Relationship between magnetic rigidity cutoff and chaotic behavior in cosmic ray time series using visibility graph and network analysis techniques

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Cosmic rays are highly energetic particles originating from astrophysical events outside the Solar System. In this study, we analyze the time series of cosmic ray flux measured by neutron detectors at 16 monitoring stations distributed worldwide. By applying visibility graph analysis, we explore the relationship between the magnetic rigidity cutoff (R_c) and the fractality exhibed from topology of the cosmic ray time series. Our results reveal a significant association between the magnetic rigidity cutoff and the fractality of the cosmic ray time series. Specifically, the analysis of visibility graphs and network properties demonstrates that the magnetic rigidity is inversely related to the magnetic rigidity cutoff. The identified relationship between magnetic rigidity and fractality provides insights into the chaotic nature of cosmic ray variations and their potential uses for predictability.

Cosmic rays, which are highly energetic particles originating from astrophysical events beyond our Solar System, have captivated scientists for many years due to their fascinating properties and their impact on the space environment. In this investigation, a novel approach based on visibility graph analysis was employed to explore the correlation between the behavior of cosmic ray time series and the magnetic rigidity cutoff. The main objective was to gain a deeper understanding of the chaotic dynamics exhibited by these variations and their potential for prediction. The outcomes unveiled a noteworthy connection between the magnetic rigidity cutoff and the fractality of cosmic ray time series, shedding further light on the intricate nature of this phenomenon and its relationship with the structure of the detector network. These findings not only contribute significantly to the field of astrophysics but also hold valuable implications for comprehending solar activity patterns and forecasting cosmic events. As humanity advances in space exploration and our understanding of astrophysical phenomena, this study assumes a important role in enhancing our knowledge of cosmic rays and their behavior within our vast cosmic environment.

I. INTRODUCTION

Cosmic rays, highly energetic particles originating from astrophysical phenomena such as supernovae, neutron stars, and black holes[1,2,20], traverse space and penetrate the Earth's atmosphere. Interacting with interstellar matter and atmospheric molecules, cosmic rays initiate a cascade of subatomic particles within the lower atmosphere. Due to the Earth's magnetic field, many of these particles deviate from their original path, while others, particularly those of lower energy, dissipate upon colliding with atmospheric molecules.

On the surface of the Earth, neutron detectors continually gauge the intensity of cosmic rays[13,14]. These detectors measure the frequency of collisions involving neutrons and atmospheric particles, rendering them highly responsive to cosmic ray fluctuations. Consequently, neutron detectors serve as valuable tools for investigating geophysical and climatic variations. Their continuous monitoring has yielded significant findings, including the identification of links between solar activity[7,15,16] and terrestrial temperature[3,10,22].

Magnetic rigidity cutoff refers to the phenomenon where charged cosmic rays with lower energy or momentum are unable to penetrate the Earth's magnetic field and reach the Earth's surface. Charged particles, such as protons and other nuclei, traveling through space are subject to the influence of magnetic fields encountered along their paths. The strength of Earth's magnetic field varies with location and altitude, causing charged particles to experience varying degrees of deflection. As the energy of a cosmic ray decreases, its ability to overcome the magnetic field's deflecting force diminishes, and there exists a critical magnetic rigidity below which the particle is effectively blocked from reaching the Earth's surface. The magnetic rigidity cutoff is crucial in cosmic ray studies, as it influences the observed cosmic ray flux at different locations on Earth and aids scientists in understanding the energy distribution and origin of cosmic rays in our universe.

In a recent study published at Sierra-Porta et. al. (2022)[17,18] has been used the multifractal detrented fluctuation analysis (MFDFA)[9] technique to investigate the relationship between geomagnetic rigidity cutoff (R_c of neutron monitor stations) with multifractal variability and behavior in the time series of the cosmic ray flux on Earth. The data used for the study were collected from 32 neutron monitoring stations distributed around the world with rigidity cutoff in the [0.0,15] GV interval. The objective is to determine the intrinsic relationship of the fractality measure for the cosmic ray time series with the rigidity cutoff of each monitoring station, which is known to correlate with the geographical latitude of the location of the cosmic ray monitor station.

The results in above mentioned research using MFDFA, show that there is some bias in the multifractal properties of the cosmic ray time series associated with the latitude of the monitoring stations. A significant relationship is obtained between the R_c for different behaviors and the Hurst exponent of the series corresponding to the counts at the neutron monitoring stations. An inverse relationship was explicitly found by authors between the Hurst exponent of different neutron monitor stations in relation to the magnetic rigidity cutoff R_c, such that $H(q) = mR_c + B$ with m and B constants and q is the degree exponent in the fluctuation neutron monitor time series $\left(F_q(n) = \left(\frac{1}{N/n} \sum_{i=1}^{N/n} F(n,i)^q \right)^{1/q} \right)$ for different scales n, where multifractal systems scale as a function $F_q(n) \propto n^{H(q)}$, with $H(q)$ is known as Hurts exponent.

In this new study, the technique of visibility graph analysis (VGA) and network analysis has been used to investigate the relationship between magnetic cutoff rigidity and chaotic dynamic of cosmic ray time series. VGA is a technique used to transform the time series into a graph of nodes and edges, where the nodes represent the values of the series and the edges represent the visibility between two nodes. The technique allows quantifying the topology of the graph, such as its degree, density, diameter, among others metrics. Therefore, it provides a different way to analyze the structure of the time series.

On the other hand, network analysis is used to quantify the properties of a complex system in terms of its connectivity and topology. With this technique, we can identify subsets of highly connected nodes, called modules or communities, which allows us to better understand the structure and function of the system under study.

Through the analysis of visibility graphs and network analysis, we have found significant results that corroborate the results found in Sierra-Porta et. al. (2022)[18] regarding the relation between magnetic rigidity and intrinsic dynamic of cosmic ray time series, which to our knowledge, after a thorough review of the existing literature, has not been used in this context.

This study shows that the VGA and network analysis technique is useful in the analysis of the structure and complexity of cosmic ray time series, and can provide valuable information on the link between magnetic rigidity, multifractality and time series topology. The results obtained from this study may have important implications in understanding the chaotic and complex nature of cosmic ray time series and their impact on the Earth.

II. METHODS AND DATA

A. Data for this study

The data used in this study are obtained from Izmiram, a recognized scientific resource specializing in the investigation of solar activity, particle physics, and Earth- and space-related phenomena. The source of the data can be found at the website `http://cr0.izmiran.ru/common/links.htm#Neutron\%20Monitor`. Izmiram collects data through a network of neutron monitors distributed in different geographic locations, which are instruments specifically designed to measure cosmic ray radiation reaching the Earth's surface.

In this study, explicit data from cosmic ray counts collected by neutron detectors provided by Izmiram are used. These data represent measurements of cosmic ray intensity at the Earth's surface. The neutron detectors are strategically distributed in different geographic locations, allowing a broad and diversified view of cosmic radiation to be obtained. Each latitude is also characterized by magnetic rigidity, also known as charged particle magnetic rigidity, which refers to the ability of a charged particle to resist deflection of its trajectory in a magnetic field (see Table I). In more technical terms, magnetic rigidity is defined as the ratio of the particle's linear momentum to its electric charge. Magnetic rigidity depends on the amount of kinetic energy of the particle and the strength of the magnetic field through which it moves. The greater the particle's kinetic energy or magnetic field, the greater its magnetic rigidity, and it is typically expressed in units of gigavolts (GV) or teravolts (TV) for high-energy particles.

	Station	Lat (N)	Lon (E)	Alt. (m.a.s.l.)	Rigidity (GV)
1	Moscow	55.47	37.32	200	2.43
2	Newark	39.68	104.25	50	2.4
3	Novosibirsk	54.48	83	163	2.91
4	Oulu	65.05	25.47	15	0.8
5	Rome	41.86	12.47	0	6.27
6	Thule	76.5	111.3	26	0.3
7	Yakutsk	62.01	129.43	105	1.65
8	Lomnicky	49.2	20.22	2634	3.84
9	Irkutsk	52.37	100.55	2000	3.64
10	Inuvik	68.36	46.28	21	0.3
11	FortSmith	60.02	68.07	180	0.3
12	Esoi	33.3	35.8	2055	10.75
13	Athens	37.97	23.78	260	8.53
14	Apatity	67.57	33.4	181	0.65
15	Hermanus	34.43	19.23	26	4.58
16	Barentsburg	78.03	14.13	51	0.01

TABLE I. Comparative table about characteristic and properties of neutron monitors stations used in this study.

In addition, the used data is concerning to solar cycle 24, which begins in 2008-12 and ends in 2019-12. In this case we use daily resolution for the data, which implies that each series we study has about 4000 instances (see Fig. 1).

B. Visibility and network methods

The visibility graph analysis technique[21] is used to analyze time series and provides a different way of understanding their structure. In VGA, the time series is transformed into a graph, where each value of the series becomes a node and edges are created to determine whether each pair of nodes is visible to each other. A pair of nodes is visible only if there are no other nodes that are higher than them and located between them. In this way the structure of the time series is constructed in a network.

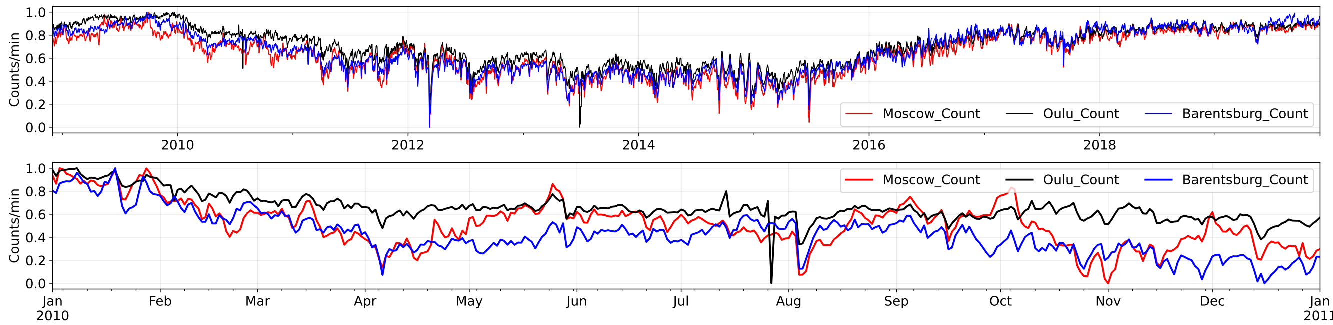

FIG. 1. Top: Neutron monitor data from three stations (Moscow, Oulu and Barentsburg) at different locations around the world in [0,1] scale over the entire data period of this study 2008-2020. Bottom: Same data from the same stations but zoomed in 2010 to establish differences between measurements at the same [0,1] scale.

Once the visibility network is constructed, its different properties can be explored. For example, the density of the network reflects the ratio of existing edges to the possible total. Modification in the network density can indicate changes in the structure of the time series and how its values are correlated. The degree of the node in the network represents the number of nodes to which it is connected. How the density of edges is distributed between nodes with different degrees can provide information about the complexity of the time series.

Studying the structure of visibility graphs can help to understand the complexity of the time series and its possible changes over time. It also provides information that can be used to feed additional analysis techniques, including network analysis.

Network analysis[6,23], moreover, is a technique used to study the connectivity and intrinsic relationships within complex systems. Networks are composed of nodes that represent the components of the system and connections between nodes that represent the interactions between them.

Networks can be analyzed from different perspectives. For example, communities can be shown in which nodes are grouped together due to their intrinsic connections. The analysis can also be carried out through centrality measures, which show the relative importance of a node in the distribution of information through the network.

Horizontal visibility graph (HVG) in a time series is formally defined as the existence of visibility between two data points, i and j, if and only if each bar k between i and j has a value x_k less than both x_i and x_j $(x_k \in [x_i;x_j])$, $i,j \ll \infty$. This implies that the values of the intermediate bars are below the values of data points i and j. This condition establishes a visibility relationship between the data points and provides information about the structure of the time series.

To construct a HVG from a one-dimensional time series, the following steps are followed. The first step is to select a one-dimensional time series of length N. Each data point in the time series becomes a node in the network. For each pair of data points, it is checked if there is horizontal visibility between them. Horizontal visibility means that there are no other points in the time series obstructing the line of sight between the two points.

If a pair of points has horizontal visibility, an edge is added between the corresponding nodes in the network. In mathematical terms, we can represent the time series as a set of points $X = x_1, x_2, x_3, \ldots, x_N$, where x_i is the value of the data point at position i. Then, to verify the horizontal visibility between two points x_i and x_j, the following conditions must be satisfied:

- All points x_k, where $i < k < j$, must be above the line joining points x_i and x_j. That is, for all $k, i < k < j$, it is satisfied: $x_k > y_i + (x_j - y_i)\frac{(k-i)}{(j-i)}$.
- All points x_m, where $j < m < i$, must lie below the line joining points x_i and x_j. That is, for all $m, j < m < i$, it is satisfied: $x_m < y_i + (x_j - y_i)\frac{(m-i)}{(j-i)}$.

If both conditions are satisfied, then the points x_i and xj are connected in the HVG by an edge. This process is repeated for all pairs of points in the time series, resulting in the construction of the horizontal visibility graph.

Once the nodes and links have been identified according to the above description, the corresponding undirected network graph can be uniquely determined (see Fig. 2). From this network, some practical data can be extracted for the time series representing the dynamical system under study. In particular, as discussed analytically in Lacasa and Toral (2010)[11] and Luque et al. (2009)[12], when the number of nodes with degree k, $N(h)$, follows an exponential relationship $N(h) \sim e^{-\alpha_0 h}$ in the network graph obtained using the HVG algorithm, the parameter $\alpha_c = ln(3/2) = 0.404$ determine if the time series system correspond to a uncorrelated ($\alpha_0 < \alpha_c$) or correlated ($\alpha_0 >= \alpha_c$) process[12].

More precisely, the system under study is characterized by chaotic dynamics when α_0 is less than α_c ($\alpha_0 < \alpha_c$). In the region around α_c, an edge of chaos behavior is observed. For values of α greater than α_c ($\alpha > \alpha_c$), the system is stable and predictable.

The above analysis uses the concept of horizontal visibility to establish relationships between data points in a time series. These relationships are represented by an undirected network graph, and the distribution of the nodes is analyzed as a function of their degree h. The presence of an exponential relationship in this distribution allows the establishment of a

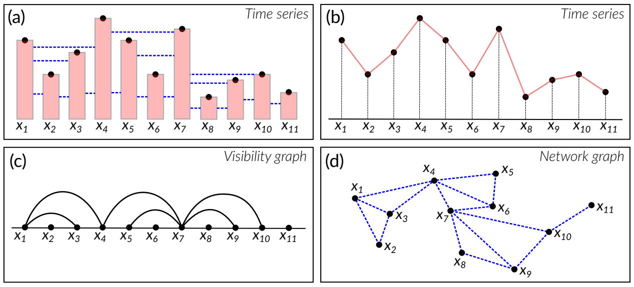

FIG. 2. Schematic visibility and network graph approach to analyze time series. In the horizontal visibility graph analysis a time series (b) can be seen as nodes of rectangles visible from each other without interruption (a). In (c) the visible connections are represented and the corresponding graph is shown in (d).

threshold α_c that determines whether the system is chaotic, exhibits edge of chaos behavior, or is stable and predictable. This approach provides a tool to characterize the dynamics of complex systems from time series data.

III. RESULTS AND ANALYSIS

In the following, we will describe the steps from considering the original series of cosmic ray counts to calculating the power law exponents in the degree of connectivity of the network graphs using the horizontal visibility algorithm and the construction of the network diagram.

We start with the series of cosmic ray counts obtained for 16 neutron stations at different latitudes. The horizontal visibility algorithm is used to construct a new data series from the original series. In this algorithm, each element in the original series is compared with the elements before it, and the number of elements visible from that viewpoint is recorded. These values are used to construct the horizontal visibility series. From the horizontal visibility series, the cumulative histogram is constructed. This involves sorting the values in the series in ascending order and counting how many values are less than or equal to each value in the series. The cumulative histogram shows how the values are distributed in the series.

A log-log scatter plot is then generated using the values in the cumulative histogram. In this graph, the x-axis represents the degree values and the y-axis represents the cumulative frequency of those values. By plotting the points on a logarithmic scale, one can better visualize the degree distribution and determine whether it follows a power law. On the log-log plot, if the points follow approximately a straight line, it indicates that the degree distribution follows a power law. To calculate the power law exponent, a linear regression fit is made to the points on the graph. The slope coefficient of the fitted line represents the degree exponent α_0 in the power law. These steps are repeated for each of the 16 series of cosmic ray counts at different latitudes. Each series will generate its own cumulative histogram, log-log plot, and degree exponent calculation.

To characterize the degree distribution we define a function that can be called iteratively to determine the negative log-likelihood value. The key idea in formulating this function is that it must contain two elements: the first is the model building equation (here, simple linear regression). The second is the log value of the probability density function (here, the log PDF of the normal distribution). Since we need the negative log-likelihood, it is obtained by simply negating the log-likelihood.

Minimize the negative log-likelihood of the generated data using the minimize method. The maximum likelihood estimation (MLE) method arrives at the final optimal solution after 35 iterations. The model parameters, intercept, regression coefficient and standard deviation fit well with those obtained using the MLE approach. Therefore, we can use the ordinary least squares (OLS) method to determine the model parameters and use them as a reference to evaluate the maximum likelihood estimation approach.

The use of OLS has been used in view of the observation of some problems already observed with proper power-law fitting using standard optimization methods (see for example: Small, M. et. al. (2007)[19], and Goldstein, M. L. et. al. (2004)[8]).

The results of these steps are shown in Figure 3 for the power law exponents corresponding to each of them. This allows us to analyze the degree distribution in the cosmic ray neutron detector network and to better understand its structure and connectivity.

Based on the results obtained, we can make the following observations. The power law exponents for the 16 series of cosmic ray counts vary between 2.5 and 3.7. This indicates that the degree distribution in the cosmic-ray neutron detector network follows a power law, and nodes with higher degree are relatively more prevalent compared to a random distribution. We have observed that the largest exponents coincide with the detector locations for small latitudes (East), or smaller values of rigidity cutoff, while the smallest exponents are found at higher latitudes, or larger values of rigidity cutoff. This suggests that the network structure and connectivity of neutron detectors may vary with geographic location.

We have performed a linear regression to explore the relationship between power-law exponents and latitudes. The results indicate that there is a significant association between these variables (determination coefficient, r2-score=0.92 and root mean squared error for regression of 0.0795,

$$\alpha_0 = \frac{m}{1+(R_c/p)^{-q}} \pm \varepsilon, \qquad (1)$$

with $m = 3.712 \pm 0.144$, $p = 96.330 \pm 0.024$, $q = 0.408 \pm 0.097$, and $\varepsilon = 0.079$ as can be show in Fig. 4. The negative above expression indicates that as latitude increases, the degree exponent tends to decrease. The cutoff at ~ 3.7 indicates the expected value of the degree exponent for a zero latitude. The results obtained here can be considered as complementary the conclusions of a previous study[17] for the identification of higher fractality associated with conditions other than magnetic rigidity.

As has been suggested by[12] according to the predictability ability of the series for critical exponent of 0.404 that can be used to discriminate between series, that is, series with a

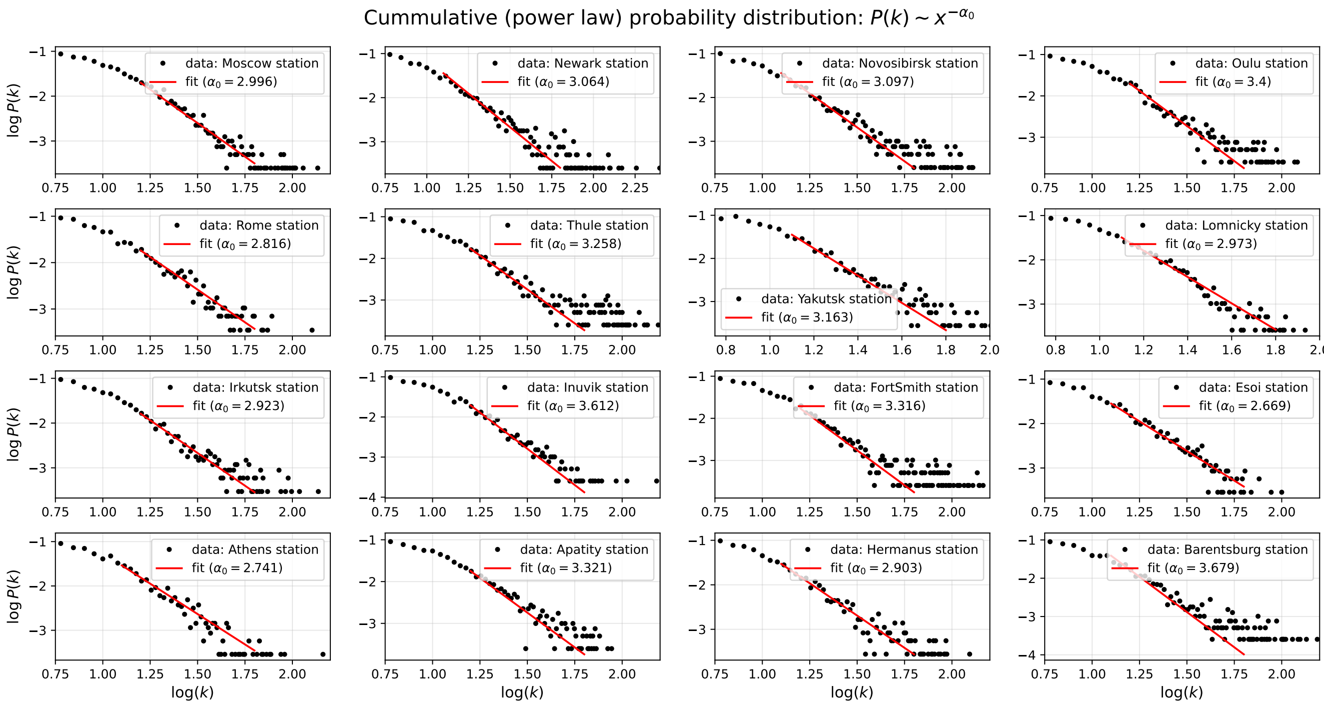

FIG. 3. Cumulative degree distribution for the 16 cosmic rays neutron monitor data stations using HVGs and the degree exponents α_0.

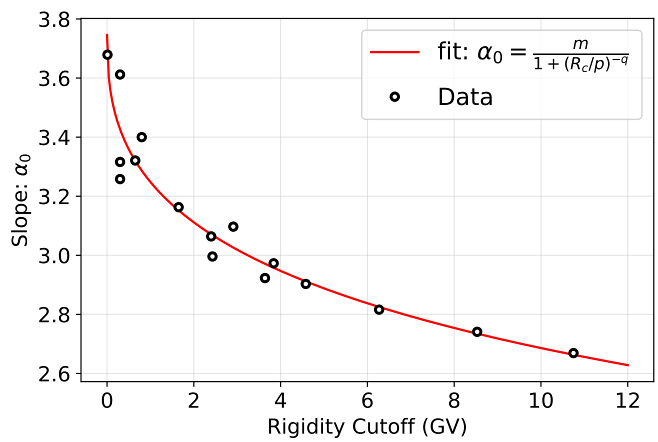

FIG. 4. Degree exponents for the 16 cosmic rays neutron monitor data stations relating with magnetic rigidity cutoff and latitude.

degree exponent greater than 0.404 have more predictable behavior, while series with a degree exponent less than 0.404 may have more chaotic behavior. All exponents are greater than α_c, however we can conclude that the least predictable series are those found in small latitude locations.

The relationship described between the exponent of the power law (scaling factor of the network nodes of cosmic ray counts at the detectors) and the inverse of magnetic rigidity in the equation reflects a dependence of the degree distribution in the network generated by the horizontal visibility algorithm on the magnetic rigidity of cosmic rays. Here are some considerations on how these variables might be related and how to interpret the equation.

The presence of magnetic rigidity in the denominator suggests that as magnetic rigidity increases, the contribution of this term decreases, and the scale tends to increase. This could indicate that the properties of the network generated by the horizontal visibility algorithm are influenced by the cosmic rays ability to penetrate the Earth's magnetic field.

The physical significance of the equation lies in the fact that magnetic rigidity is related to a cosmic rays ability to overcome magnetic barriers. A lower rigidity implies a greater ability to traverse the magnetic field, which could be associated with higher variability or complexity in the time series of cosmic rays.

The equation suggests a nonlinear relationship shaping the curve. This functional form could be an empirical representation of the observed relationship between magnetic rigidity and the degree distribution in the network generated by the horizontal visibility algorithm.

An increase in the value of the degree slope may suggest a higher proportion of nodes with elevated degrees in the analysis network. This phenomenon potentially hints at increased complexity or heterogeneity in the time series of cosmic rays, possibly reflecting alterations in the dynamics of cosmic events generating these behavior. However, it is essential to emphasize that this interpretation requires a more in-depth analysis and consideration of other factors. While the rise in the degree slope could be associated with higher levels of complexity, further research is needed to substantiate this hypothesis. It is crucial to acknowledge potential confounding variables and limitations in our analytical approach. This result, while suggesting a potential relationship, warrants deeper exploration and careful examination in future investigations.

As demonstrated in O. A. Danilova et. al. (2023)[5] (also see: W. Chu (2016)[4]), there exists a substantial influence of certain heliospheric and solar wind parameters, such as the solar wind speed and the interplanetary magnetic field. Additionally, certain geomagnetic activity indices, including the Dst index (known to modulate cosmic ray intensities at Earth's surface), significantly contribute to variations in the latitudinal behavior of cosmic ray geomagnetic thresholds. The correlation of rigidity cutoff variations with electromagnetic parameters exhibits variability that is station-dependent and follows a discernible pattern.

Finally, the relationship between the exponent of the power law and magnetic rigidity in the provided equation suggests an intriguing dependence between the structure of the network generated by the horizontal visibility algorithm and the physical properties of cosmic rays. The equation may serve as an empirical expression capturing this observed relationship in experimental data, but a precise interpretation would require a more detailed analysis of specific dataset properties and underlying physics.

The relationship between the scaling factor and the nodes in cosmic ray time series can be understood by considering how the scaling factor influences the degree distribution in the network generated by the horizontal visibility algorithm. In the context of horizontal visibility, the scaling factor is related to the ability to detect significant changes in the time series, thus affecting the connection between nodes in the network.

The scaling factor in the context of horizontal visibility influences the distance at which two points in the time series are considered visible to each other. A larger scaling factor implies that a more significant change in the data is required for two points to be considered mutually visible.

The relationship between the scaling factor and nodes is also linked to the complexity of the time series. A properly chosen scaling factor can reveal relevant patterns and events in the series, while an inappropriate scaling factor might obscure or overlook important features. Furthermore, since α_0 somewhat establishes the predictability of the series, then it is interesting to note that cosmic ray counts at stations with low rigidity will be more complicated to predict than those housed at higher rigidity.

The findings from our previous article Sierra-Porta. et. al. (2022)[18], particularly the observed relationship between cutoff rigidity and the Hurst exponent using multifractal detrented fluctuation analysis, indicate a significant connection between the geographical latitude of cosmic ray detectors and the multifractal behavior of cosmic ray flux. The inverse relationship observed, where higher cutoff rigidity corresponds to a lower Hurst exponent, suggests that the chaotic nature of the cosmic ray series is influenced by the rigidity and geographical location of monitoring stations.

Now, it appears that there might be a multifaceted interplay involving the factor of scale, cutoff rigidity, and the Hurst exponent. The inverse relationship between the scaling factor and rigidity, as well as between cutoff rigidity and the Hurst exponent, suggests a potential interconnectedness.

The scaling factor influences the network structure, reflecting the visibility of events in the time series. An inverse relationship with cutoff rigidity could imply that cosmic rays with higher rigidity are associated with more prominent and distinguishable events in the time series. In the meantime, the inverse relationship between cutoff rigidity and the Hurst exponent suggests that as the rigidity increases, the long-term correlations in the cosmic ray series decrease. This implies that cosmic rays with higher rigidity exhibit less persistent behavior, possibly due to the influence of Earth's magnetic field.

The observed bias in the chaotic nature of the cosmic ray series associated with the latitude of monitoring stations, as well as the inverse relationship between rigidity and the Hurst exponent, may be connected through the factor of scale. The interconnected results suggest that the multifractal behavior of cosmic rays is not only influenced by their geographical detection location but also by the rigidity of the cosmic rays. The inverse relationships indicate a complex interplay between the physical properties of cosmic rays, the geographic distribution of detectors, and the analytical methods employed.

IV. CONCLUSIONS

In summary, this study analyzed the degree distribution in the cosmic ray neutron detector network from the Izmiram data using a power law to relate the degree of connectivity of the nodes in the series and found that the degree exponents varied such that the results revealed a significant association between degree exponents and latitudes, where larger exponents were found at smaller latitude locations (east) and smaller exponents at higher latitudes. This confirms the results of a previous study and the finding suggests that the structure and connectivity of the detector network may be influenced by geographical factors. In addition, all the series considered are quite predictable, which has been studied extensively as modulation with solar activity. Cosmic ray count series with higher degree exponents (lower latitudes) were found to be more predictable compared to those with lower exponents (higher latitudes), which exhibited potentially more chaotic behavior.

Although this study presents an initial exploration of the intricate relationship between cosmic ray geomagnetic thresholds and various factors influencing them, it is clear that further research is needed to achieve a more complete understanding. Our findings suggest that the inclusion of additional variables related to parameters that capture the dynamics of the interplanetary medium could improve the accuracy of correlations and models. Considering the complex interplay between cosmic ray variations and solar wind parameters, as well as geomagnetic perturbations, a refined approach encompassing a wider range of influencing factors is recommended. Future studies should seek to incorporate a more comprehensive set of variables to unravel the nuanced dynamics governing cosmic ray behavior and establish more robust correlations and predictive models.

ACKNOWLEDGMENTS

Because of the nature of the methods used in this study, there are no particular limitations to mention for the proposed analysis, although, probably, one of the major difficulties is to obtain complete cosmic ray count series with the least amount of missing data. In this study, we wanted to use more points and longer series by covering more solar cycles. However, due to the particular form of horizontal visibility analysis the codes used to obtain the results are done sequentially so a longer series implies much more time and computational power. For now the results imply clear conclusions. We thank Izmiram for the availability of the data.

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DATA AVAILABILITY STATEMENT

A repository with codes, jupyter notebooks and data for analysis and the original data can be found at: `https://github.com/sierraporta/VisibilityGraphAnalysisCosmicRays/tree/main`.

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