PAPER • OPEN ACCESS

# Power flow solution in direct current grids using the linear conjugate gradient approach

To cite this article: O D Montoya *et al* 2020 *J. Phys.: Conf. Ser.* **1448** 012016



Journal of Physics: Conference Series **1448** (2020) 012016 doi:10.1088/1742-6596/1448/1/012016

# Power flow solution in direct current grids using the linear conjugate gradient approach

**O D Montoya[1], A F Escobar[1], and V M Garrido[1]**

[1] Programa de Ingeniería Eléctrica, Universidad Tecnológica de Bolívar, Cartagena de Indias, Colombia

E-mail: o.d.montoyagiraldo@ieee.org

**Abstract.** The Colombian power system is being modified by the large-scale integration of renewable energy resources and energy storage systems, in conjunction with the microgrid concept that originates the possibility of alternating and direct current grids or hybrid between them. Here, we propose a classical gradient conjugate method to solve linear algebraic equations without matrix inversions, to address the power flow problem in electrical direct current networks with constant power loads, to contribute with the paradigm of microgrids operated in direct current. This methodology can be applied to the power flow equations since the admittance matrix is positive definite and diagonal dominant which guarantees convergence of the power flow problems. Numerical simulations evidence the applicability of the gradient conjugate method to solve power flow problems in direct current networks with radial and mesh topologies. All the simulations are conducted in MATLAB software version 2017$a$ licensed by the Universidad Tecnológica de Bolívar, Colombia.

## 1. Introduction

Direct current (DC) networks are promissory networks to provide electrical service at medium and low-voltage levels with low power losses and low voltage deviations [1]. One of the main advantages of using DC grids is that neither reactive power nor frequency are presented, since voltage and currents are direct current signals; which simplifies their control strategies and make them efficient in comparison to the classical alternating current counterparts [2, 3]. Direct current networks can be analyzed from statical and dynamical point of view, and these analyses depend on the period of time of interest; for example statical analysis involves optimization procedures that requires minutes in some cases to be solved, and its results are typical employed in tertiary control scenarios to define the set points of controllers [4–6]; notwithstanding, dynamical analysis is classically applied for periods of time of milliseconds and is used to make stability analyses on the DC network to define switching actions in converters [7]. Here, we are interested on studying DC grids in steady state conditions, which implies that nonlinear algebraic equations related with power flow problem need to be solved [8].

In specialized literature power flow problem has been widely studied for alternating current (AC) and DC grids by applying multiple numerical methods such as Gauss-Seidel [1], Newton-Raphson [9, 10], successive approximations and Taylor-based approaches [2], graph-based methods [11–13] or convex approximations [14], among others. The main idea to solve power flow problem in electrical networks is to determine the state variables of the system (voltage in all the nodes) to make different electrical analysis, such as study of power losses and voltage regulation measurements [15]. In this sense, this paper addresses the problem of power flow analysis in DC grids with mesh (multiple slack nodes) structures, avoiding matrix inversions with the linear gradient conjugate method under a recursive procedure to minimize the final voltage error. To compare our proposed approach a classical Gauss-Seidel method and its accelerate version are used [8].

In the Colombian power system context, this research is interesting since it addresses the problem of power flow analysis in the paradigm of direct current networks and it is connected with the new regulation policies about renewable energy resources and distributed energy resources, and their integration into the Colombian power system [16]. On the other hand, it is important to mention that DC grids are efficient in comparison to the AC grids, since reactive power and frequency concepts are not presented any longer. This clearly simplifies their controls, improves voltage profiles and reduce power losses on the grid, among others.

Regarding the above, section 2 presents the nonlinear formulation of the power flow problem; section 3 presents the classical linear conjugate gradient method adapted to solve the power flow problem as a linear equivalent model, which is embedded into a recursive procedure that deals with the nonlinear solution; section 4 presents the test system and the numerical validation, using a 69-nodes test feeder with two slack nodes; section 5 presents the concluding remarks as well as possible future works in power flow analysis, followed by acknowledgments and list of references.

## 2. Power flow formulation

The power flow problem in direct current networks can be modeled as a nonlinear non-convex set of algebraic equations, when grid operates under steady state conditions [4]. To obtain its mathematical model second Kirchhoff's law and the first Tellegen's theorem are applied to each closed-loop trajectory and at each node of the network [2]. The power flow problem takes the form presented in Equation (1).

$$p_k^g - p_k^d = v_k \sum_{j=1}^{n} G_{kj} v_j, \; \forall k \in \mathcal{N}, \tag{1}$$

where $p_k^g$ and $p_k^d$ are the power generation and demand connected at node $k$, $v_k$ is the voltage profile at node $k$, while $v_j$ is the value of the voltage at node $j$, $G_{ij}$ is the $kj^{th}$ component of the conductance matrix that relates nodes $k$ and $j$, respectively. Note that $n$ is the number of nodes of the network, which are contained in the set $\mathcal{N}$.

Observe that Equation (1) shows the power balance per node of the network; nevertheless, it can be written in compact form by separating the voltage controlled nodes in $\mathcal{S}$ from the rest of demand nodes $\mathcal{D}$ as presented by Equation (2) and Equation (3), respectively [2].

$$p_g = \mathbf{diag}(v_g)\left[\mathbf{G}_{gg} v_g + \mathbf{G}_{gd} v_d\right], \tag{2}$$
$$-p_d = \mathbf{diag}(v_d)\left[\mathbf{G}_{dg} v_g + \mathbf{G}_{dd} v_d\right], \tag{3}$$

where $p_g$ and $p_d$ are vectors with all power generation in voltage controlled nodes and power consumption in all demand nodes, repetitively; $v_g$ and $v_d$ are voltage profile in slack nodes (constant values) and demand nodes (variables), respectively; while $\mathbf{G}_{gg}$, $\mathbf{G}_{gd}$, $\mathbf{G}_{dg}$ and $\mathbf{G}_{dd}$ are subcomponents of the admittance matrix that relates slack and demand nodes, or combination of them. Observe that $\mathbf{diag}(v_g)$ and $\mathbf{diag}(v_d)$ are diagonal matrices with the components of the vectors $v_g$ and $v_d$. These matrices are positive definite, since $v_{\min} \leq v_k \leq v_{\max}, \; \forall k \in \mathcal{N}$.

Observe that Equation (2) is linear, since $v_g$ are perfectly known and $p_g$ are free variables that absorb any possible power consumption in all the demand nodes [2]; while Equation (3) remains nonlinear and non-convex due to the products between demand voltage nodes $v_d$. Note that the main interest in power flow analysis for direct current grids is to find the set of demand voltage *i.e.*, $v_d$ that satisfies Equation (3). For doing so, in the next section we explore the classical linear conjugate gradient method [17].

Journal of Physics: Conference Series **1448** (2020) 012016 doi:10.1088/1742-6596/1448/1/012016

## 3. Linear conjugate gradient method

Linear conjugate gradient method is a well-known and largely used method to solve linear systems of equations with the form $\mathbf{A}x = b$, without inverting directly the $\mathbf{A}$ matrix [17]. To reach this objective, an iterative procedure is followed. The main condition (necessary) to apply this method is that the $\mathbf{A}$ matrix be positive definite and diagonal dominant; which is exactly the case of the $\mathbf{G}_{dd}$ matrix in Equation (3) [2]. Note that if we define $\mathbf{A} = \mathbf{G}_{dd}$, $x = v_d$ and $b(x) = -\mathbf{diag}^{-1}(x)p_d - \mathbf{G}_{dg}v_g$, then, Equation (3) can be rewritten as Equation (4) as follows.

$$\mathbf{A}x = b(x), \tag{4}$$

when $b$ is a vector that depends on the values of the vector $x$. Note that Equation (4) can be solved iteratively by using the linear gradient conjugate method, as demonstrated in [1, 13]. The set of power flow equations converges to their solution if Banach fixed point theorem is applied on Equation (1) when it is represented as a successive iterative approximation procedure. Algorithm 1 shows the application of the linear gradient conjugate method applied on Equation (3).

**Algorithm 1.** Application of the linear conjugate gradient method for power flow solution.
**Data.** Read data of the network. Define the starting point of $x_0 = \mathbf{1}$ and $v = x_0$.
**for** $y = 0 : y_{\max}$ **do**
  Define $r_0 = b(x_0) - \mathbf{A}x_0$.
  **if** $||r_0|| \leq \epsilon$ **then**
    Return $v = x_0$ as the solution of the power flow problem.
    **break**.
  **else**
    Define $p_0 = r_0$.
    **for** $z = 0 : z_{\max}$ **do**
      $\alpha_z = \frac{r_z^T r_z}{p_z^T \mathbf{A} p_z}$.
      $x_{z+1} = x_z + \alpha_z p_z$.
      $r_{z+1} = r_z - \alpha_z \mathbf{A} p_k$.
      **if** $||r_{z+1}|| \leq \epsilon$ **then**
        **break**.
      **end**
      $\beta_z = \frac{r_{z+1}^T r_{z+1}}{r_z^T r_z}$.
      $p_{z+1} = p_z + \beta_z p_k$.
    **end**
  **end**
  **if** $||x_{z+1} - v|| \, \epsilon$ **then**
    Return the solution of the power flow problem $v = x_{z+1}$.
    **break**.
  **else**
    $v = x_{z+1}$ and $x_0 = v$.
  **end**
**end**
**Result.** Return the solution of the power flow problem $v$.

Algorithm 1 presents the necessary steps for implementing the linear conjugate gradient algorithm for solving nonlinear equations with the form presented by Equation (4). Note that it works with the residual to approximate the solution of the variables contained in $x$ that minimizes this residual to an acceptable convergence error.

## 4. Test system and numerical validation

### *4.1. Test system*

To validate the proposed linear conjugate gradient method for power flow analysis in DC networks, a 69-nodes test feeder is employed. The electrical configuration of this test system is depicted in Figure 1. All the electrical parameters of this test system can be consulted in [18] and for simulation purposes we employ 12.66 kV and 100 kW as voltage and power bases; in addition, for simplicity, slack node operates with a voltage of 1.00 p.u. Note that we select this radial topology, since it is the worst case in power flow analysis and shows that the proposed method is able to deal with this type of grids.

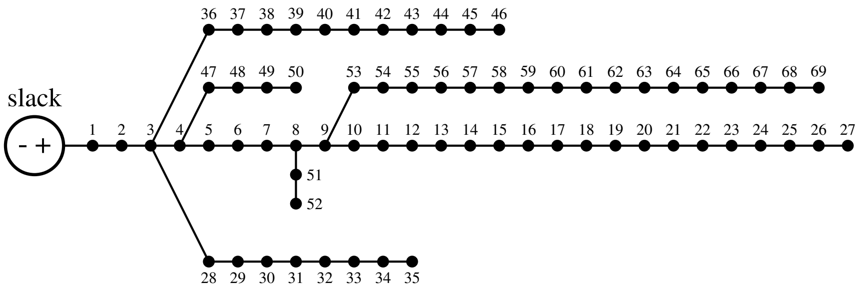


**Figure 1.** Electrical connection among nodes in the 69-nodes test feeder.

### *4.2. Numerical validation*

The simulations were performed in a desk-computer with an Intel(R) Core(TM) i5-3550 processor at 3.50 GHz, 8 GB RAM, running a 64-bits Windows 7 Professional operating system by using MATLAB software 2017$a$ licensed by the Universidad Tecnológica de Bolívar, Colombia. As comparative methods, we employ the classical Gauss-Seidel method [1] and its accelerate version, since these do not employ inverses during its iterative procedures. Table 1 reports the computational time of each method after 100 consecutive iterations, as well as final total iterations and power losses, respectively.

**Table 1.** Comparative results between Gauss-Seidel methods and the linear conjugate approach.

| Method | Average time (ms) | Number of iterations | Power loss (p.u) |
|---|---|---|---|
| Classical Gauss-Seidel | 3535.7402 | 46264 | 1.5385 |
| Accelerate Gauss-Seidel | 44.9792 | 546 | 1.5385 |
| Linear conjugate gradient | 39.3775 | 13 | 1.5385 |

From the results shown in Table 1 we can confirm that the classical Gauss-Seidel method without accelerating factor takes more than 3500 ms to solve the power flow problem for the 69-nodes test feeder, while its accelerate counterpart takes about 45 ms to solve it. Nevertheless, for this test system, the proposed method only takes about 39.4 ms to reach the same solution, being the speediest method for power flow analysis without matrix inversions. In addition, note that all these methods reach the same power losses in the grid, which implies that, in numeric terms, all of them are equivalent for power flow analysis in DC grids with constant power loads, and its selection depends on the grid requirements and computational resources available. It is important to mention that the computational time is strong, related with the number of iterations required to achieve the solution of the power flow problem; in this sense, note that the proposed linear conjugate gradient only takes 13 iterations to reach the solution of the problem, while the accelerate Gauss version takes 42 times the number of iterations of the proposed method to do the same task (see column 3 in Table 1). This confirms the efficiency and robustness of our proposal in comparison with Gauss-Seidel methods.

Note that we decide not to implement recent developed successive approximation method [2] or Newton based approaches [9], since they require matrix inversions and the main idea of this paper is to avoid those procedures. Additionally, although the numerical example presented is radial, we do not use graph-based methods for solving power flow analysis [11, 13], as they are limited to a unique voltage controlled node, while our approach based on linear conjugate gradient is general, *i.e.*, it can be used for radial and mesh grids with one or multiple slack nodes indistinctly.

## 5. Conclusion and future work

A new application of the classical linear gradient conjugated method for solving nonlinear power flow equations in direct current networks was presented in this paper. The solution of this problem was reached by a recursive procedure that works iteratively with the conjugate gradient inside a loop. To demonstrate the efficiency of the proposed approach, it was compared with the classical Gauss-Seidel method and its accelerate version. Our proposal shows faster convergence times in terms of the processing time requirements for power flow solution, compared with classical numerical methods that avoid matrix inversions. These results are promissory to integrate renewable energy resources and energy storage systems and it can be used for proposing expansion/construction plants of new power systems in the Colombian context. In addition, as future work it will be possible to apply the nonlinear conjugate gradient method for power flow analysis, avoiding recursive solutions, which may help to reduce the computational time in the power flow solution even for alternating and direct current grids.

## References

[1] Garces A 2017 Uniqueness of the power flow solutions in low voltage direct current grids *Electr. Power Syst. Res.* **151** 149
[2] Montoya O D, Garrido V M, Gil-González W, and Grisales-Noreña L F 2019 Power flow analysis in DC grids: Two alternative numerical methods *IEEE Trans. Circuits Syst. II: Express Briefs* **66(11)** 1865
[3] Montoya O D, Grisales-Noreña L F, González-Montoya D, Ramos-Paja C, and Garces A 2018 Linear power flow formulation for low-voltage DC power grids *Electr. Power Syst. Res.* **163** 375
[4] Simpson-Porco J W, Dorfler F, and Bullo F 2015 On resistive networks of constant-power devices *IEEE Trans. Circuits Syst. II: Express Briefs* **62(8)** 811
[5] Nasirian V, Moayedi S, Davoudi A, and Lewis F L 2015 Distributed cooperative control of DC microgrids *IEEE Trans. Power Electron.* **30(4)** 2288
[6] Li X, Guo L, Li Y, Hong C, Zhang Y, Guo Z, Huang D, and Wang C 2018 Flexible interlinking and coordinated power control of multiple DC microgrids clusters *IEEE Trans. Sustain. Energy* **9(2)** 904
[7] Parhizi S, Lotfi H, Khodaei A, and Bahramirad S 2015 State of the art in research on microgrids: A review *IEEE Access* **3** 890
[8] Montoya O D, Gil-González W, and Grisales-Noreña L F 2018 Optimal power dispatch of DGs in DC power grids: A hybrid Gauss-Seidel-genetic-algorithm methodology for solving the OPF problem *WSEAS Transactions on Power Systems* **13(13)** 335
[9] Garcés A 2018 On the convergence of Newton's method in power flow studies for DC microgrids *IEEE Trans. Power Syst.* **33(5)** 5770
[10] Montoya O D, Gil–González W, and Grisales–Noreña L F 2018 Linear–based Newton–Raphson approximation for power flow solution in DC power grids *IEEE 9th Power, Instrumentation and Measurement Meeting (EPIM)* (Salto: IEEE)
[11] Montoya O D, Grisales-Noreña L F, and Gil-González W 2019 Triangular matrix formulation for power flow analysis in radial DC resistive grids with CPLs *IEEE Trans. Circuits Syst. II: Express Briefs* **Early Access** 1
[12] Shen T, Li Y, and Xiang J 2018 A graph-based power flow method for balanced distribution systems *Energies* **11(18)** 1
[13] Montoya O D 2019 On the existence of the power flow solution in DC grids with CPLs through a graph-based method *IEEE Trans. Circuits Syst. II: Express Briefs* **Early Access** 1
[14] Li J, Liu F, Wang Z, Low S H, and Mei S 2018 Optimal power flow in stand-alone DC microgrids *IEEE Trans. Power Syst.* **33(5)** 5496
[15] Grisales-Noreña L F, Gonzalez Montoya D, and Ramos-Paja C 2018 Optimal sizing and location of distributed generators based on PBIL and PSO techniques *Energies* **11(1018)** 1
[16] León-Vargas F, García-Jaramillo M, and Krejci E 2019 Pre-feasibility of wind and solar systems for residential self-sufficiency in four urban locations of Colombia: Implication of new incentives included in law 1715 *Renewable Energy* **130** 1082
[17] Dag H and Alvarado F L 1997 Toward improved uses of the conjugate gradient method for power system applications *IEEE Trans. Power Syst.* **12(3)** 1306
[18] Montoya O D, Gil-González W, and Garces A 2019 Power flow approximation for DC networks with constant power loads via logarithmic transform of voltage magnitudes *Electr. Power Syst. Res.* **175** 105887