

Genericity of Homeomorphisms with Full Mean Hausdorff Dimension

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Abstract—It is well known that the presence of horseshoes leads to positive entropy. If our goal is to construct a continuous map with infinite entropy, we can consider an infinite sequence of horseshoes, ensuring an unbounded number of legs.

Estimating the exact values of both the metric mean dimension and mean Hausdorff dimension for a homeomorphism is a challenging task. We need to establish a precise relationship between the sizes of the horseshoes and the number of appropriated legs to control both quantities.

Let N be an n -dimensional compact Riemannian manifold, where $n \geq 2$, and $\alpha \in [0, n]$. In this paper, we construct a homeomorphism $\phi : N \rightarrow N$ with mean Hausdorff dimension equal to α . Furthermore, we prove that the set of homeomorphisms on N with both lower and upper mean Hausdorff dimensions equal to α is dense in $\text{Hom}(N)$. Additionally, we establish that the set of homeomorphisms with upper mean Hausdorff dimension equal to n contains a residual subset of $\text{Hom}(N)$.

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