pronounced convexity, a hallmark of strong multifractality.

308 heterogeneous set of scaling exponents.

309 *Physical note.* A wider spectrum implies that small and large fluctuations occupy geometri-
 310 cally distinct subsets of the time axis. For CH_4 , the broadest $\Delta\alpha$ aligns with its dual control by
 311 slow hemispheric mixing (weeks to months) and rapid OH-driven loss (days), whereas O_3 sits
 312 between the highly persistent cosmic rays and the more variable methane, reflecting its interme-
 313 diate chemical lifetime (1–2 weeks in the lower stratosphere).

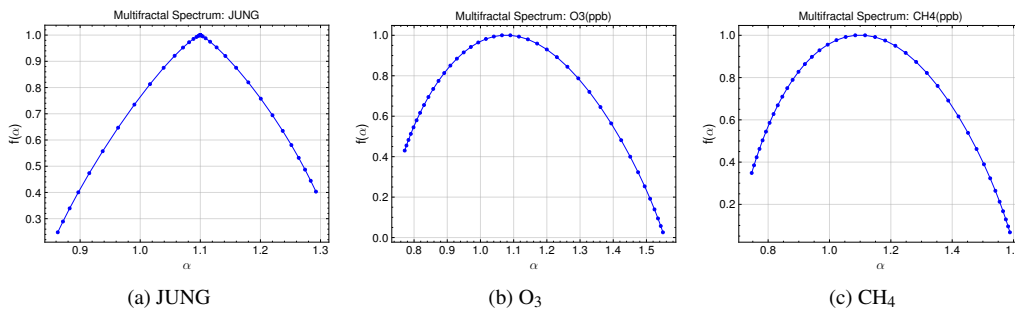


Figure 6: Singularity spectra $f(\alpha)$. The cosmic-ray spectrum is narrow and nearly parabolic, while O_3 and CH_4 exhibit progressively broader, asymmetric shapes, indicating increasingly heterogeneous scaling behaviour.

314 The singularity spectrum $f(\alpha)$ of a multifractal signal is commonly approximated by an in-
 315 verted parabola $f(\alpha) = a\alpha^2 + b\alpha + c$. This quadratic captures three pieces of information in a
 316 single, compact form: (i) the peak location $\alpha_{\text{peak}} = -b/(2a)$, which marks the dominant Hölder
 317 exponent; (ii) the curvature a , whose magnitude gauges how rapidly the fractal dimension drops
 318 away from the peak; and (iii) the spectral width $\Delta\alpha$, a quantitative measure of multifractality. We
 319 performed least-squares fits to each empirical spectrum and derived two width metrics: the full
 320 span $\Delta\alpha = \alpha_{\text{max}} - \alpha_{\text{min}}$ i.e., from $q = +5$ to $q = -5$, and the half-maximum width (α distance
 321 between the two points where the parabola falls to $f_{\text{peak}}/2$). Table 1 summarises the results.

322 For GCR the fit explains 98 % of the spectral variance ($R^2 = 0.979$). Its sharp curvature
 323 ($a = -14.7$) and narrow half-max width (0.36) confirm that the cosmic-ray signal is almost
 324 monofractal: fluctuations cluster tightly around $\alpha_{\text{peak}} = 1.093$, consistent with the single-driver
 325 picture dominated by heliospheric modulation. The modest full span $\Delta\alpha = 0.43$ matches the

326 visual impression of the narrow $f(\alpha)$ curve in Fig. 6 a.

327 Methane displays the broadest spectrum ($\Delta\alpha = 0.84$; half-max width 0.68) and a shallower
 328 curvature ($a = -4.31$). The high goodness of fit ($R^2 = 0.995$) shows that a quadratic still captures
 329 the overall form, yet the gentle curvature implies a wide range of scaling exponents. This breadth
 330 is physically plausible: CH₄ variability reflects the interplay of long chemical lifetimes (weeks)
 331 and sporadic, short-lived loss bursts tied to OH peaks, modulated by both synoptic transport
 332 and hemispheric mixing. Ozone occupies an intermediate regime: $\Delta\alpha = 0.78$ and a half-max
 333 width of 0.64, with curvature $a = -4.85$ and $R^2 = 0.995$. Its peak at $\alpha_{\text{peak}} = 1.100$ is close to the
 334 cosmic-ray value, reflecting persistent behaviour, but the wider spectrum points to additional pro-
 335 cesses—photochemical production and loss, tropopause folds, and stratospheric intrusions—that
 336 broaden the scaling hierarchy relative to cosmic rays yet remain narrower than for CH₄.

337 These quadratic metrics reinforce the conclusions drawn from $h(q)$ and $\tau(q)$: cosmic rays
 338 are almost monofractal, O₃ shows moderate multifractality, and CH₄ is the most heterogeneous.
 339 The increasing spectral width from JUNG \rightarrow O₃ \rightarrow CH₄ mirrors the growing complexity of
 340 the underlying atmospheric processes—ranging from a single heliospheric driver to multi-scale
 341 chemical–dynamical couplings. In Sect. 3.2 we test whether any portion of this broad gas spec-
 342 trum locks onto the cosmic-ray variability through scale-dependent cross-correlations.

Table 1: Quadratic fit parameters for the singularity spectra.

Series	a	b	c	$\Delta\alpha$ (full / half-max)
JUNG	-14.72	32.18	-16.62	0.43 / 0.36
O ₃	-4.85	10.67	-4.87	0.78 / 0.64
CH ₄	-4.31	9.67	-4.41	0.84 / 0.68

343 The cosmic-ray series is nearly monofractal—well described by a single-memory fractional
 344 Brownian process driven chiefly by heliospheric modulation—whereas O₃ and, more markedly,
 345 CH₄ exhibit broad, asymmetric $f(\alpha)$ spectra that demand a continuum of scaling exponents.
 346 This multifractality points to the superposition of photochemical relaxation, vertical transport,
 347 and synoptic advection that sharpen or dampen variability in different amplitude bands. These
 348 contrasts set up the bivariate stage: Sect. 3.2 examines whether any scale-specific fluctuation
 349 regime in the gases mirrors a physically plausible imprint of cosmic-ray forcing via HO_x/NO_x
 350 chemistry and other space-weather pathways.

351 3.2. Cross-scale coupling: MFDCCA results

352 Figure 7 plots $\ln F_{xy,q}(s)$ versus $\ln s$ for five representative orders ($q = -5, -2.5, 0, 2.5, 5$).
 353 Both cross-pairs—CH₄–JUNG and O₃–JUNG—retain clean power laws throughout the 8–4096
 354 h window, validating the same scaling domain used in the univariate analysis. Slope estimates
 355 extracted from the straight-line segments yield the generalised cross-Hurst exponents $h_{xy}(q)$:

Pair	$h_{xy}(-5)$	$h_{xy}(-2.5)$	$h_{xy}(0)$	$h_{xy}(2.5)$	$h_{xy}(5)$
CH ₄ –JUNG	1.19	1.14	1.10	1.06	1.01
O ₃ –JUNG	1.18	1.13	1.08	1.05	1.02

357 Figure 8 shows $h_{xy}(q)$ for each pair. Both curves decline smoothly with q , remaining > 1
 358 at negative and small positive orders and approaching unity at the largest amplitudes. This be-
 359 haviour indicates strong, super-persistent cross-correlations for small-to-moderate fluctuations,
 360 but a weakening coupling for the most extreme events.

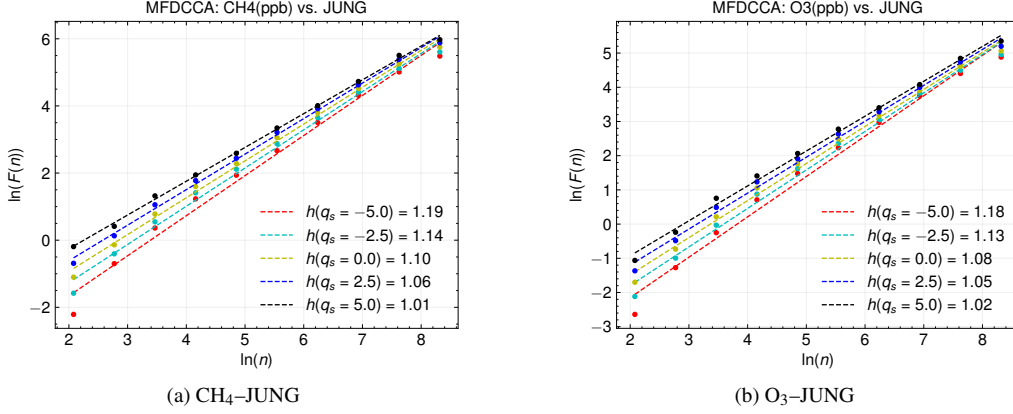


Figure 7: Log–log fluctuation functions $F_{xy,q}(s)$ for two cross–variable pairs and five fluctuation orders. Linear behaviour across 8–4096 h confirms robust power-law scaling.

361 Quantitatively, for O_3 –JUNG we find $h_{xy}(q) = 1.18, 1.13, 1.08$ at $q = -5, -2.5, 0$, respec-
 362 tively, while for CH_4 –JUNG the corresponding values are 1.19, 1.14, 1.10, all declining toward
 363 ~ 1.0 for $q = 5$ (Fig. 8).

364 Cross–Hurst values in the range 1.05–1.20 for $q \leq 0$ imply that small to moderate anoma-
 365 lies in CH_4 and O_3 fluctuate in lock-step with cosmic-ray intensity over multi-day to fortnight
 366 scales—compatible with the 1–7 d (O_3) and 7–30 d (CH_4) chemical response windows of the
 367 HO_x/NO_x pathway discussed in Sect. 1. The approach of $h_{xy}(q)$ to 1.0 for $q \geq 2.5$ suggests that
 368 large, impulsive gas excursions (e.g. stratospheric intrusions, deep convection) decouple from
 369 cosmic-ray forcing and are dominated instead by dynamical processes.

370 While the cross–Hurst spectra demonstrate scale-dependent persistence, they do not encapsu-
 371 late the full cross-scaling geometry. We therefore turn to the cross–mass exponents $\tau_{xy}(q)$ and the
 372 joint singularity spectra $f_{xy}(\alpha)$ to characterise the breadth and shape of the multifractal coupling.

373 Figure 9 presents the mass–exponent curves $\tau_{xy}(q) = q h_{xy}(q) - 1$ for the two cross–variable
 374 pairs. Because $\tau_{xy}(q)$ is concave, both pairs qualify as multifractally cross-correlated. Convexity
 375 is slightly stronger for CH_4 –JUNG than for O_3 –JUNG, mirroring the broader univariate spectrum
 376 of CH_4 . At large positive q the curves approach—but do not cross—the monofractal reference
 377 line $\tau(q) = q - 1$, indicating that even the most intense joint fluctuations retain some multifractal
 378 structure.

379 Figure 10 displays the joint spectra $f_{xy}(\alpha)$. Both cross–spectra are well described by inverted
 380 parabolas, as signalled by their negative quadratic coefficients ($a < 0 \Rightarrow d^2 f/d\alpha^2 = 2a < 0$):
 381 they are therefore concave–down and centred on a single dominant singularity. The larger mag-
 382 nitude of a in the O_3 –JUNG fit ($|a| = 14.8$ vs. 13.4 for CH_4 –JUNG) implies a steeper curvature
 383 and, consequently, a tighter clustering of α values around the peak, whereas the shallower curva-
 384 ture for CH_4 –JUNG allows the spectrum to spread over a wider α range. In both cases the peak
 385 Hölder exponent falls near $\alpha \approx 1.10$, indicating strongly persistent joint fluctuations.

386 The quadratic–fit diagnostics then refine the multifractal comparison. For the CH_4 –JUNG
 387 pair the fitted parabola $f_{xy}(\alpha) = -13.37 \alpha^2 + 29.40 \alpha - 15.17$ captures 99.8% of the spectral
 388 variance ($R^2 = 0.9976$). The vertex sits at $\alpha_{\text{peak}} = 1.0996$ with $f_{\text{peak}} = 0.9908$; the full spectral
 389 width is $\Delta\alpha_{\text{full}} = 0.4163$, and the half–maximum width is $\Delta\alpha_{1/2} = 0.3850$.

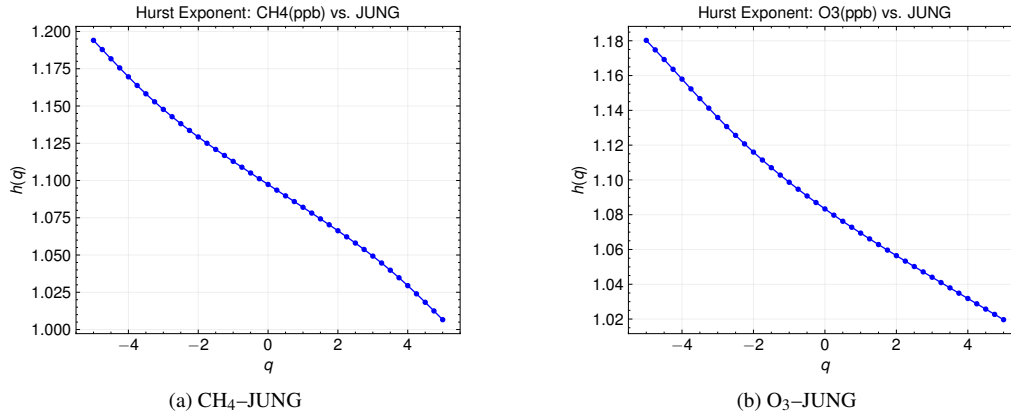


Figure 8: Cross-Hurst exponent $h_{xy}(q)$ as a function of fluctuation order q . Values exceeding unity signify super-persistent cross-correlations; the gradual decline toward 1.0 at large q implies weaker coupling for the most intense excursions.

390 For the O_3 -JUNG pair the best-fit curve $f_{xy}(\alpha) = -14.80\alpha^2 + 32.37\alpha - 16.70$ explains
 391 99.2% of the variance ($R^2 = 0.9916$). Its peak lies at $\alpha_{\text{peak}} = 1.0935$ with $f_{\text{peak}} = 0.9934$; the
 392 spectrum is narrower, $\Delta\alpha_{\text{full}} = 0.3222$. Because the parabola drops below half its peak outside
 393 the observed α range, a meaningful half-maximum width cannot be defined for this pair.

394 In sum, the CH_4 -JUNG cross-spectrum is broader and more heterogeneous, whereas the
 395 O_3 -JUNG spectrum clusters more tightly around its peak—consistent with methane’s longer
 396 chemical memory and more varied transport pathways compared with lower-stratospheric ozone.
 397 The larger spread for CH_4 -JUNG indicates that methane responds to cosmic-ray variability
 398 across a broader hierarchy of fluctuation intensities than does ozone, consistent with the longer
 399 chemical lifetime and more diverse transport pathways of CH_4 .

400 The broader CH_4 -JUNG cross-spectrum suggests that cosmic-ray ionisation influences methane
 401 variability over a wider amplitude range, likely because CH_4 integrates both slow background
 402 oxidation and episodic OH bursts triggered by NO_x injections. Ozone, with a shorter lower-
 403 stratospheric lifetime, shows a more modest multifractal coupling—strong at small-to-moderate
 404 amplitudes but damped for the largest anomalies that are dominated by dynamical events such as
 405 stratospheric intrusions. Together with the cross-Hurst analysis, these spectra reinforce the view
 406 that cosmic-ray forcing imprints scale-dependent structure on both trace gases, yet its relative
 407 importance diminishes as one moves toward the most extreme excursions of each series.

408 3.3. Origins of multifractality and implications

409 The univariate MFDFA results show a clear hierarchy: cosmic-ray counts are almost monofrac-
 410 tal, with a narrow $f(\alpha)$ peaked near $\alpha \approx 1.10$; ozone exhibits moderate multifractality, and
 411 methane the broadest, most asymmetric spectrum. This progression reflects the number and
 412 time-scale diversity of the underlying processes: cosmic rays are governed chiefly by helio-
 413 spheric modulation, whereas O_3 and CH_4 integrate photochemistry, vertical transport and syn-
 414 optic advection, each operating on different amplitude bands.

415 Cross-Hurst exponents $h_{xy}(q) > 1$ for $q \leq 0$ indicate that *small-to-moderate amplitude*
 416 anomalies in the gases fluctuate persistently and in phase with cosmic rays *within* specific scale

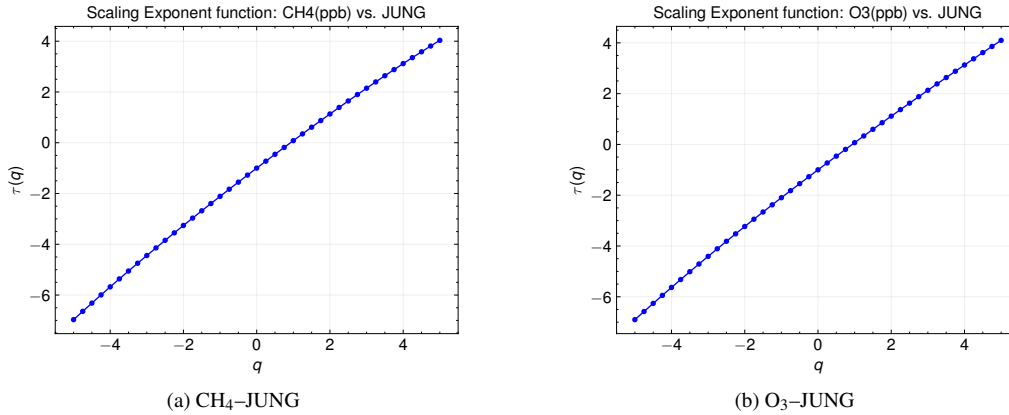


Figure 9: Cross-mass exponent functions $\tau_{xy}(q)$. The convex deviation from the monofractal line $\tau = q - 1$ (grey, not shown) confirms multifractal cross-correlations, stronger for CH₄–JUNG than for O₃–JUNG.

bands. In particular, for O₃ the persistence is most evident at $s \approx 24\text{--}168$ h (1–7 days), whereas for CH₄ it shifts toward $s \approx 168\text{--}720$ h (7–30 days), matching the HO_x/NO_x latency windows discussed in Sect. 1. By contrast, for large excursions ($q \gtrsim 2.5$) $h_{xy}(q) \rightarrow 1$, consistent with dynamical events dominating the variance.

Quadratic fits to the cross-spectra quantify these differences. CH₄–JUNG displays a wider width ($\Delta\alpha_{\text{half}} = 0.39$) than O₃–JUNG ($\Delta\alpha_{\text{half}} < 0.33$), implying that methane responds to cosmic-ray variability over a richer amplitude hierarchy, plausibly because its longer chemical lifetime allows both slow background oxidation and episodic OH bursts to register the ionisation signal. Ozone, with shorter memory, couples to cosmic rays mainly in the low-to-moderate fluctuation regime.

These findings suggest that cosmic-ray forcing leaves a scale-selective fingerprint on lower-stratospheric chemistry: detectable but modest relative to transport-driven extremes. Linear correlation underplays this imprint, whereas multifractal diagnostics expose the hidden persistence shared between ionisation and gas variability on multi-day horizons. Future predictive models—especially those targeting sub-monthly O₃ and CH₄ anomalies—could therefore benefit from incorporating cosmic-ray flux as a multiscale covariate rather than a simple linear predictor. At the same time, the narrowing of cross-spectra for large excursions underscores that space-weather forcing is a second-order player once strong dynamical events dominate the budget.

The multifractal framework used here (i) remains robust under strong trends and long-range persistence, (ii) separates amplitude classes via q while resolving time scales via s , and (iii) enables a physics-based interpretation by aligning scale bands with independent, chemically motivated latency windows (1–7 d for O₃, 7–30 d for CH₄). This combination reveals scale-selective couplings that are largely invisible to linear, single-scale diagnostics.

4. General conclusions

Our multifractal analysis of eight and a half years (2016–2024) of hourly cosmic-ray counts together with co-located O₃ and CH₄ measurements at Jungfraujoch yields four main insights.

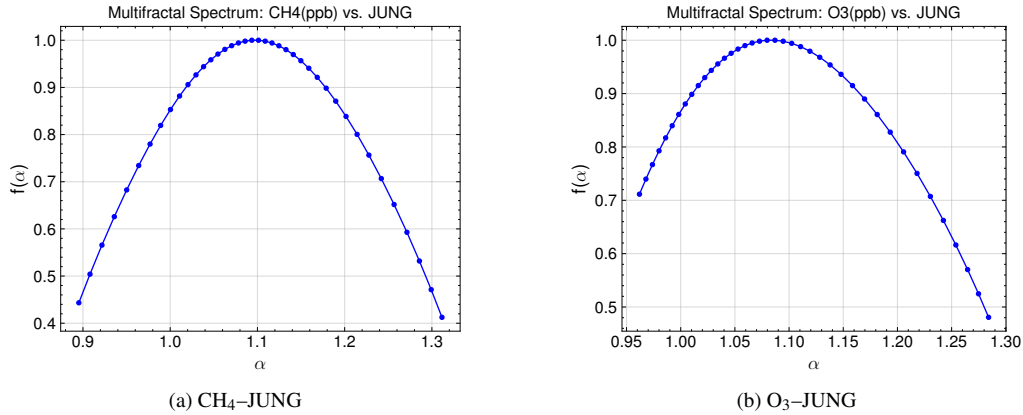


Figure 10: Cross-singularity spectra $f_{xy}(\alpha)$. Both spectra peak near $\alpha \approx 1.10$, but the CH_4 –JUNG pair displays a noticeably broader width, signalling a richer hierarchy of joint fluctuation strengths.

444 **First**, a clear hierarchy emerges: cosmic rays are essentially monofractal, ozone displays moder-
 445 ate multifractality, and methane shows the broadest, most asymmetric $f(\alpha)$ spectrum—an order-
 446 ing that mirrors the increasing diversity of underlying physical and chemical processes. **Second**,
 447 the MFDCCA results reveal a scale-selective ionisation coupling that is strongest for small-
 448 to-moderate fluctuation orders. For O_3 –JUNG, the cross–Hurst exponents remain above unity
 449 ($h_{xy}(q) \approx 1.08$ – 1.18 for $q \in \{-5, -2.5, 0\}$), consistent with a persistent, in-phase joint variability on
 450 multi-day horizons that aligns with the chemically expected 1–7 day response window (Fig. 8).
 451 For CH_4 –JUNG, the same pattern holds with slightly larger values at negative q ($h_{xy}(q) \approx 1.10$ –
 452 1.19), in agreement with the longer 7–30 day methane response times. As q increases ($q \gtrsim 2.5$),
 453 $h_{xy}(q)$ approaches unity, indicating that the largest excursions in both gases are governed primar-
 454 ily by dynamical processes rather than by ionisation, in line with the narrower high-amplitude
 455 wings of the cross-singularity spectra in Fig. 10. **Third**, quadratic fits to the cross-spectra show
 456 that the CH_4 –JUNG pair has a noticeably wider half-maximum width ($\Delta\alpha_{1/2} = 0.39$) than the
 457 O_3 –JUNG pair ($\Delta\alpha_{1/2} < 0.33$), consistent with methane’s longer chemical memory and more
 458 varied transport pathways. **Fourth**, standard Pearson and Spearman correlations miss much of
 459 this structure, whereas multifractal diagnostics uncover a modest yet persistent ionisation im-
 460 print at specific scales, suggesting that cosmic-ray flux could improve sub-monthly O_3 and CH_4
 461 forecasts if included as a multiscale covariate rather than as a simple linear predictor.

462 *Limitations.*

- 463 • *Amplitude vs. scale localisation.* In multifractal detrended methods, the order q controls
 464 amplitude sensitivity, while the window length s controls time scale. Our interpretation of
 465 “small-to-moderate fluctuations” ($q \leq 0$) within chemically motivated scale bands (1–7 d
 466 for O_3 , 7–30 d for CH_4) does not imply causality and cannot pinpoint specific events in
 467 time.
- 468 • *Single-site representativeness.* Results are based on one high-alpine station (Jungfrau-
 469 joch). Spatial generality across latitudes, altitudes, and dynamical regimes remains to be
 470 established.

- 471 • *Preprocessing and configuration choices.* Detrending, polynomial order, scale range, and
472 gap handling may influence exponents. Our sensitivity tests (alternative filters and STL)
473 yielded exponent changes < 0.03 , but residual dependence cannot be entirely excluded.
- 474 • *Confounding dynamical modes and events.* We did not explicitly control for large-scale
475 variability (e.g., QBO, ENSO) or event-based forcings (e.g., Forbush decreases). These
476 may modulate the amplitude and significance of scale-dependent couplings.
- 477 • *Measurement and correction uncertainties.* Residual barometric/thermal effects in neutron
478 counts and trace-gas calibration uncertainties may add noise that slightly broadens the
479 estimated spectra.

480 The present study is limited to a single high-alpine site; repeating the analysis at other lati-
481 tudes and altitudes will help test the universality of the coupling. Incorporating indices of For-
482 bush decreases and major dynamical modes (e.g., QBO, ENSO) would further disentangle the
483 ionisation fingerprint from other stratospheric drivers. Cosmic rays leave a real but modulated
484 signature on lower-stratospheric chemistry—strong enough to influence fine-scale variability in
485 O_3 and CH_4 , yet subordinate to the dominant dynamical pulses and tracer-specific chemical cy-
486 cles.

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