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## A Novel Full Boundary Element Formulation for Transient Analysis of Elastic Membranes Coupled to Acoustics Fluids

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# A Novel Full Boundary Element Formulation for Transient Analysis of Elastic Membranes Coupled to Acoustics Fluids

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**Abstract.** A full time-direct Boundary Element Formulation for the dynamic analysis of elastic membranes coupled to acoustics fluid is presented. The elastic membranes is modeled using the classical linear elastic pre-stretched membrane theory. The acoustic fluid is modeled using the acoustic-wave equation for homogeneous, isotropic, inviscid and irrotational fluids. Elastostatic fundamental solution is used in the boundary element formulation for the elastic membrane. The boundary element formulation for the acoustic fluid is based on the fundamental solution of three dimensional Poisson equation. Domain integrals related to inertial terms and those related with distributed pressure on membrane, were treated using the Dual Reciprocity Boundary Element Method. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles and dynamic pressure at fluid-structure interfaces. The time integration is carried out using the Newmark method. Results obtained shows the accuracy and efficiency of the proposed boundary element formulation.

## 1. Introduction

Acoustic problems in real life applications involve considerations of structural and acoustic part together and thus calls for a coupled vibroacoustic treatment rather a pure acoustic approach. Thus, whenever an elastic structure is in contact with a fluid, the structural vibrations and the acoustic pressure field in the fluid are influenced by the mutual vibro-acoustic coupling interaction [1].

The analysis of fluid-structure coupled systems is a challenging and complex task. In general, the use of experimentation or numerical methods represent the two uniques alternatives to obtain approximate solutions for these kind of problems. However, numerical methods based on domain discretization requires refined meshes for high frequency problems, since the length of the elements should be proportional to the size of the wavelength. This means a more time-consuming model.

The boundary element method (BEM) is a modern numerical technique which has enjoyed increasing popularity over the last two decades, and is now an established alternative to traditional



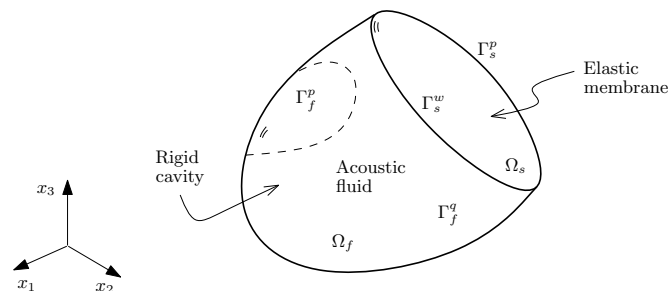
computational methods of engineering analysis [2, 3]. The main advantage of the BEM is its unique ability to provide a complete solution in terms of boundary values only, with substantial savings in modelling effort.

Since consolidation of the boundary element method (BEM) as reliable numerical method for structural and fluid analysis, linear vibrations of structures coupled with an internal fluid has been carried out using hybrid BEM-FEM formulations. In these formulations BEM is used to model the fluid media and the FEM to model the structural response [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The main advantage of such formulations lies in a substantial reduction in the number of the degrees of freedom in the discretization of the fluid domain. However, in this formulations is necessary to discretize the entirely structure due to the use of the FEM.

Few BEM-BEM coupled formulations for fluid structure-interaction has been published [18, 19, 20, 21, 22, 23]. However, despite the fact that the BEM has been used for dynamic analysis of membrane structures and for analyses of acoustic fluids, to the best of authors knowledge, these formulations do not have been used for the fluid-structure interaction problem analysis using a full boundary element formulation for such purpose.

In this work, a new full boundary element formulation for the transient dynamic analysis of acoustic fluids coupled to elastic membranes is presented. Membranes are modeled using a boundary element formulation based on the linear elastic membrane theory under small deflection. The acoustic fluid is modeled using a boundary element formulation for the three-dimensional acoustic wave equation. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles at fluid-structure interfaces. Domain integrals on both, fluid and structure equations, were treated using the Dual Reciprocity Boundary Element Method. The developed formulation was used to study the linear vibration response of elastic membranes coupled to acoustic fluids.

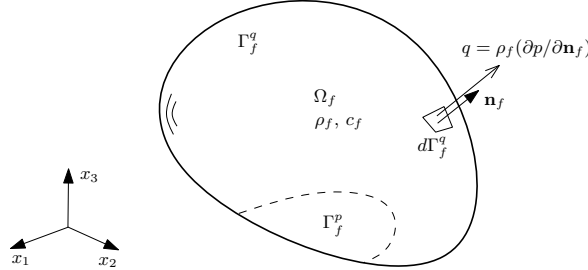
## 2. Structure subjected to a fluid pressure loading



**Figure 1.** Fluid-structure interaction problem

Consider a partially opened cavity  $\Omega_f$  with rigid walls and a flexible elastic membrane  $\Omega_s$  with mass density  $\rho_s$  and thickness  $h$  (see figure 1). Cavity contains an homogeneous and isotropic acoustic fluid with mass density  $\rho_f$ . The membrane vibrations and the acoustic pressure field in the fluid are influenced by the mutual vibro-acoustic coupling interaction. In this work, the vibro-acoustic coupling interaction is modeled using an eulerian formulation where the acoustic response is described by the pressure, while the membrane response is described by the transversal displacement field.

2.1. Acoustic wave equation



**Figure 2.** Acoustic fluid domain

The dynamic pressure of an ideal inviscid fluid under small perturbations in a spatial region  $\Omega_f$  confined by the boundary surface  $\Gamma_f = \Gamma_f^p \cup \Gamma_f^q$ , is governed by the wave equation (see figure 2) [24]:

$$p_{,\alpha\alpha} = \frac{1}{c_f^2} \ddot{p} \tag{1}$$

In this equation  $p$  is the fluid pressure,  $c_f^2 = \kappa/\rho_f$  stands for wave propagation velocity,  $\kappa$  is the bulk modulus. Above equation can be modified to include the effect of presence of an acoustic source. Double dot represents a second time derivative. Indicical notation is used throughout this work. Greek indices vary from 1 to 2 and Roman indices from 1 to 3. The boundary and initial conditions for equation (1) in the time interval  $[0, t^*]$  are:

$$\begin{aligned} p(\mathbf{x}, t) &= \hat{p}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_f^p, \quad t \in [0, t^*] \\ q(\mathbf{x}, t) &= n_i^f p_{,i} = \hat{q}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_f^q, \quad t \in [0, t^*] \\ p(\mathbf{x}, 0) &= \hat{p}_0(\mathbf{x}), \quad \dot{p}(\mathbf{x}, 0) = \hat{\dot{p}}_0(\mathbf{x}), \quad \mathbf{x} \in \Omega_f \end{aligned} \tag{2}$$

where  $n_i^f$  are the components of the outward normal vector  $\mathbf{n}_f$  at boundary. and  $t$  denotes time.

2.2. Dynamic equation of an elastic membrane

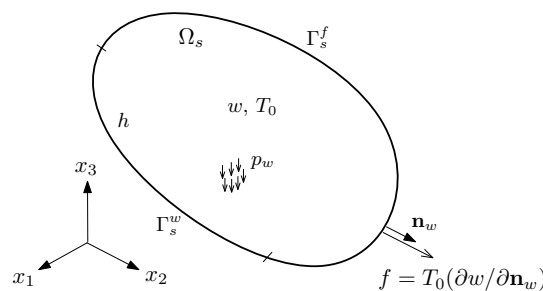
Now consider a linear elastic membrane with thickness  $h$  occupying the spatial domain  $\Omega_w$  confined by the boundary  $\Gamma_s = \Gamma_s^w \cup \Gamma_s^f$  (see figure 3). An initial tension  $T_0$  is uniformly applied to the membrane. In this work, the small deflection elastic membrane theory is considered. Thus, differential equation describing the trasversal displacement  $w(\mathbf{x}, t)$  of this membrane in the time interval  $[0, t^*]$  is given by:

$$w_{,\alpha\alpha} + \frac{p_w}{T_0} = \frac{1}{c_w^2} \ddot{w} \tag{3}$$

where  $c_w^2 = T_0/\rho_s$  and  $p_w(\mathbf{x}, t)$  is a distributed pressure applied over the membrane. The boundary and initial conditions for this equation are given by:

$$\begin{aligned}
 w(\mathbf{x}, t) &= \hat{w}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_s^w, \quad t \in [0, t^*] \\
 f(\mathbf{x}, t) &= n_i^w w_{,i} = \hat{f}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_s^f, \quad t \in [0, t^*] \\
 w(\mathbf{x}, 0) &= \hat{w}(\mathbf{x}), \quad \dot{w}(\mathbf{x}, 0) = \hat{\dot{w}}(\mathbf{x}), \quad \mathbf{x} \in \Omega_s
 \end{aligned}
 \tag{4}$$

In these expressions,  $n_i^w$  are the components of the outward normal vector  $\mathbf{n}_w$  at boundary.



**Figure 3.** Elastic membrane domain

### 3. Boundary element equations

#### 3.1. Boundary element equations for acoustic wave equation

The derivation of the integral formulation for equation (1) is based on the application of the Boundary Element Method to the acoustic wave equation as presented in [3]. Thus, by using the weighted residual method and making use of the Green's identity, the following equation is obtained:

$$\begin{aligned}
 c(\mathbf{x}')p(\mathbf{x}') + \int_{\Gamma_f} Q(\mathbf{x}', \mathbf{x})p(\mathbf{x})d\Gamma_f &= \int_{\Gamma_f} P(\mathbf{x}', \mathbf{x})q(\mathbf{x})d\Gamma_f \\
 &+ \frac{1}{c_f^2} \int_{\Omega_f} P(\mathbf{x}', \mathbf{X})\ddot{p}(\mathbf{X})d\Omega_f
 \end{aligned}
 \tag{5}$$

In this equation,  $\mathbf{x}'$  and  $\mathbf{x}$  represent collocation and field points, respectively;  $P(\mathbf{x}', \mathbf{x})$  and  $Q(\mathbf{x}', \mathbf{x})$  are fundamental solutions for pressure and gradient pressures for three dimensional acoustic problems, respectively, as presented in [3]. The value of  $c(\mathbf{x}')$  is equal to  $\frac{1}{2}$  when  $\mathbf{x}'$  is located on a smooth boundary.

In order to treat the domain integral, the Dual Reciprocity Boundary Element Method (DRM)

is used as presented in [25]. In this way, equation (5) can be re-written as:

$$c(\mathbf{x}')p(\mathbf{x}') + \int_{\Gamma_f} Q(\mathbf{x}', \mathbf{x})p(\mathbf{x})d\Gamma_f = \int_{\Gamma_f} P(\mathbf{x}', \mathbf{x})q(\mathbf{x})d\Gamma_f + \frac{1}{c^2} \sum_{j=1}^{NDRF} \ddot{\alpha}_j(t) \left[ c_i \hat{P}_{ij} + \int_{\Gamma_f} Q(\mathbf{x}', \mathbf{x})\hat{P}_j d\Gamma_f - \int_{\Gamma_f} P(\mathbf{x}', \mathbf{x})\hat{Q}_j d\Gamma_f \right] \quad (6)$$

In this equation,  $NDRF$  represents the number of total DRM collocation points used in the fluid;  $\hat{P}_j(\mathbf{x}', \mathbf{x})$  and  $\hat{Q}_j(\mathbf{x}', \mathbf{x})$  are the particular solutions to equivalent homogeneous equation (1). These particular solutions were obtained considering the function  $f_j(r) = 1 + r_j$  for the approximation of  $\ddot{p}(t)$ , as presented in [25]. Coefficients  $\ddot{\alpha}_j$  are related to  $\ddot{p}$  through:  $\ddot{p} = F_{ij}\ddot{\alpha}_j(t)$ , where  $F_{ij}$  is a matrix of coefficients, obtained by taking the value of  $\ddot{p}(t)$  at different DRM points.

To discretize boundary surfaces of the acoustic medium,  $N$  boundary quadrilateral elements were used and  $p(\mathbf{x})$  and  $q(\mathbf{x})$  were assumed to be constant over each element and equal to their values at the mid-element node. Thus, the discretized form of equation (6) is given by:

$$c_i(\mathbf{x}')p(\mathbf{x}', t) + \sum_{k=1}^N \left[ \int_{\Gamma_k} Q(\mathbf{x}', \mathbf{x})d\Gamma \right] p_k(t) - \sum_{k=1}^N \left[ \int_{\Gamma_k} P(\mathbf{x}', \mathbf{x})d\Gamma \right] q_k(t) = \frac{1}{c_f^2} \sum_{j=1}^{NDRF} \ddot{\alpha}_j \left[ c_i(\mathbf{x}')\hat{P}_{ij}(\mathbf{x}', \mathbf{x}) + \sum_{k=1}^N \int_{\Gamma_k} Q(\mathbf{x}', \mathbf{x})\hat{P}_j(\mathbf{x}', \mathbf{x})d\Gamma - \sum_{k=1}^N \int_{\Gamma_k} P(\mathbf{x}', \mathbf{x})\hat{Q}_j(\mathbf{x}', \mathbf{x})d\Gamma \right] \quad (7)$$

Applying this equation at each collocation point, the following linear system of equations is obtained:

$${}^f\mathbf{M}\ddot{\mathbf{p}} + {}^f\mathbf{H}\mathbf{p} = {}^f\mathbf{G}\mathbf{q} \quad (8)$$

where  ${}^f\mathbf{M}$  is the fluid mass matrix,  ${}^f\mathbf{H}$  and  ${}^f\mathbf{G}$  are boundary element influence matrices;  $\mathbf{p}$  and  $\mathbf{q}$  are vectors of nodal pressures and normal derivative of pressure, respectively.

### 3.2. Boundary element equations for an elastic membrane

The derivation of the integral formulation for equation (3) is based on the application of the BEM to the membrane equation as presented in [3]. Thus, by using the weighted residual method, and making use of the Green's identity, the integral formulation for equation (3) is given by:

$$c(\mathbf{x}')w(\mathbf{x}', t) + \int_{\Gamma_s} T(\mathbf{x}', \mathbf{x})w(\mathbf{x}, t)d\Gamma_s - \int_{\Gamma_s} W(\mathbf{x}', \mathbf{x})f(\mathbf{x}, t)d\Gamma_s = -\frac{1}{T_0} \int_{\Omega_s} W(\mathbf{x}', \mathbf{x})p_w(\mathbf{x}, t)d\Omega_s + \frac{1}{c_w^2} \int_{\Omega_s} W(\mathbf{x}', \mathbf{x})\ddot{w}(\mathbf{x}, t)d\Omega_s \quad (9)$$

where  $\mathbf{x}$  and  $\mathbf{x}'$  are field and collocation points respectively,  $W(\mathbf{x}', \mathbf{x})$  and  $T(\mathbf{x}', \mathbf{x})$  are fundamental solutions for displacement and traction, respectively as given in [2].  $c(\mathbf{x}')$  is the jump term arising from the terms of  $O(1/r)$  in the kernel  $T(\mathbf{x}', \mathbf{x})$ .

In this work, the DRM was used to transform domain integrals related to inertial terms into boundary integrals. In this way, this equation (9) can be re-written as [25]:

$$\begin{aligned}
c_i(\mathbf{x}')w(\mathbf{x}', t) + \int_{\Gamma_s} T(\mathbf{x}', \mathbf{x})w(\mathbf{x}, t)d\Gamma_s - \int_{\Gamma_s} W(\mathbf{x}', \mathbf{x})f(\mathbf{x}, t)d\Gamma_s = \\
-\frac{1}{T_0} \sum_{j=1}^{NDRM} \alpha_j(t) \left[ c_i(\mathbf{x}')\hat{W}_j(\mathbf{x}, t) + \int_{\Gamma_s} T(\mathbf{x}', \mathbf{x})\hat{W}_j(\mathbf{x}', \mathbf{x})d\Gamma_s \right. \\
\left. - \int_{\Gamma_s} W(\mathbf{x}', \mathbf{x})\hat{T}_j(\mathbf{x}', \mathbf{x})d\Gamma_s \right] \\
+\frac{1}{c_w^2} \sum_{j=1}^{MDRM} \beta_j(t) \left[ c_i(\mathbf{x}')\hat{W}_j(\mathbf{x}, t) + \int_{\Gamma_s} T(\mathbf{x}', \mathbf{x})\hat{W}_j(\mathbf{x}', \mathbf{x})d\Gamma_s \right. \\
\left. - \int_{\Gamma_s} W(\mathbf{x}', \mathbf{x})\hat{T}_j(\mathbf{x}', \mathbf{x})d\Gamma_s \right] \quad (10)
\end{aligned}$$

where  $NDRM$  represent the total number of DRM collocation used in the membrane;  $\hat{T}_j(\mathbf{x}', \mathbf{x})$  and  $\hat{W}_j(\mathbf{x}', \mathbf{x})$  are the particular solutions to equivalent homogeneous equation (3). These particular solutions were obtained considering the function  $f_j(r) = 1 + r_j$  for the approximation of  $\ddot{w}(t)$  and  $p_w(t)$  terms, as presented in [25]. Coefficients  $\alpha_j$  are related to  $p_w(t)$  through:  $p_w = A_{ij}\alpha_j(t)$ , where  $A_{ij}$  is a matrix of coefficients, obtained by taking the value of  $p(t)$  at different DRM points. Similarly, coefficients  $\beta_j$  are related to  $\ddot{w}$  through:  $\ddot{w} = B_{ij}\beta_j(t)$ , where  $B_{ij}$  is a matrix of coefficients, obtained by taking the value of  $\ddot{p}(t)$  at different DRM points.

Applying this equation at each collocation point, the following linear system of equations is obtained:

$${}^s\mathbf{M}\ddot{\mathbf{w}} + {}^s\mathbf{H}\mathbf{w} = {}^s\mathbf{G}\mathbf{f} - {}^s\mathbf{B}\mathbf{p}_w \quad (11)$$

Here  ${}^s\mathbf{M}$  is the membrane mass matrix,  ${}^s\mathbf{H}$  and  ${}^s\mathbf{G}$  are the influence matrices,  ${}^s\mathbf{B}$  is the influence matrix related with distributed pressure  $\mathbf{p}_w$  applied over the membrane.

#### 4. Fluid-structure coupling equations

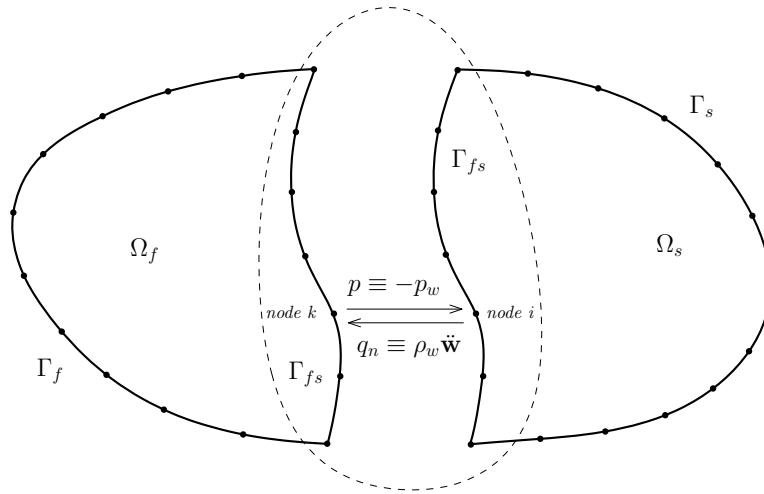
Fluid-structure coupling equations are given by compatibility considerations about normal pressure and dynamic pressure force acting at the fluid-structure interface. Mathematically, these conditions can be written as follows [26]:

$$\mathbf{n}_f \cdot \nabla p = \mathbf{q}_n \equiv -\rho_w \mathbf{C}_w \ddot{\mathbf{w}} \quad (12)$$

$$\mathbf{p}_w \equiv -\mathbf{C}_f \mathbf{p} \quad (13)$$

That is, pressure gradient acting on the fluid-structure interface  $\Gamma_{fs}$  are related to normal acceleration of the plate and the acoustic pressure is equilibrated with pressure on the membrane (see figure 4). In these equations,  $\mathbf{C}_w$  and  $\mathbf{C}_f$  represent connectivity matrices joining fluid and structural degree of freedom at fluid-structure interface.

Replacing equations (12) and (13) into equations (8) and (11) we obtain the coupled fluid-



**Figure 4.** Acoustic fluid domain

structure equation problem:

$$\begin{bmatrix} {}^s\mathbf{M} & \mathbf{0} \\ \mathbf{S} & {}^f\mathbf{M} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{w}} \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} {}^s\mathbf{H} & -\mathbf{A} \\ \mathbf{0} & {}^f\mathbf{H} \end{bmatrix} \begin{Bmatrix} \mathbf{w} \\ \mathbf{p} \end{Bmatrix} = \begin{bmatrix} {}^s\mathbf{G} & \mathbf{0} \\ \mathbf{0} & {}^f\mathbf{G}_{ff} \end{bmatrix} \begin{Bmatrix} \mathbf{t} \\ \mathbf{q} \end{Bmatrix} \quad (14)$$

In this equation, the off-diagonal sub-matrices  $\mathbf{A} = {}^s\mathbf{B}_{fs}\mathbf{C}_f$  and  $\mathbf{S} = \rho_w {}^f\mathbf{G}_{fs}\mathbf{C}_w$  are fluid-membrane coupling matrices.  ${}^f\mathbf{G}_{fs}$  and  ${}^f\mathbf{G}_{ff}$  are sub-matrices of  ${}^f\mathbf{G}$  related with degrees of freedom defined on the interface  $\Gamma_{fs}$ .  ${}^s\mathbf{B}_{fs}$  is a sub-matrix of  ${}^s\mathbf{B}$  related with pressure terms defined in  $\Omega_s$ .

Equations (14) can be rewritten in a general way as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{H}\mathbf{u} = \mathbf{G}\mathbf{r} \quad (15)$$

where,  $\ddot{\mathbf{u}} = \{\ddot{\mathbf{w}}, \ddot{\mathbf{p}}\}^T$ ,  $\mathbf{u} = \{\mathbf{w}, \mathbf{p}\}^T$  and  $\mathbf{r} = \{\mathbf{f}, \mathbf{q}\}^T$ .

## 5. Time integration

The two integration schemes which were tested to obtain the time response for the equation (15) are described below.

### The Houbolt Method

The Houbolt integration scheme is an explicit unconditionally stable algorithm based on backward-type finite difference formula with error of order  $O(\Delta\tau^2)$ . The most important aspect of this method, when compared to other time integration methods based on central difference approximations or Newmark scheme, is the introduction of artificial damping which truncates the influence of higher modes in the response. Here the acceleration is approximated as:

$$\ddot{\mathbf{u}}^{\tau+\Delta\tau} = \frac{1}{\Delta\tau^2}(2\mathbf{u}^{\tau+\Delta\tau} - 5\mathbf{u}^{\tau} + 4\mathbf{u}^{\tau-\Delta\tau} - \mathbf{u}^{\tau-2\Delta\tau}) \quad (16)$$



where  $\Delta\tau$  represents the time-step. Writing equation (15) at time  $\tau + \Delta\tau$ :

$$\mathbf{M}\ddot{\mathbf{u}}^{\tau+\Delta\tau} + \mathbf{H}\dot{\mathbf{u}}^{\tau+\Delta\tau} = \mathbf{G}\mathbf{r}^{\tau+\Delta\tau} \quad (17)$$

and substituting equation (16) into equation (17), we have:

$$(2\mathbf{M} + \Delta\tau^2\mathbf{H})\mathbf{u}^{\tau+\Delta\tau} - \frac{1}{\Delta\tau^2}\mathbf{G}\mathbf{r}^{\tau+\Delta\tau} = \mathbf{M}(5\mathbf{u}^\tau - 4\mathbf{u}^{\tau-\Delta\tau} - \mathbf{u}^{\tau-2\Delta\tau}) \quad (18)$$

The above equation allows the calculation of the distribution of  $\mathbf{u}$  at time  $\tau + \Delta\tau$  by using boundary conditions at that time and information from three previous time steps. This algorithm requires a special starting procedure in which initial conditions for  $\mathbf{u}$  are employed to calculate  $\mathbf{u}^1$  and  $\mathbf{u}^2$ .

### The Newmark Method

In this method the acceleration is approximated as:

$$\ddot{\mathbf{u}}^{\tau+\Delta\tau} = \frac{1}{\beta\Delta\tau^2} [\mathbf{u}^{\tau+\Delta\tau} - \mathbf{u}^\tau] + \frac{1}{\beta\Delta\tau}\dot{\mathbf{u}}^\tau - \frac{(1-2\beta)}{2\beta}\ddot{\mathbf{u}}^\tau \quad (19)$$

substituting equation (19) into equation (15), we have:

$$\left( \frac{\mathbf{M}}{\beta\Delta\tau^2} + \mathbf{H} \right) \mathbf{u}^{\tau+\Delta\tau} - \mathbf{G}\mathbf{r}^{\tau+\Delta\tau} = \mathbf{M} \left[ \frac{1}{\beta\Delta\tau^2}\mathbf{u}^\tau - \frac{1}{\beta\Delta\tau}\dot{\mathbf{u}}^\tau + \frac{(1-2\beta)}{2\beta}\ddot{\mathbf{u}}^\tau \right] \quad (20)$$

## 6. Numerical examples

The results shown below for the different examples were obtained mainly from the Newmark method since the Houbold method, in general, does not converge. Hereinafter HBEM and NBEM will refer to the combination of BEM with the Houbolt and the Newmark method, respectively.

### 6.1. Box-shaped structure containing an acoustic fluid subjected a Heaviside load coupled to a membrane

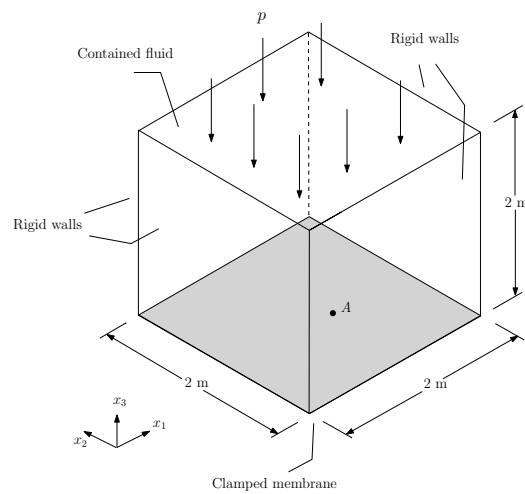
The first example consists of a partially opened box-shaped structure with rigid walls and a flexible elastic membrane with  $c_w = 0.7071$  m/s as presented in figure 5. The structure contains an acoustic fluid with mass density  $\rho_f = 1.21$  kg / m<sup>3</sup> and a bulk's modulus  $\kappa = 139876$  Pa. A distributed unit pressure load  $p(t) = -1.0$  Pa is applied over the face of the structure located at  $x_3 = 2$  m. Pressure gradients  $q = n_i^f p_{,i}$  are considered zero at the other faces. Initial conditions for this problem are  $\mathbf{u}(\mathbf{x}, 0) = \dot{\mathbf{u}}(\mathbf{x}, 0) = 0$ .

To validate the BEM solution, a 3D finite element code (FEM) for this problem was developed. The meshes used for discretization are resumed at the table below.

These collocation points are coincidents with fluid collocation points located in the fluid-membrane interface.

### 6.2. Box-shaped structure containing an acoustic fluid subjected an harmonic load coupled to a membrane

For the second example, only the Heaviside pressure load was changed for a harmonic load defined as  $p(t) = -\sin(\omega t)$  Pa. Here both the frequency  $\omega$  and the amplitude was set to one.



**Figure 5.** Box-shaped structure coupled to a membrane

**Table 1.** FEM Meshes

Mesh	Number of 3D elements	Number of fluid nodes	Number of 2D elements	Number of membrane nodes
FEM 6x6x6	216	343	36	49
FEM 8x8x20	1280	1701	64	81
FEM 12x12x20	2880	3549	144	169
FEM 16x16x20	5120	6069	256	289

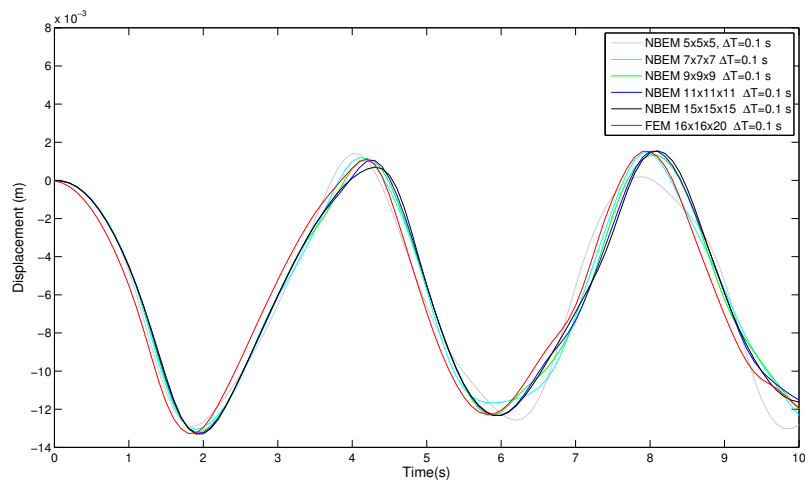
**Table 2.** BEM Meshes

Mesh	Number of 2D quadrilateral elements	Number of interior collocation points	Number of 2D constant elements	Number of interior collocation points
BEM 5x5x5	125	125	20	25
BEM 7x7x7	343	125	28	49
BEM 9x9x9	729	125	36	81
BEM 11x11x11	1331	125	44	121
BEM 15x15x15	3375	125	60	225

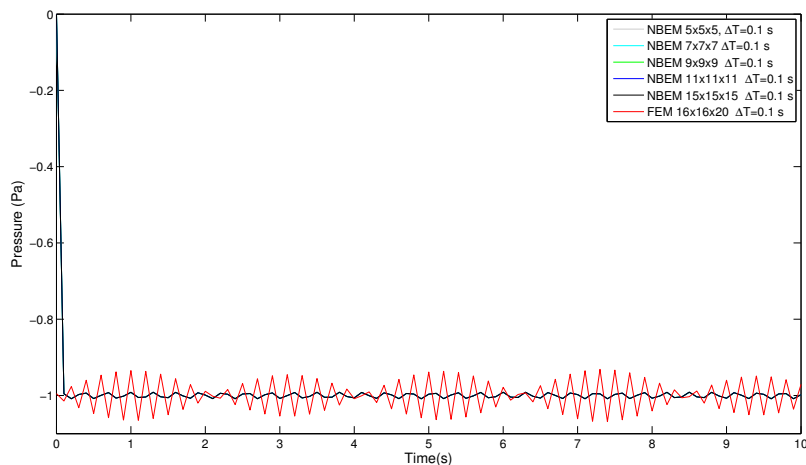
*6.3. Channel-shaped structure containing an acoustic fluid coupled to an inclined membrane*

In this example lets to analyze a partially opened Channel-shaped structure with rigid walls containing an acoustic fluid coupled to a inclined flexible elastic membrane located at the right side of the channel. The membrane wave propagation velocity, the acoustic fluid mass density and the fluid compressibility are respectively  $c_w = 0.7071$  m/s,  $\rho_f = 1.21$  kg / m<sup>3</sup> and  $\kappa = 139876$  Pa. A distributed harmonic pressure load  $p(t) = -sin(wt)$  Pa is applied over the membrane, see the figure 1. Pressure gradients  $q = n_i^f p, i$  considered zero at the rigid walls. Initial conditions for this problem are  $\mathbf{u}(\mathbf{x}, 0) = \dot{\mathbf{u}}(\mathbf{x}, 0) = 0$ .

The convergence in time for the displacement and pressure, using a refined mesh are shown below in figures 13 and 14. Figure 13 shows that the behavior of displacement over time as well as for pressure in figure 14 is periodic.



**Figure 6.** Displacement time history at point A(0.25,-0.25,-1) using DR-BEM with Newmark method

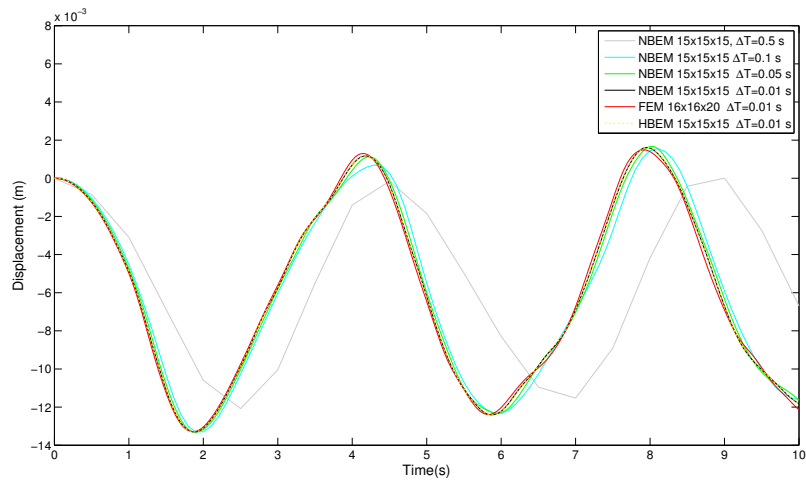


**Figure 7.** Pressure time history at point A(0.25,-0.25,-1) using DR-BEM with Newmark method

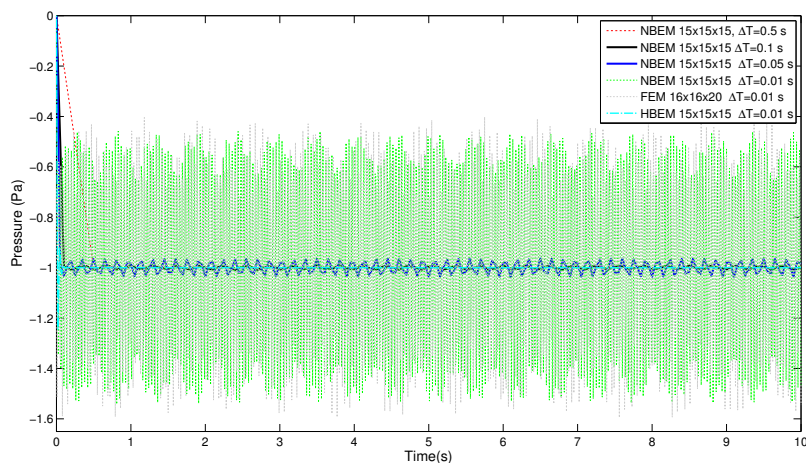
#### 6.4. Channel-shaped structure containing an acoustic fluid coupled to a membrane

The last example consists of a partially opened Channel-shaped structure with rigid walls containing an acoustic fluid coupled to a flexible elastic membrane located at top face of the channel. All properties, pressure load and boundary conditions remain as before, see the figure 15.

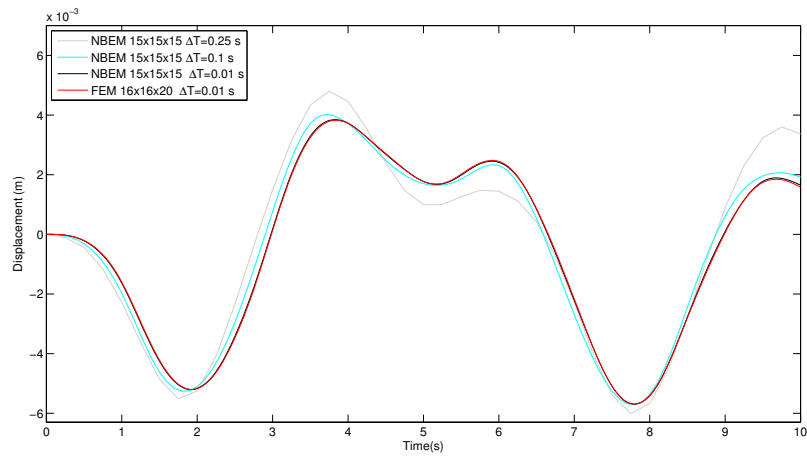
The convergence in time for the displacement and pressure, using a refined mesh are shown below in figures 16 and 17. It can be observed that the time history for displacement is lightly different compared with the inclined membrane, i.e., the shape of the wave is similar and only the amplitude changes, because the dimensions are different, and as it was expected, the longer membrane has higher displacement at the center .



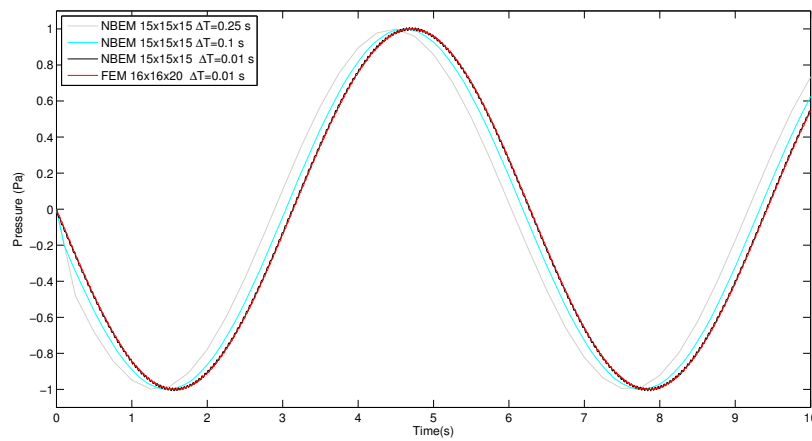
**Figure 8.** Displacement time history at point A(0.25,-0.25,-1) using DR-BEM with Newmark or Houbolt method



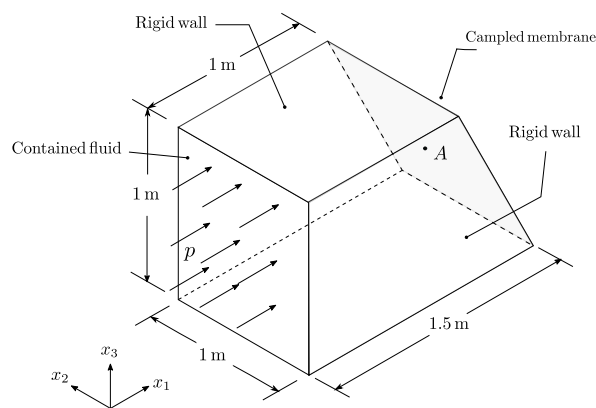
**Figure 9.** Pressure time history at point A(0.25,-0.25,-1) using DR-BEM with Newmark or Houbolt method



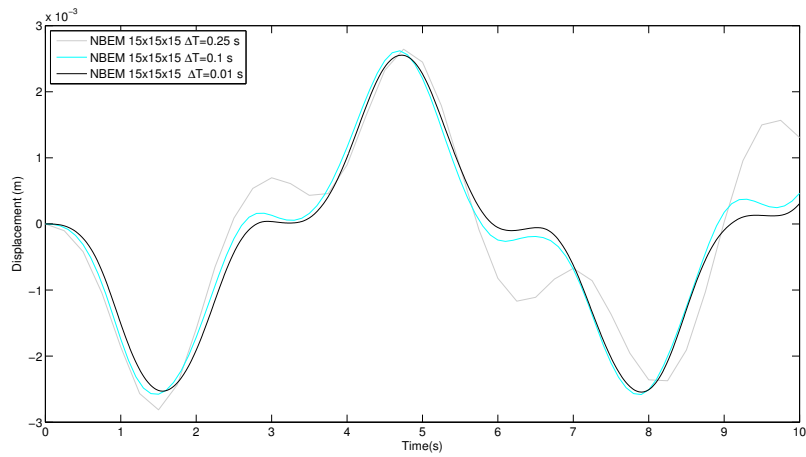
**Figure 10.** Displacement time history at point A(0.25,-0.25,-1) using DR-BEM with Newmark method



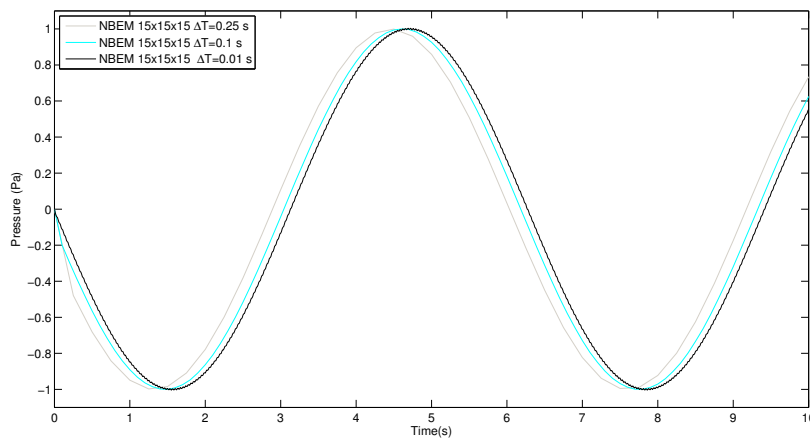
**Figure 11.** Pressure time history at point A(0.25,-0.25,-1) using DR-BEM with Newmark method



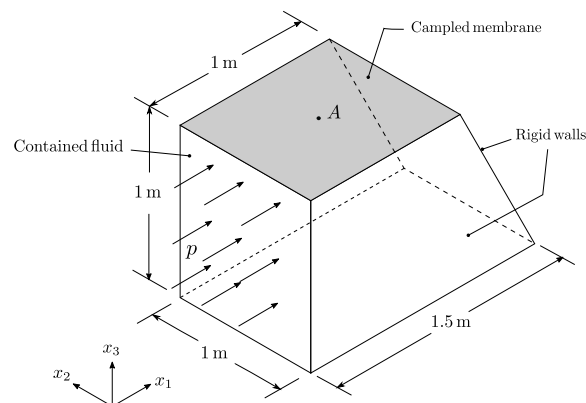
**Figure 12.** Channel 1



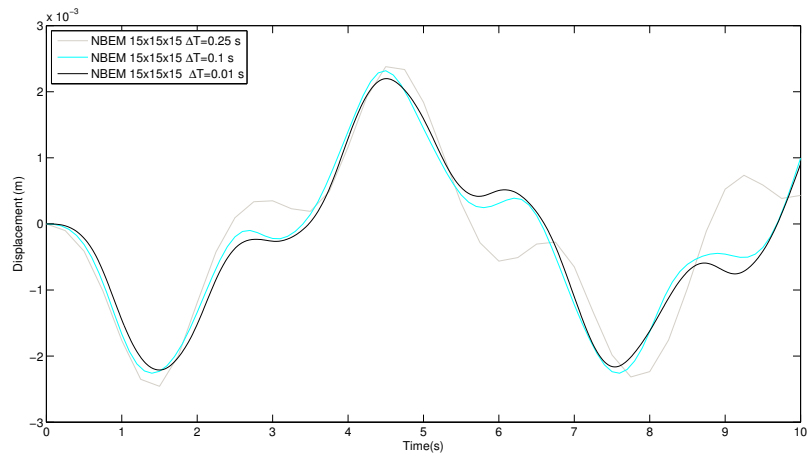
**Figure 13.** Displacement time history at point A(1.25,0.5,0.5) using DR-BEM and Newmark method



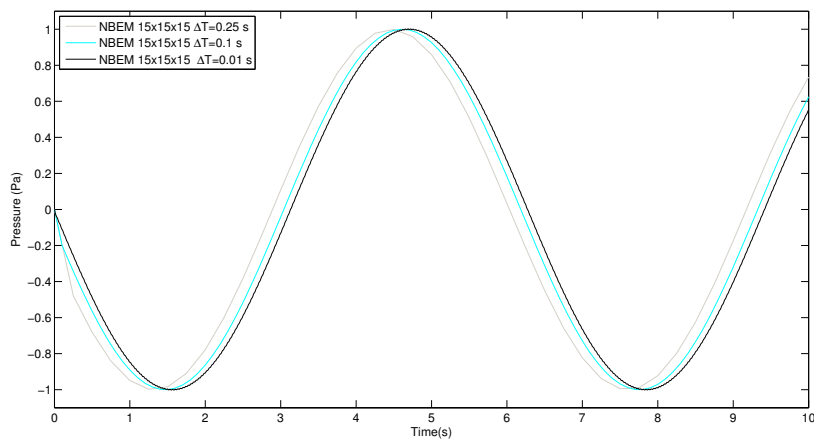
**Figure 14.** Pressure time history at point A(1.25,0.5,0.5) using DR-BEM and Newmark method



**Figure 15.** Channel 2



**Figure 16.** Displacement time history at point A(0.5,0.5,1) using DR-BEM and Newmark method



**Figure 17.** Pressure time history at point A(0.5,0.5,1) using DR-BEM and Newmark method

## 7. Conclusions

A new full boundary element formulation for the transient dynamic analysis of acoustic fluids coupled to elastic membranes is presented. Membranes were modeled using a boundary element formulation based on the linear elastic membrane theory under small deflection. The acoustic fluid was modeled using a boundary element formulation for the three dimensional acoustic wave equation. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles at fluid-structure interfaces. Domain integrals on both, fluid and structure equations, were treated using the Dual Reciprocity Boundary Element Method. Results show good agreement with those obtained from finite element models, turning the proposed formulation in an alternative numerical engineering tool for the dynamic analysis of acoustic fluids coupled flexible elastic membranes.

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