VRP Model with Time Window, Multiproduct and Multidepot

José Ruiz-Meza\textsuperscript{1,2,}\textsuperscript{*}, Isaid Montes\textsuperscript{1}, Arnoldo Pérez\textsuperscript{1}, and María Ramos-Márquez\textsuperscript{3}

\textsuperscript{1}Industrial Engineering Program, Corporación Universitaria del Caribe, Sincelejo, Colombia
\textsuperscript{2}Faculty of Engineering, Universidad de La Sabana, Chía, Colombia
\textsuperscript{3}Faculty of Engineering, Universidad Tecnológica de Bolívar, Cartagena, Colombia
\textsuperscript{*}Corresponding author. E-mail: jose.ruizm@cecar.edu.co

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With the increase in the transfer of products in supply chains, the organization of routes requires a complex allocation insofar as different environmental variables are considered, and VRP models are an efficient tool for the solution of routing systems of low, medium and high complexity. In this paper, we developed a vehicle routing model with hard time window, multidepot, multiproduct and heterogeneous fleet for the minimization of the distance travelled. We applied the model to a case study of a company that distributes water bottles and bales in which we made a new distribution of delivery schedules by order applied Pareto analysis. We obtained optimal computational results using exact methods in a very short computational time and minimizing the distance to 35.08\% of the current route.

Keywords: Pareto analysis; mathematical model; vehicle routing; optimization

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1. Introduction

The need to comply with the requirements demanded by customers, has led companies to innovate every day in order to remain competitive in the market, one of the essential aspects being on-time delivery of the products to their final or partial destination, which is affected by the usual transport problems \cite{1}. The transport process can be translated as one of the most important aspects in the supply chains, because your organization can suppose the success of a company, but, at the same time, it is one of the highest logistic costs \cite{2}.

This process has several variables that must be considered when defining the best way to carry out the movement of goods at any link in the supply chain, be it supply, internal transport in production or in distribution. Within these variables we have the travel time, the distance traveled, the availability of vehicles, the capacity, the allocated costs, among others \cite{3}.

In this sense, the VRP (Vehicle Routing Problem) Models are an efficient tool to provide solutions to transport problems that consider various variables oriented to the real context \cite{4}, and which serve as support for decision-making when wanting to obtain answers oriented to optimization \cite{2}.

According to this, with the implementation of a VRP Model, competitive advantages are generated in the company, which translate into lower costs and efficiency in the transport process \cite{5}.

In this research, a mixed-integer linear programming mathematical model was developed, which, in consideration of the contextualized problem of a distribution company of bottles and water packs, presented the variants of multi depot, multi product, hard time windows and heterogeneous vehicle fleet to optimize the distance traveled.

Due to the complexity of the model, we decided to perform a Pareto analysis to prioritize the set of customers. In addition, we decided to propose a new organization of delivery schedules, in order to obtain optimal answers in low computational times, through exact methods.

The main contributions of this article are as follows: i. On the basis of the traditional vehicle routing problem, we considers many factors such as hard time windows, multiple depots, multiple products and heterogeneous fleet to solve a real scene of product delivery in supply chain. ii. We carried out a Pareto analysis, organizing the customers...
in an accumulated sum according to the total of products they demand selecting as few customers as possible, but they have a large number of orders. At the same time, we designed the order can be prioritized to be delivered from the amount of minutes assigned to the time window. These two points make the problem more realistic and the calculation time of the problem is shorter.

The remainder of this study is organized as follows. We present a brief literature review in Section 2. In Section 3 provides a mathematical model of VRP multideposit, multiproduct, hard time windows and heterogeneous fleet for the problem identified. The computational analyses are analyzed through a case study conducted at the distribution company of bottles and water packs in Section 4. Section 5 summarizes the conclusions and discussions. Finally, some lines for further research are provided in Section 6.

2. Vehicle Routing Problems

2.1. Overview of the VRP

The first work developed on routing problems was carried out by [6], who solved a fuel distribution problem. However, the work that included the phrase "vehicle routing" for the first time was the one proposed by [7]. Prior to these authors [8] constructed an effective algorithm for the solution of this type of problems, called savings algorithm or Clarke and Wright, considering that they are part of the Np-Hard family, which cannot be resolved in a polynomial time, and increase in complexity as the number of nodes grows, making the exact methods inefficient [9]. Basically, the vehicle routing problem consists of the assignment of one or several vehicles to certain routes for the fulfillment of customer demands, which start from a deposit and return to it at the end of the route [10].

Whenever it comes to modeling a real routing problem (i.e., organizations with product distribution processes to meet their customers’ requirements), it is necessary to consider a set of variants that have been included, throughout history, in the development of this type of problems. Generally, considerations are addressed to customers, vehicles and depots [11].

In this sense, the main variants of the VRP —as described by [12] could be grouped into two types: homogeneous VRP, where all the nodes handle exactly the same characteristics; and heterogeneous VRP, where these characteristics are different (e.g., type of vehicles, capacity, time windows, stochastic, and depots).

Within these two great classifications, several variants that can be addressed individually or articulated with others emerge, according to the problem that is trying to solve. [10] illustrated different variants that have been worked on throughout the history of VRP. Among them, heterogeneous fleet HFVRP [13], time windows VRPTW [14], multidepot MDVRP [15], capacitated CVRP [16], stochastic SVRP [17], pickups and deliveries VRPPD [18], and new green variants such as the EMVRP (VRP with energy minimization) and the PRP (pollution routing problem) introduced by [19] and [20] respectively.

2.2. Variants of the VRP applied to the model

As mentioned above, the VRP has a large number of variants that increase complexity and computational time to obtain solutions. In this article, we developed a VRP model that considers multiple products, multiple depots, hard time windows and heterogeneous and capacitated vehicle fleet.

The multi depot variant (MDVRP) models a situation where the vehicles assigned to the routes can start from different depots or supply points, to comply with the visits established to several customers located in different points of an area and, in the end, they must return to the warehouse from which they left. Situation that is presented repeatedly in various organizations and logistics operations [24].

Time windows (VRPTW) intend to model a routing problem where it is necessary to consider the service times $s_i$ that the vehicle takes to perform downloads and/or loads in the same node [24]. Likewise, each customer is associated with a time window $[a_i, b_i]$, in which the service must be performed, considering the time of arrival at the node [11]. The Time Windows can be soft or hard. The first ones (VRPSTW) allow the violation of the arrival times,
generating a penalty that translates into costs [14]. In the case of hard time windows (VRPHTW), it is established that they cannot be violated [25, 26].

The first types of VRP assumed a fleet of homogeneous vehicles. However, in real-world transport practical problems, companies use different types of vehicles in their fleet, which have different load capacities to meet the demand of their customers [23]. With this, it is necessary to address the problem of routing with heterogeneous fleet (HFVRP), with which it is tried to satisfy the classic restrictions of the VRP by means of the use of a limited or unlimited fleet of vehicles with capacities different [13]. The organizations aim to diversify their products to increase their market share and competitiveness [27]. Therefore, it is normal that, in the transport process, the capacity of the vehicles must be configured in order to order various types of products. For this type of problems, the multi-product route variation is applied, which has been very little addressed in the literature reviews from the main divisions of the VRP. The multiproduct routing problem model a system in which the delivery of several products to a set of customers must be satisfied, so the capacity of the vehicle must be configured according to the specifications of each product, to meet the demands [28, 29].

3. Description of the Mathematical Model

The model can be represented through a directed graph $G = (N, A)$, where the set of vertices $N$ would be divided into two subsets that would be $N_c = \{n_1, n_2, n_3, ..., n_c\}$ the customers and $N_e = \{n_{c+1}, n_{c+2}, n_{c+3}, ..., n_{c+e}\}$ the headquarters of the company. Fig. 1.

![Fig. 1. Graphic representation of multiproduct and multi-depot VRP](image)

The model considers a heterogeneous fleet denoted by $k = \{1, 2, 3, ..., k\}$, which has a limited capacity denoted by $Q_{km}$. These vehicles must perform the service to each customer $i$, in a service time $S_i$, to collect a demand $d_{im}$, of products $m$, in a hard time window $[a_i, b_i]$. Similarly, each edge $(i,j) \in A$, where $i,j \in N$ and $i \neq j$, is related to a travel time $TT_{ij}$ and a travel distance $C_{ij}$. In the case of companies, no collection service is being offered, so for the set of $N_e$, we have $S_i = 0$ and at the same time $d_i = 0$.

The main objective is to minimize the distances traveled by the vehicles, considering that any route made by a vehicle must start and finish at the same company. The service must be performed within the time window established for each customer. Otherwise, the vehicle must visit another customer where it can perform the service. The maximum capacity of the vehicles assigned to the routes must be respected. A vehicle is assigned to a single company.

Next, the notation of the constructed mathematical model is detailed.

### Indexes and Sets

- $i = j = h$: Node index
- $m$: Product index
- $k$: Vehicle index
- $N_c$: Set of customers $\{1, 2, 3, ..., c\}$
- $N_e$: Set of companies $\{1, 2, 3, ..., e\}$
- $K$: Fleet of vehicles $\{1, 2, 3, ..., k\}$
- $P$: Set of products $\{1, 2, 3, ..., m\}$

### Parameters

- $C_{ij}$: Travel distance from node $i$ to node $j$
- $d_{im}$: Demand from node $i$ for the product $m$
- $TT_{ij}$: Travel time to go from node $i$ to node $j$
- $S_i$: Customer service time $i$
- $Q_{km}$: Vehicle capacity $k$ for product $m$
- $T_R$: Maximum route time for the vehicle $k$
- $M$: Very large number to avoid negativity and selection of customer-to-customer points

### Variables

- $X_{ijk} = 1$ If the pair of nodes $i$ and $j$ are in the route of vehicle $k$
- $X_{ijk} = 0$ other
- $T_i$: Time in which the vehicle arrives at the Node $i$
- $Y_{ijkm}$: Quantity of product $m$ delivered by a vehicle $k$ from $i$ to $j$
- $R_k$: Calculation of vehicle route time $k$

3.1. Formulation of mixed integer linear programming

The proposed model has the following characteristics: multiobjective, multiproduct, heterogeneous fleets, capacitated and hard time windows. Where the objective function that aims to minimize the distance traveled by vehicles in the assigned routes.

**Objective function**

$$\text{Min} Z = \sum_i \sum_j \sum_k C_{ij} X_{ijk} \quad (1)$$
The capacity constrain is imposed in 8. Constrains given
The model is subject to the following constraints:
Finally, the types of variables are determined in 14, 15 and
route time is determined in equation 11 and limited by
First customer and between customers respectively. Vehicle
a vehicle makes an arc
is set to zero at constraint 6. Constrains 7 ensure that if
exactly one deposit. Constrain 5 ensures the service in each
customer is visited exactly once by a vehicle. Constrain 3
ensures that each customer is visited and left by the same
vehicle. Constrain 12. Time window constraint are embodied in 13.
Finally, the types of variables are determined in 14, 15 and
16.

4. Computational results
We solved the model by mixed integer linear programming
with initial test instances and real instances. The model was
programmed using GAMS and was solved by obtaining
optimal solutions through the CPLEX library. A computer
with Intel® Core™ i7-5500 2.4 GHz CPU, 4.00 GB RAM, 64 Bit operating system and Windows 7 Professional was
used.

For the initial test instances, we consider a set of nodes
N = 6, where Nc = {1, 2}, vehicles k = 2, and products
m = 2 to perform model validation. Similarly, we consider
the capacity of the vehicles, the arrival times before, arrivals
after, maximum route time, service time and demands, as
shown in Table 1.

The solutions obtained with these instances have an opti-
mal distance of 4.33 km, complying with all the deliveries
in each node, in a total time of 30 minutes for the vehicle k1
and 33 minutes for k2, with a GAP of 0%. Fig. 2.

![Fig. 2. Solution of the model for constructed instances](image)

4.1. Solutions with real instances
We developed the model to be applied in a distribution
company of treated water bottles and bales, which is lo-
cated in the municipality of Sincelejo, Sucre, and has two
deposits: the main one, where the entire administrative
area is located, and the other, as an auxiliary warehouse.
In total, the company has approximately 470 customers,
made up of premises, companies, institutions and residen-
tial houses. Due to the complexity of the model, considered
as NP- hard [30] we decided to carry out a Pareto analysis,
organizing the customers in an accumulated sum, accord-
ing to the total of products they demand, selecting only 46
customers.

In this sense, two constrains was applied for the Pareto
analysis. First, using information of previous six month we
calculate the total product sold and divided the total sales
made to each of the 470 customers by the total orders to
obtain the percentage representation. Subsequently, with
Table 1. Instances built for validation of the model.

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Vehicle capacity</th>
<th>Nodes</th>
<th>Arrivals before</th>
<th>Arrivals after</th>
<th>Service time</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>m1=30</td>
<td>N_n=1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>m1=0,m2=0</td>
</tr>
<tr>
<td></td>
<td>m2=10</td>
<td>N_n=2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>m1=0,m2=0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N_n=3</td>
<td>20</td>
<td>25</td>
<td>5</td>
<td>m1=3,m2=0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N_n=4</td>
<td>16</td>
<td>20</td>
<td>5</td>
<td>m1=3,m2=1</td>
</tr>
<tr>
<td>K2</td>
<td>m1=15</td>
<td>N_n=5</td>
<td>14</td>
<td>22</td>
<td>5</td>
<td>m1=2,m2=0</td>
</tr>
<tr>
<td></td>
<td>m2=5</td>
<td>N_n=6</td>
<td>20</td>
<td>25</td>
<td>5</td>
<td>m1=2,m2=0</td>
</tr>
</tbody>
</table>

the cumulative sum of percentage we identified that 187
customer correspond to 80% Fig.3.

To calculate the service time we take the data for each
customer at the locations accompanying the current dis-
tribution route. We observed a problem derived from the
scheduling of delivery schedules, for which a new organiza-
tion of schedules with 1-hour slots was proposed, counted
after starting the workday for reception of orders. There
would be exactly 20 minutes to get the routes thrown by
the model and an interval of 50 minutes for the vehicle
to make the route. If an order cannot be delivered in the
requested time, it can be sent in the next order, taking into
account that the company has a maximum delivery period
of 3 hours in compliance with its quality policy. This or-
der can be prioritized to be delivered from the amount of
minutes assigned to the Time window. Table 2

Given the complexity of the model and the number of
customers, an optimal solution was obtained with a GAP of
0% at 13.15 hours, with the use of 3 of 4 vehicles available
with which a total of 1751 was covered. Table 3.

The efficiency of the company consists in the organi-
zation of the daily routing, which is why, from the new
organization of schedules, the model can generate optimal
solutions with a small set of customers. This type of robust
models present results in less computational time, being
worked in a clustered way and not in a holistic way.

To verify the efficiency of the model in a clustered man-
nner, we selected a real reference route formed by the cus-
tomers \( N_C = \{1, 2, 3, ..., 8\} \) that are served by the two de-
pots \( N_e = \{1, 2\} \). The current route has a distance traveled
of 10.8 km in a total time of 44.8 minutes. Fig.4.

In the previous route, only two vehicles were used. We
assume the availability of the four vehicles that the com-
pany has, whose capabilities are evidenced in the previous
Table 3, so that the model made the selection of the quan-
tity and type of vehicle needed to optimize the route. For
the eight customers, the demands of the products were
\( m_1 = 3, 3, 2, 1, 6, 1, 1, 6 \) and \( m_2 = 2, 0, 0, 2, 0, 0, 0, 1 \). The
model showed an optimal result with a traveled distance
of 7.011 km a total time of 78 minutes (28 minutes for the
vehicle k1 and 50 minutes for k2 ) and a GAP of 0%. Fig.5.

5. Conclusions and discussions

The organization of routes is one of the most common prob-
lems in transport processes, due to the several variables
that must be considered, from the type of vehicles to the
type of cargo and the conditions for unloading. With the
development of this research, we built a vehicle routing
model that groups the variants of multideposit, multiprod-
uct, heterogeneous fleet and hard time windows, and eval-
uate it with real data from a company that distributes water
bottles and bales.

The complexity of the model was observed, when try-
ing to program the routing for all the prioritized customers
by means of the Pareto analysis, because, when being con-
sidered Np-hard, with the increase of the nodes, the com-
putational time for the execution of the model increases
exponentially therefore, we conclude that it is better to
work in a clustered manner by order of orders, through
a new organization of schedules and deliveries that was
proposed. To check the efficiency and optimality of the
model, we select an order and accompany the route. Subse-
quently, we modeled the order obtaining an optimal route
with a minimization of 35.08% of the distance traveled,
evidencing that there is a notable improvement with the
routing proposed by the designed model. For all this,
we can see that the model developed in this research is consid-
ered an efficient tool, not only in terms of solving routing
problems with small instances, but can be used as a basis
for comparison to evaluate the performance and quality
of solutions approximate methods such as heuristics and
metaheuristics.

In this sense, when operations research (OR) is applied
to solve real-life problems (i.e., as in the case of product
distribution), it is common to design Np-hard mathemat-
ical models that become more complex as the size of in-
put data increases. For these situations different solution
approaches can be generated such as the application of
Fig. 3. Pareto diagram for 470 customers.

Table 2. Organization of order and service schedules.

<table>
<thead>
<tr>
<th>Start</th>
<th>End</th>
<th>Time of organization of the route</th>
<th>Time of departure of the vehicle</th>
<th>Maximum time delivery order (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m.</td>
<td>9:00 a.m.</td>
<td>20</td>
<td>09:20 a.m.</td>
<td>10:10 a.m.</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>10:00 a.m.</td>
<td>20</td>
<td>10:20 a.m.</td>
<td>11:10 a.m.</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>11:00 a.m.</td>
<td>20</td>
<td>11:20 a.m.</td>
<td>12:00 a.m.</td>
</tr>
<tr>
<td>11:00 a.m.</td>
<td>12:00 p.m.</td>
<td>20</td>
<td>2:20 p.m.</td>
<td>3:10 p.m.</td>
</tr>
<tr>
<td>2:00 p.m.</td>
<td>3:00 p.m.</td>
<td>20</td>
<td>3:20 p.m.</td>
<td>4:10 p.m.</td>
</tr>
<tr>
<td>3:00 p.m.</td>
<td>4:00 p.m.</td>
<td>20</td>
<td>4:20 p.m.</td>
<td>5:10 p.m.</td>
</tr>
<tr>
<td>4:00 p.m.</td>
<td>5:00 p.m.</td>
<td>20</td>
<td>5:20 p.m.</td>
<td>6:00 p.m.</td>
</tr>
<tr>
<td>5:00 p.m.</td>
<td>6:00 p.m.</td>
<td>20</td>
<td>8:20 a.m.</td>
<td>9:10 a.m.</td>
</tr>
</tbody>
</table>

Table 3. Instances built for validation of the model

<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>Vehicle capacity</th>
<th>Time windows (min) Before</th>
<th>Route time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>m1=30 m2=10</td>
<td></td>
<td>195</td>
</tr>
<tr>
<td>K2</td>
<td>m1=30 m2=10</td>
<td>5</td>
<td>204</td>
</tr>
<tr>
<td>K3</td>
<td>m1=15 m2=5</td>
<td>180</td>
<td>209</td>
</tr>
<tr>
<td>K4</td>
<td>m1=15 m2=5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4. Current route measured in terms of distance and travel time

Fig. 5. Route obtained by the model.
approximate methods (heuristics and metaheuristics) that generate efficient solutions [31]. In the area of metaheuristics, the use of hybrid algorithms based on approximate and exact methods are a field of wide exploitation [32, 33] because they provide the advantages of each approach as the guarantee of obtaining quality solutions [34].

However, another solution approach is to design strategies that allow the problem to be relaxed in order to obtain optimal solutions using exact methods. In this paper, we developed a mathematical model Np-hard that generates solution in high computational time (13.15 hours) applied to the case study of the company considering all customers prioritized in Pareto analysis. In order to relax the problem and generate quick solutions for the calculation of short routes, we established an order time organization scheme that allowed us to make the company’s delivery times more efficient and at the same time minimize the distances traveled.

6. Recommendations and future research

To obtain efficient solutions with large instances in a low computational time, the application of heuristic or metaheuristic algorithms is recommended, such as the hybrid algorithms that present good solutions for this type of problem. In the case of future research, we recommend the inclusion of costs in order to have better criteria for vehicle selection and route assignment. Likewise, we consider that the use of ecological variants is necessary due to the environmental problem derived to a large extent by the emission of greenhouse gases (GHG), in which transport presents an important contribution. On the other hand, we consider giving greater importance to the problems of routing multiproducts, because in the real distribution systems in the supply chains, it is very common to find this type of variables.

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