Minimization of the distribution operating costs with D-STATCOMS: A mixed-integer conic model

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ABSTRACT

This paper addresses the problem of determining the optimal location and operation of distribution static synchronous compensators (D-STATCOMs) in power distribution networks. Metaheuristic algorithms have been conventionally used to solve this mixed-integer nonlinear programming problem. In contrast, we propose a mixed-integer second-order cone programming (MI-SOCP) model that guarantees global optimum and fast convergence of the algorithms in optimization solvers. The model seeks to minimize the annual installation cost and the operating cost, subject to active and reactive power balance constraints, voltage regulation bounds, devices capabilities, and the number of D-STATCOMs available to be connected. The multiobjective nature of the problem is also analyzed and solved using the proposed MI-SOCP model. An extensive set of simulations on the IEEE-33 nodes test system demonstrate the advantages of this approach compared to conventional heuristic algorithms and with solutions given by algebraic modeling software.

1. Introduction

Energy loss in distribution networks is a challenge that must be overcome by utility companies to improve efficiency in an increasingly competitive market. Operating costs associated with this loss must be shared between end-users and distribution companies based on the regulatory policies of each country [1]. Therefore, it is necessary to install active components such as variable capacitors, distribution static synchronous compensators (D-STATCOMs), batteries, and distributed generators [2]. However, in multiple applications with batteries and renewable energy resources, the total costs of the energy losses can increase owing to the optimal integration objective of these devices being different from the energy losses reduction; thus, it can be a combination of environmental and economic performance indicators [3]. Although all of these devices significantly reduce energy loss, capacitor banks and D-STATCOMs have rates of return in a few years and long useful life [4]. When compared to capacitor banks, D-STATCOMs have additional advantages such as dynamic reactive power compensation and the possibility to generate or absorb reactive power depending on the requirements of the grid [5,6]. However, the D-STATCOM must be placed and operated correctly to obtain measurable improvements in energy loss at reasonable investment costs.

An optimization model for the location, sizing, and operation of D-STATCOMs in active distribution networks is required in this context. Such a model must include mixed-integer variables with nonlinear and non-convex constraints related to the power flow equations [7]. The model must consider these nonlinear equations since $r \approx x$ in power distribution networks, making the DC-power flow approximation inapplicable. Due to its high complexity, the problem has been classically solved using sensitivity analysis [8] and metaheuristic algorithms [9]. Authors of [10] presented an application of the approximate quadratic conic model to determine the nodes where the D-STATCOMs are to be installed, while their optimal sizes were decided using a conic power flow formulation. Numerical results demonstrated the efficiency of the proposed approach when compared with metaheuristic methods. Fernández et al. in [11], proposed a heuristic optimization algorithm for locating D-STATCOMs in radial low-voltage distribution grids by listing the first ten nodes with the worst voltage unbalance. Each of these nodes was assigned a D-STATCOM with a size of 30 kVA. The objective function minimized the total grid power losses, and the node that allowed the highest reduction of losses was selected as the best node. Authors of [12], and [13] presented an application of the classical particle swarm optimization method to determine the location and size of D-STATCOMs in distribution grids. The objective functions consider

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voltage profile improvement and power loss minimization. A complete revision with the main approaches to the studied problem can be found in [14] and the references therein.

A similar approach was proposed in [9], which combines the classical Chu & Beasley genetic algorithm (CBGA) with the second-order cone programming formulation of the power flow. However, due to the random nature of the CBGA, it is not possible to ensure the global optimum. In addition, since the second-order cone formulation is entrusted with the minimization of the total energy losses for each nodal combination and size of D-STATCOMs (optimal power losses), the final grid operative costs increase compared to the previous metaheuristic approach based on the discrete–continuous version of the vortex-search algorithm reported in [15].

This paper proposes a different approach by using a mixed-integer second-order cone approximation. This model considers the nonlinear nature of the power flow equations and the discrete variables associated with the placement of new components. A linear approximation for the cubic cost function is also proposed to obtain a mixed-integer convex model solvable by commercial optimization packages.

The problem may also be considered as a multiobjective model. This type of model is usually solved using heuristic algorithms. However, in the present study, the convex formulation is used to obtain a Pareto frontier without resorting to cumbersome heuristic methods [16].

The contributions of this study are fourfold:

(1) The mixed-integer nonlinear programming model is transformed into a conic approximation that is effortlessly solved by CvxPy to find the global optimum.
(2) The multiobjective problem concerning the investment and operating costs is solved through the conic-approximation using weighting factors. This approach demonstrates the conflict between the two objectives. Multiobjective problems generally employ metaheuristic algorithms, even though exact techniques in convex optimization models are capable of solving them.
(3) The effect of varied number of D-STATCOMs available for installation is evaluated for zero to five new components.
(4) Numerical results demonstrate the effectiveness and robustness of the proposed model compared to several packages for mixed-integer nonlinear optimizations available in the General Algebraic Modeling system (GAMS) as well as the metaheuristic optimization approach based on the discrete–continuous version of the vortex-search algorithm provided in [15].

The rest of the paper is organized as follows: Section 2 presents the conventional optimization model, which is nonlinear, non-convex, and mixed-integer. Section 3 demonstrates the proposed approximations that allow obtaining a mixed-integer second-order model. The proposed test system is described in Section 4. The numerical results are presented in Section 4, followed by conclusions in Section 5.

2. Problem definition

A D-STATCOM is a regulating device based on power electronics voltage-source converter, which adds flexibility to the power distribution network. Compared to a variable capacitor, a D-STATCOM is a more flexible device since it injects the exact amount of reactive power according to the requirements of the grid [17]. In addition, it can supply additional functions such as harmonic filtering and dynamic compensation. It may also consume reactive power in the event of a surplus.

The mathematical model for determining the optimal location and size of D-STATCOMs in power distribution networks is a nonlinear mixed-integer extension of the optimal power flow (OPF) [15], where continuous variables are related to voltages and powers, and new binary variables are included to represent the location of a D-STATCOM in a particular node. The grid is represented by an oriented graph $\mathcal{G} = \{\mathcal{N}, \mathcal{B}\}$, where $\mathcal{N}$ is the set of nodes and $\mathcal{B} \in \mathcal{N} \times \mathcal{N}$ is the set of branches [18]. We adopt a branch model in the complex domain for operation in a time horizon $\mathcal{T}$. Fig. 1 depicts the variables associated with a generic branch $l = (km) \in \mathcal{B}$ for a time $t \in \mathcal{T}$.

The power flow from node k to m (s^s_{lt}) is different from the power flow from node m to k (s^r_{lt}), as given in (1) and (2) respectively:

$$s^s_{lt} = v_{kt} y^*_l \left(v_{kt} - v_{mt}\right)^* \tag{1}$$

$$s^r_{lt} = v_{mt} y^*_l \left(v_{mt} - v_{kt}\right)^* \tag{2}$$

where v_{kt} and v_{mt} are the complex voltages in the nodes k and m for the time t, and $(\cdot)^*$ represents the complex conjugate. Total power loss in each branch is given by the sum of s^s_{lt} and s^r_{lt}, as given in (3):

$$s^{\text{loss}}_{lt} = s^s_{lt} + s^r_{lt} = y^*_l \left(v_{kt}v^*_{kt} - v_{kt}v^*_{mt} - v_{mt}v^*_{kt} + v_{mt}v^*_{mt}\right) \tag{3}$$

Note that (1) to (3) are nonlinear non-convex equations that make the problem NP-hard.

A node-to-branch incidence matrix A is defined and split into two components such that $A = A^+ + A^-$, where A^+ contains the positive values of A and A^- the negative values. Thus, the balance equation is formulated as follows:

$$s^g_k - s^d_k + jq^c_k = \sum_{l \in L} \left(A^+_{kl} s^s_l + A^-_{kl} s^r_l\right) \tag{4}$$

In this equation and in the rest of this paper, superscripts represent the type of variable, namely g for generation, d for demand, and c for compensation given by the D-STATCOMs.

The objective to minimize is the annual operative costs (f^{annual}), which include the annual cost associated with the energy losses (f^{loss}) and the annualized costs of investment in new D-STATCOMs (f^{invest}), as given in (5). (This objective function was formulated based on the recommendations of the authors of [19] and [20].):

$$\begin{aligned} &\min f^{\text{annual}} = f^{\text{loss}} + f^{\text{invest}} \\ &f^{\text{loss}} = \sigma T \sum_{l \in \mathcal{B}} \sum_{t \in \mathcal{T}} \text{real}\left\{\left(s^s_{lt} + s^r_{lt}\right)\right\} \Delta t \\ &f^{\text{invest}} = \xi T \sum_{k \in \mathcal{N}} h_k \left(\alpha h^2_k + \beta h_k + \gamma\right) \end{aligned} \tag{5}$$

where Δt is the length of the period in the time horizon; h_k represents the optimal size for a D-STATCOM connected at node k; T is the period of time under analysis (usually one year); ξ is a positive constant associated with the annualization costs of investment in new D-STATCOMs; and α, β, and γ are the cubic, quadratic, and linear coefficients of the investment cost in new D-STATCOMs [15]. The rest of the model is given in (6); some previously presented constraints are repeated for the sake of completeness.

$$\begin{aligned} s^g_{kt} - s^d_{kt} + jq^c_{kt} &= \sum_{l \in L} \left(A^+_{kl} s^s_{lt} + A^-_{kl} s^r_{lt}\right), \forall k \in \mathcal{N}, \forall t \in \mathcal{T} \\ s^s_{lt} &= v_{kt} y^*_l \left(v_{kt} - v_{mt}\right)^*, \forall l \in \mathcal{B}, \forall t \in \mathcal{T} \\ s^r_{lt} &= v_{mt} y^*_l \left(v_{mt} - v_{kt}\right)^*, \forall l \in \mathcal{B}, \forall t \in \mathcal{T} \\ v_{0t} &= v^{\text{nom}} e^{j0}, \forall t \in \mathcal{T} \\ \left\|v_{kt}\right\| &\leq v^{\max}, \forall k \in \mathcal{N}, \forall t \in \mathcal{T} \\ 0 \leq h_k &\leq z_k q^{\max}, \forall k \in \mathcal{N} \\ -h_k \leq q^c_{kt} &\leq h_k, \forall k \in \mathcal{N}, \forall t \in \mathcal{T} \\ \sum_{k \in \mathcal{N}} z_k &\leq \eta, \\ z_k &\in \{0, 1\} \; \forall k \in \mathcal{N}, \end{aligned} \tag{6}$$

where q^c_{kt} represents the reactive power of a D-STATCOM connected at node k at time t; v^{nom} is the nominal voltage at the substation (usually 1 in per-unit); $v^{\max}$ represents the regulation bound for all nodes; $q^{\max}$ is the maximum size allowed for each D-STATCOM; z_k is a binary variable that defines if a D-STATCOM is connected at node k; and η is the number of D-STATCOMs available for installation. Eq. (4) defines the complex power balance at each node, and Eqs. (1) and (2) show the sending and receiving power flows at each line,

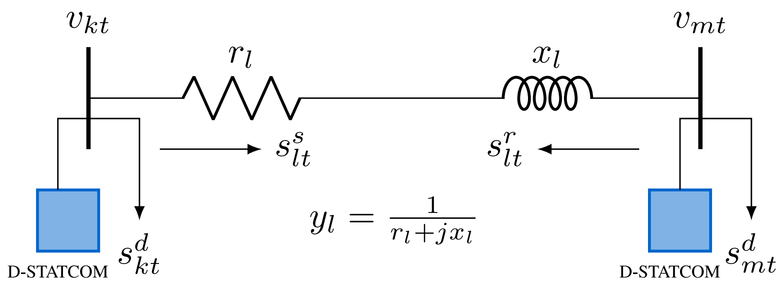

Fig. 1. Representation of a generic branch in a power distribution network.

which is defined as a nonlinear function of the complex voltages. All node voltages are maintained within the allowed limits, and voltage at the substation is set to its nominal value; other box-type constraints determine the nominal size for a D-STATCOM connected at node k and its daily operative dispatch, respectively. Finally, the maximum number of D-STATCOMs available for installation is limited by η.

Model (6) is undoubtedly complicated. It is a mixed-integer nonlinear programming (MINPL) problem in the complex domain, where nonlinearities appear in Eqs. (1), (2), and (5). The model is NP-hard; hence, the metaheuristic approach is common in the scientific literature. Nevertheless, some constraints are already convex, and other parts can be approximated to convex constraints, particularly second-order cones. The transformation of the MINLP model into a Mixed-Integer Second-Order Cone Programming (MI-SOCP) equivalent is presented in the next section.

3. Approximated mixed-integer convex formulation

We employ a second-order cone to approximate the complex power flows [18]. Moreover, the cubic form of the annualized investment costs is linearized using the values of extreme costs. Both transformations are described in detail below.

3.1. Conic transformation of power flow equations

We use a standard second-order cone approximation for the power flow equations [21]. The two auxiliary variables are defined as follows: $u_{kt} = v_{kt}v_{kt}^* = \|v_{kt}\|^2$ and $w_{lt} = v_{kt}v_{mt}^*$. These new variables are substituted in (1) and (2), obtaining affine functions (7) and (8):

$$s_{lt}^s = y_l^* \left(u_{kt} - w_{lt}\right) \tag{7}$$

$$s_{lt}^r = y_l^* \left(u_{mt} - w_{lt}^*\right) \tag{8}$$

Note that u is a real variable, whereas w is complex. These variables are related by the following expressions, which remain non-convex.

$$\begin{aligned} w_{lt} &= v_{kt}v_{mt}^* \\ w_{lt}w_{kmt}^* &= v_{kt}v_{mt}^*v_{mt}v_{kt}^* \\ \|w_{lt}\|^2 &= \|v_{kt}\|^2\,\|v_{mt}\|^2 \\ \|w_{lt}\|^2 &= u_{kt}u_{mt} \end{aligned}$$

This hyperbolic constraint can be represented as follows:

$$\begin{aligned} \|w_{lt}\|^2 &= u_{kt}u_{mt} \\ \|w_{lt}\|^2 &= \frac{1}{4}\left(u_{kt}+u_{mt}\right)^2 - \frac{1}{4}\left(u_{kt}-u_{mt}\right)^2 \\ \left(u_{kt}-u_{mt}\right)^2 + \|2w_{lt}\|^2 &= \left(u_{kt}+u_{mt}\right)^2 \\ \left\| \begin{matrix} 2w_{lt} \\ u_{kt}-u_{mt} \end{matrix} \right\| &= u_{kt}+u_{mt} \end{aligned}$$

This expression is still non-convex but can be approximated to a convex constraint by relaxing the equality in order to define a second-order cone:

$$\left\| \begin{matrix} 2w_{lt} \\ u_{kt}-u_{mt} \end{matrix} \right\| \leq u_{kt}+u_{mt}$$

In this way, the power flow equations approximate the convex constraints. This relaxation is precise under well-defined conditions, as was demonstrated in [22].

3.2. Linear equivalent function for the D-STATCOM costs

The size of D-STATCOMs in power distribution networks is less than 2000 kVAr. Thus, the investment costs may be approximated to a linear function without losing accuracy. A Taylor's expansion around $h_k = 0$ is applied, which produces the following expression:

$$f^{\text{invest}} = T\xi \sum_{k\in N} \gamma h_k$$

A convex function can also be obtained using a linear or quadratic regression applied to the set of data generated after evaluating the size of the D-STATCOMs in a range from 0 kVAr to 2 MVAr from (5). The precision of this approximation is analyzed in the results section.

3.3. Proposed MI-SOCP model

For the sake of completeness, the proposed mixed-integer second-order cone equivalent formulation for the problem of determining the optimal location and size of D-STATCOMs in medium-voltage distribution networks is presented in (9).

$$\begin{aligned} &\min f^{\text{annual}} = f^{\text{loss}} + f^{\text{invest}} \\ &\quad f^{\text{loss}} = \sigma T \sum_{l\in\mathcal{B}}\sum_{t\in\mathcal{T}} \text{real}\left\{\left(s_{lt}^s + s_{lt}^r\right)\right\}\Delta t \\ &\quad f^{\text{invest}} = T\xi \sum_{k\in\mathcal{N}} \gamma h_k \\ &\quad s_{kt}^g - s_{kt}^d + jq_{kt}^c = \sum_{l\in\mathcal{B}} \left(A_{kl}^+ s_{lt}^s + A_{kl}^- s_{lt}^r\right),\ \forall k\in\mathcal{N}, \forall t\in\mathcal{T} \\ &\quad s_{lt}^s = y_l^*\left(u_{kt} - w_{lt}\right),\ \forall l\in\mathcal{B}, \forall t\in\mathcal{T} \\ &\quad s_{lt}^r = y_l^*\left(u_{mt} - w_{kmt}^*\right),\ \forall l\in\mathcal{B}, \forall t\in\mathcal{T} \\ &\quad u_{0t} = \left(v^{\text{nom}}\right)^2,\ \forall t\in\mathcal{T} \\ &\quad \left\| \begin{matrix} 2w_{lt} \\ u_{kt}-u_{mt} \end{matrix} \right\| \leq u_{kt}+u_{mt},\ \forall k\in\mathcal{N}, \forall t\in\mathcal{T} \\ &\quad u^{\min} \leq u_{kt} \leq u^{\max},\ \forall k\in\mathcal{N}, \forall t\in\mathcal{T} \\ &\quad 0 \leq y_k \leq z_k q^{\max},\ \forall k\in\mathcal{N} \\ &\quad -h_k \leq q_{kt}^c \leq h_k,\ \forall k\in\mathcal{N}, \forall t\in\mathcal{T} \\ &\quad \sum_{k\in\mathcal{N}} z_k \leq \eta \\ &\quad z_k \in \{0,1\},\ \forall k\in\mathcal{N} \end{aligned} \tag{9}$$

where $u^{\min} = (v^{\min})^2$ and $u^{\max} = (v^{\max})^2$.

3.4. Multiobjective model

The objective function described in (5) has two objectives that are in conflict; therefore, while one objective is improved, the other objective gets worse. Hence, the utility company must have several options from which they might choose the best solution according to

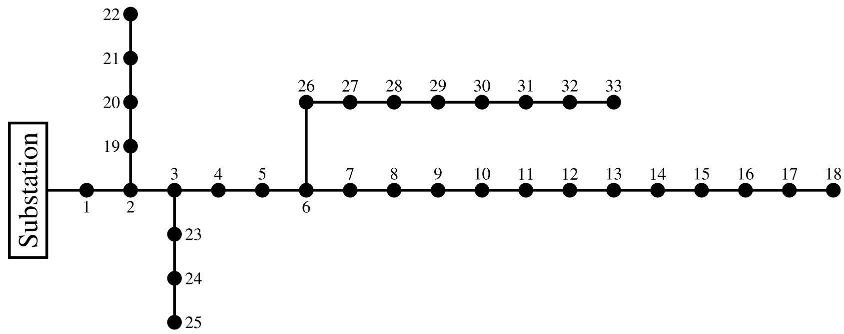

Fig. 2. Single line diagram for the IEEE 33-bus test system.

Table 1
Electrical parameters of the IEEE 33-bus system.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0477	100	60
2	3	0.4930	0.2511	90	40
3	4	0.3660	0.1864	120	80
4	5	0.3811	0.1941	60	30
5	6	0.8190	0.7070	60	20
6	7	0.1872	0.6188	200	100
7	8	1.7114	1.2351	200	100
8	9	1.0300	0.7400	60	20
9	10	1.0400	0.7400	60	20
10	11	0.1966	0.0650	45	30
11	12	0.3744	0.1238	60	35
12	13	1.4680	1.1550	60	35
13	14	0.5416	0.7129	120	80
14	15	0.5910	0.5260	60	10
15	16	0.7463	0.5450	60	20
16	17	1.2890	1.7210	60	20
17	18	0.7320	0.5740	90	40
2	19	0.1640	0.1565	90	40
19	20	1.5042	1.3554	90	40
20	21	0.4095	0.4784	90	40
21	22	0.7089	0.9373	90	40
3	23	0.4512	0.3083	90	50
23	24	0.8980	0.7091	420	200
24	25	0.8960	0.7011	420	200
6	26	0.2030	0.1034	60	25
26	27	0.2842	0.1447	60	25
27	28	1.0590	0.9337	60	20
28	29	0.8042	0.7006	120	70
29	30	0.5075	0.2585	200	600
30	31	0.9744	0.9630	150	70
31	32	0.3105	0.3619	210	100
32	33	0.3410	0.5302	60	40

Table 2
Daily behavior of the active and reactive power demands.

Period	Act. (pu)	React. (pu)	Period	Act. (pu)	React. (pu)
1	0.1700	0.1477	25	0.4700	0.3382
2	0.1400	0.1119	26	0.4700	0.3614
3	0.1100	0.0982	27	0.4500	0.3877
4	0.1100	0.0833	28	0.4200	0.3434
5	0.1100	0.0739	29	0.4300	0.3771
6	0.1000	0.0827	30	0.4500	0.4269
7	0.0900	0.0831	31	0.4500	0.4224
8	0.0900	0.0637	32	0.4500	0.3647
9	0.0900	0.0702	33	0.4500	0.4226
10	0.1000	0.0875	34	0.4500	0.3081
11	0.1100	0.0728	35	0.4500	0.2994
12	0.1300	0.1214	36	0.4500	0.3336
13	0.1400	0.1231	37	0.4300	0.3543
14	0.1700	0.1390	38	0.4200	0.3399
15	0.2000	0.1410	39	0.4600	0.4234
16	0.2500	0.1998	40	0.5000	0.4061
17	0.3100	0.2497	41	0.4900	0.3820
18	0.3400	0.3224	42	0.4700	0.3820
19	0.3600	0.3263	43	0.4500	0.3887
20	0.3900	0.3661	44	0.4200	0.2751
21	0.4200	0.3585	45	0.3800	0.3383
22	0.4300	0.3316	46	0.3400	0.2355
23	0.4500	0.4187	47	0.2900	0.2301
24	0.4600	0.3652	48	0.2500	0.1818

Table 3
Parameters for annual calculation of the investment costs in STATCOMs.

Par.	Value	Unit	Par.	Value	Unit
σ	0.1390	US$kWh	T	365	Days
Δ_h	0.50	h	α	0.30	US$/MVAr^3
β	−305.10	US$/MVAr^2	γ	127380	US$/MVAr
k_1	6/2190	1/Days	k_2	10	Years

its requirements. In order ensure these options, the objective function presented in (5) is rewritten as follows:

$$f^{\text{annual}} = \omega f^{\text{loss}} + (1-\omega) f^{\text{invest}}$$

where $\omega \in [0, 1]$ is the weight factor, which allows varying the weight in each objective function.

4. Test system

Numerical experiments were performed on the IEEE 33-bus test system [23]. It is a radial test feeder composed of 33 buses and 32 lines, where the substation bus is located at node 1, which is operated with a voltage rate of 12.66 kV. The electrical configuration of the test system and its electrical parameters are given in Fig. 2 and Table 1, respectively.

To evaluate the daily performance of the distribution network, we considered the active and reactive power curves presented in [15], which are reproduced in Table 2 for the sake of completeness. Note that these active and reactive power curves must be scaled by 2 in the numerical implementation since their normalizations were made to have a maximum of 0.5.

The parametric information of STATCOMs is reported in Table 3. Some of these data were borrowed from [20] and [24].

5. Computational validation

The numerical implementation of the proposed MI-SOCP model was carried out in MATLAB 2020b with CVX [25] and Gurobi solver [26]. A PC with an AMD Ryzen 7 3700 2.3 GHz processor and 16.0 GB RAM running on a 64-bit version of Microsoft Windows 10 was used. In addition, for comparative purposes, the exact MINLP model was solved through GAMS with BONMIN and COUENNE solvers, and the results were compared with the metaheuristic-based approach reported in [15]

Three simulation scenarios were analyzed:

S1: Comparison of the proposed approach with GAMS and the metaheuristic approach reported in [15], considering that there are three STATCOMs available for installation.

Table 4
Comparison chart between the proposed MI-SOCP and other models reported in the literature.

Method	f^{annual} (US$/year)	f^{loss} (US$/year)	f^{invest} (US$/year)
Ben. case	112740.90	112740.90	0.00
COUENNE	103945.49	94900.15	9045.34
BONMIN	98936.36	90742.84	8193.52
DCVSA	97284.49	89314.22	7970.27
MI-SOCP	96767.69	86875.11	9989.58

Table 5
Optimal location and sizes of the D-STATCOMs for the comparative and proposed methods.

Method	Location	Size (kVAr)	Time (s)
BONMIN	$\{18, 19, 33\}$	$\{171.15,\ 0.00,\ 472.69\}$	83.47
COUENNE	$\{6, 7, 12\}$	$\{296.47,\ 165.64,\ 248.4\}$	1370.509
DCVSA	$\{14, 30, 32\}$	$\{159.90,\ 359.10,\ 107.20\}$	63.86
MI-SOCP	$\{14, 30, 32\}$	$\{196.22,\ 439.80,\ 141.20\}$	190.50

S2: Evaluation of the effect of varying the number of STATCOMs available for installation from 0 to 5 devices on the total annual operative costs of the network.

S3: Conformation of the Pareto front, considering that the costs of the STATCOMs and the annual energy loss costs are objectives in conflict.

5.1. Comparison with existing literature reports (S1)

The scenario **S1** compared the proposed MI-SOCP model solved in CVX+Gurobi to the solution obtained using two different solvers available in GAMS as well as to the discrete version of the vortex search algorithm reported in [15]. Table 4 presents the results for each method separating the components of the objective function.

The proposed MI-SOCP formulation found the best solution (global optimum) of the studied problem with an objective function equivalent to US$ 96775.37 per year. This value was obtained after evaluating the MI-SOCP solution in the exact costs function. Compared to the benchmark case, a reduction of 14.17% was observed. The model in GAMS was confined to a local optimum, where the BONMIN solver provided an annual reduction of about 12.24% in the grid operative costs, while the COUENNE provided a reduction of only 7.80%. The DCVSA algorithm found the same set of nodes to locate the D-STATCOMS (see Table 5), i.e., nodes 14, 30, and 32; however, the sizes of the D-STATCOMs differ, which causes the annual reduction in the operative costs of the network to be stuck at US$ 97284.49, i.e., 13.71% of the benchmark case.

Note that the proposed MI-SOCP allows an improvement of US $509.12 per year when compared with the solution of the DCVSA reported in Table 4.

The main advantage of the proposed approach when compared to the DCVSA is that each evaluation of the MI-SOCP in CVX+Gurobi provided the same solution since it is an exact algorithm. In contrast, the effectiveness of the DC-VSA was reported to be 36%, after 100 consecutive executions, which implies that there exists a 64% possibility of having a sub-optimal solution with quality worse than the solution documented in Table 4.

Regarding processing times, it is essential to mention that the proposed MI-SOCP took about 190.50 s to solve the studied problem, while the DCVSA took about 63.86 s. The main advantage of the MI-SOCP solution is that it has 100% effectiveness in solving the problem, while the DCVSA has only 36% effectiveness.

Fig. 3 shows the internal rate of return (IRR) for a ten-year useful life of the D-STATCOMs considering the optimal solution given by MI-SOCP. The IRR is computed with discount rates (RDR) from 4% to 10%.

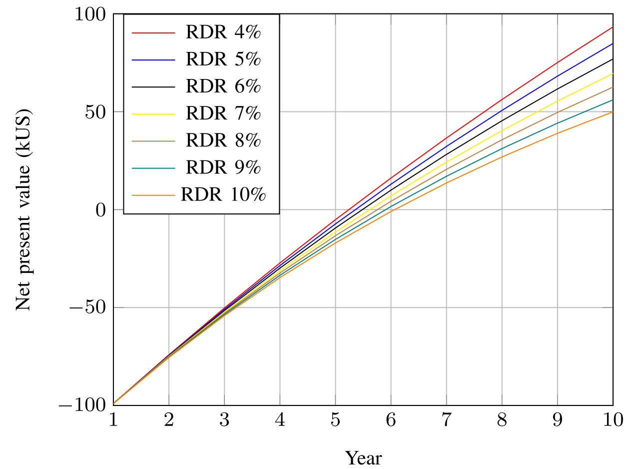

Fig. 3. Internal rate of return analysis for the installation of D-STATCOMs.

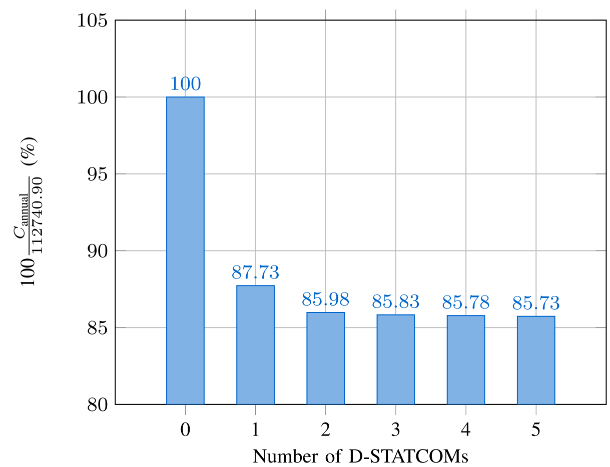

Fig. 4. Percentage of the benchmark case depending on the number of STATCOMs.

For an RDR of 4%, the net present value becomes positive after the sixth year, while for an RDR of 10%, the net present value becomes positive after the seventh year. Observe in Fig. 3 that the net present value in the tenth year is between 49.9 kUS and 93.3 kUS, which implies that the distribution company will have critical positive gains with the implementation of D-STATCOMs in its grids.

5.2. Effect of the number of STATCOMs on the annual operating cost (S2)

In this simulation scenario, i.e., **S2**, we evaluated the effect of the number of STATCOMs in the annual operating costs by varying their availability from 0 to 5 devices. The effect of the number of STATCOMs on the final annual cost of the network is reported in Fig. 4.

This graphic shows that following the location of the third STATCOM, the final objective function presented a saturation, implying that more STATCOMs did not effectively reduce the annual operative costs. Note that the difference between 3 and 5 STATCOMs was about 0.01%, i.e., 11.27 dollars per year of operation, which does not justify installing more than 3 STATCOMs in the IEEE 33-bus system.

5.3. Multiobjective evaluation (S3)

A Pareto set was obtained by applying the objective function shown in (5) and using the proposed MI-SOCP formulation. The weighting factor method was used to obtain a performance index that allowed exploring the space of the objectives, i.e., $\Omega = \left\{f^{\text{loss}}, f^{\text{invest}}\right\}$, where ω

Table 6
Pareto set using the weighting factor methodology.

Factor (ω)	C_{annual} (US$/year)	C_{loss} (US$/year)	C_c (US$/year)
0.0	112740.88	0.00	112740.88
0.1	112740.88	0.00	112740.88
0.2	106775.90	1349.25	108125.15
0.3	94333.68	5366.39	99700.07
0.4	89123.84	8094.14	97217.98
0.5	86875.11	9900.26	96775.37
0.6	85437.72	11717.81	97155.53
0.7	84374.91	13765.81	98140.72
0.8	83786.78	15507.10	99293.88
0.9	83400.95	17891.55	101292.50
1.0	83256.74	76428.00	159684.75

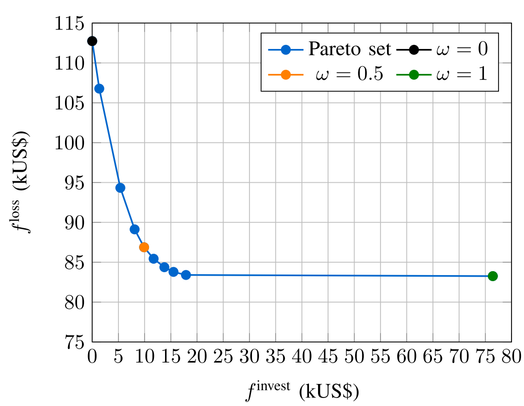

Fig. 5. Pareto set for the multiobjective optimization approach using the weighting factors approach.

is the weight factor that varies from 0 to 1 in the desired step [27]. A step of 0.1 was used, which produced the Pareto set listed in Table 6.

The following conclusions can be drawn from the results of the Pareto front in Table 6: (i) There is, effectively, a multiobjective compromise between the annual energy loss costs and the costs of investment in D-STATCOMs, i.e., the improvement of one of these objectives implies a deterioration of the other one and vice versa. Note the following observations for the two extreme solutions: for $\omega = 0$ the costs of the annual energy losses are US$ 112740.88 per year with zero investments, and for $\omega = 1$ a higher inversion is presented with a value of US$ 76428.00 per year of operation, which produces the minimum cost of energy losses; (ii) There are six solutions with annual operative costs lower than US$ 100 000, which present variations between US$ 94333.68 and US$ 84374.91 in the energy loss costs, and variations between US$ 5366.39 and US$ 13765.81 in the investments costs. These possibilities offered by the solution can be important to the utility company since it depends on its budget. It can choose the best solution by balancing the annual energy losses and the annual inversions in D-STATCOMs; (iii) The best solution provided in Table 4 (MI-SOCP model) corresponds indeed to the minimum costs reported in the multiobjective case (see $\omega = 0.5$), which confirms that a scaling factor in the objective function for the single-objective case does not affect the final result of the convex model. However, the main advantage of having a Pareto front is the set of possibilities afforded to a utility company when selecting the best option based on its investment capabilities.

For illustrative purposes, the Pareto front in Table 6 is depicted in Fig. 5, where the extreme solutions and the central solution are presented.

In Fig. 5, as previously mentioned, the most promissory solutions are contained around the central solution, i.e., $\omega = 0.5$, since these allow for the identification of the best trade-off between both objective functions with total annual operating costs lower than US$ 100.

6. Conclusions

The problem of determining the optimal location and size of D-STATCOMs in power distribution networks was addressed in this research with an MI-SOCP reformulation in the complex domain. This approach allowed a convexification of the MINLP model. Numerical results in the IEEE 33-bus system showed that the proposed MI-SOCP enables a reduction of 14.17% with regard to the benchmark case. At the same time, the best literature report, i.e., the discrete–continuous version of the vortex search algorithm stuck in a local optimum with a reduction of 13.71%, implies that the proposed method improves this solution by US$ 509.12 per year of operation.

Numerical results demonstrated that the convex approximation allows global optimum, while the exact solves in GAMS were getting stuck at a local optimum. Moreover, the proposed linear formulation for the annual operative costs of the D-STATCOMs is effective in convexifying the model since the error between the exact and the approximation function is lower than 0.078%. The optimal solution for the estimated costs in D-STATCOMs was 9900.26. At the same time, the correct value for the exact MINLP model was US$ 9892.58, i.e., an error of US$ 7.68 between the approximated and the exact models.

When the number of the D-STATCOMs was varied from 0 to 5 devices, it was evident that the annual operative cost of the grid presented a saturated behavior with an asymptote on 85.70% of the costs of the benchmark case, where the difference between three and five STATCOMs is lower than US$ 11.27 per year. This confirmed that three STATCOMs is the adequate number of devices for the IEEE 33-bus system.

The multiobjective simulation case demonstrated that the two objective functions are in conflict, which implies that the improvement of one objective deteriorates the other one. However, the weighing-based multiobjective approach found six solutions with annual operative costs lower than US$ 100 000 per year of operation, which correspond to a set of possible implementable options for the grid operator as a function of its investment resources. In addition, it was observed that $\omega = 0.50$ corresponds to the global minimum of the single-objective function case, confirming the efficiency of the proposed MI-SOCP model in ensuring the global optimum at each evaluation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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