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On the Conic Convex Approximation to Locate and Size Fixed-Step Capacitor Banks in Distribution Networks

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Abstract: The problem of the optimal siting and sizing of fixed-step capacitor banks is studied in this research from the standpoint of convex optimization. This problem is formulated through a mixed-integer nonlinear programming (MINLP) model, in which its binary/integer variables are related to the nodes where the capacitors will be installed. Simultaneously, the continuous variables are mainly associated with the power flow solution. The main contribution of this research is the reformulation of the exact MINLP model through a mixed-integer second-order cone programming model (MI-SOCP). This mixed-integer conic model maintains the nonlinearities of the original MINLP model; however, it can be solved efficiently with the branch & bound method combined with the interior point method adapted for conic programming models. The main advantage of the proposed MI-SOCP model is the possibility of finding the global optimum based on the convex nature of the power flow problem for each binary/integer variable combination in the branch & bound search tree. The numerical results in the IEEE 33- and IEEE 69-bus systems demonstrate the effectiveness and robustness of the proposed MI-SOCP model compared to different metaheuristic approaches. The MI-SOCP model finds the final power losses of the IEEE 33- and IEEE 69-bus systems of 138.416 kW and 145.397 kW, which improves the best literature results reached with the flower pollination algorithm, i.e., 139.075 kW, and 145.860 kW, respectively. The simulations are carried out in MATLAB software using its convex optimizer tool known as CVX with the Gurobi solver.

Keywords: capacitor banks; distribution networks; second-order cone programming model; power losses minimization

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1. Introduction

1.1. General Context

Electrical distribution grids are responsible for delivering electricity to all end-users at medium- and low-voltage levels in rural and urban areas [1]. These grids are typically built with radial configuration, as this topology allows reducing investment costs in conductors and grid infrastructure [2]. Additionally, they permit simplifying the protection coordination schemes [3]. Even if radial distribution configurations are economical for distribution companies, these grids present significant energy losses due to the power flow, having only one path between the substation and each final user [4]. In medium- and low voltage grids, energy losses can range from 6% to 18% of the total energy input at the substation bus. In contrast, in transmission systems, owing to their solid meshed configurations, these losses are about 2% [5].

To deal with the problem of power losses in distribution networks, utilities can use multiple alternatives, which include the topology variations and connection of shunt devices [6–8]. In the case of topology variations, the most common approach is adding different distribution lines that reconfigure the grid to obtain a configuration that reduces the power losses for a particular load condition [9]. In the case of the connection of shunt devices, the most common sources are renewable energy sources [10], battery energy storage systems [11], distribution static compensators [12], and fixed-step capacitor banks [13]. However, renewable energies and batteries are expensive elements through which investment costs cannot be recovered with power loss reduction, which implies that other grid functions must be considered to justify their installation along with the distribution network [11]. In the case of reactive power compensation with capacitor banks and distribution static compensators, the first devices are cheaper compared to the latter [14]. Additionally, their maintenance is minimum, and their useful life is greater than 25 years [15], and they have high reliability [16], which makes capacitors the most reliable and efficient strategy to reduce power losses with minimum investment costs [13].

1.2. Motivation

To reduce power losses in distribution networks, the installation of capacitor banks is a well-known solution methodology widely used by utilities for decades [13]. However, in this research, we are interested in addressing this problem with a powerful optimization methodology that will provide distribution companies with a reliable method to define the optimal location and size of the capacitor banks to be installed in their distribution grids. The main advantage of this tool is that it is based on a strong optimization theory named convex optimization that finds the global optimum at each execution, which is impossible with the conventional and largely explored metaheuristic optimizers [17].

Combinatorial methods are the preferred approach to locating and sizing capacitors in distribution networks, owing to the problem of the optimal location and sizing of capacitor banks in distribution networks when the grid power flow equation that is considered generates a mixed-integer nonlinear programming model (MINLP) [18]. The binary (also integer) variables are associated with the places (i.e., nodes) and sizes of the capacitors to be installed. In contrast, the continuous variables are related to active and reactive power flows, current flows, and voltages.

1.3. Revision of the State-of-the-Art

To solve the MINLP model that represents the problem of the optimal placement and sizing of capacitor banks in distribution networks, multiple solution methodologies have been reported in the current literature. The authors of [13] have proposed the application of the flower pollination algorithm to select and locate fixed-step capacitor banks in radial distribution networks. The objective function corresponds to the minimization of the annual grid operative costs (i.e., costs of the energy losses in one year of operation and the investment costs in capacitors). Numerical results of the test feeders composed of 33, 34, 69, and 85 nodes demonstrate the effectiveness of the proposed optimization method compared to the analytical sensitive methods, fuzzy logic algorithms, and classical genetic algorithms, among others. Gil-González et al., in [14] proposed the application of the discrete version of the vortex search algorithm to locate and select fixed-step capacitor banks in radial distribution grids. Numerical results in the IEEE 33- and IEEE 69-bus systems demonstrated the efficiency of the proposed solution method compared to the flower pollination algorithm. The authors of [3] presented the solution of the exact MINLP model of the studied problem in the general algebraic modeling system (i.e., GAMS) software. Additionally, they demonstrated that these results could be improved by applying the classical Chu & Beasley genetic algorithm. The IEEE 33- and IEEE 69-bus systems were used as test feeders with the main contribution that the proposed genetic algorithm can deal with radial and meshed distribution configurations without modifications in the solution methodology. The author of [19] presented a heuristic methodology to locate and size capacitor banks in distribution networks. The author's main contribution is the inclusion of the total harmonic distortion of the grid in the optimization model. Numerical results in the IEEE 34-, IEEE 69-, and IEEE 85-node test feeders demonstrate that total grid costs can be reduced more through the proposed algorithm compared to the classical genetic algorithm. Some additional algorithms used to solve the exact MINLP model are the modified genetic algorithms [20], artificial bee colony optimizer [21], particle swarm optimization [22], tabu search algorithm [23], gravitational search algorithm [24], and cuckoo search algorithm [25], among others.

The main characteristic of the literature mentioned above is that all of them work with a master-slave methodology. In the master stage, it solves the problem of the optimal location and selection of the capacitor banks [3]. In contrast, the slave stage is entrusted with the solution of the equivalent power flow problem [14]. Even if the master-slave methodology base is easily implementable, the usage of metaheuristic approaches poses serious disadvantages: (i) They require many parameters that must be tuned, which are typically adjusted by using the knowledge of the problem under study, thus making metaheuristics highly dependent on the programmer's abilities. (ii) Multiple simulations are required to analyze statistically and determine the average behavior of the method. This implies the impossibility of ensuring the optimal solution can be found at each test [26]. (iii) The computational efforts are typically higher when binary and continuous variables appear, which complicates the multiple simulations needed for statistical tests.

To deal with the disadvantages of metaheuristics, this research, we will contribute with a new methodology to solve the exact MINLP model by ensuring the global optimum finding properties via convex optimization, which is presented below.

1.4. Contribution and Scope

This research proposes a mixed-integer second-order cone programming (MI-SOCP) model to transform the exact MINLP model that represents the problem of the optimal siting and sizing capacitor banks in distribution networks into a mixed-integer convex one. It is worth emphasizing that the nonlinear nature of the problem is maintained due to the conic approximation of the power flow problem; however, the resulting mixed-integer programming is tractable in practice by using a branch & bound (B&B) method [27]. This method is commonly used in mixed-integer programming problems, in which a linear programming problem is solved in each branch iteration. In this case, the method is extended to MI-SOCP, in which a second-order cone optimization problem is solved in each iteration [28].

The main advantage of the proposed MI-SOCP model is that for each node explored in the B&B step, the optimal solution is guaranteed based on the convex properties of the SOCP equivalent power flow model [29]. This entails that the optimal global solution of the MI-SOCP model is guaranteed [17]. The classical IEEE 33- and IEEE 69-node test feeders are utilized to demonstrate the effectiveness and robustness of the proposed MI-SCOP model for locating and sizing capacitor banks in electrical distribution networks. The simulation results reveal that our proposal offers the best possible solutions for these systems.

It is worth mentioning that in this research, radial distribution networks are only considered in the current literature. The problem of the optimal siting and sizing of capacitor banks is typically analyzed in this type of network. However, the proposed MI-SOCP approximation can deal with meshed distribution networks because its formulation is based on the admittance nodal matrix, which contains the information of the grid nodal connections.

1.5. Document Organization

The remainder of this paper is organized as follows: Section 2 presents the exact MINLP formulation of the problem of the optimal location and sizing of capacitor banks in distribution networks. Section 3 presents the MI-SOCP reformulation of the problem via a conic representation of the power balance equations. Section 4 briefly offers the solution approach by describing the main aspects of the MI-SOCP via B&B methods. Section 5 indicates the main characteristics of the IEEE 33- and IEEE 69-node test feeders used during simulations. Section 6 presents the numerical validations as well as their analysis and discussion. Section 7 offers the main concluding remarks derived from this research work.

2. Exact MINLP Formulation

The problem of the optimal location and sizing of capacitor banks in distribution networks is a complex non-linear non-convex optimization problem. This problem combines integer and continuous variables, i.e., the MINLP model, intending to minimize the total grid power losses, subject to the power flow equations and the capacities of the different network elements. In this optimization problem, the binary variables are related to the location of the capacitor banks and the continuous variables are associated with active and reactive power injections and voltage profiles. The exact MINLP model is presented below, where it is considered that the distribution has a radial topology with a unique slack node.

2.1. Objective Function

The objective function of the problem of the optimal location and sizing of capacitor banks in distribution networks is defined in (1) (note that this equation is the complex equivalent of the classical trigonometric function to calculate power losses in distribution lines [30]).

$$\min \; p_{\text{loss}} = \text{real}\left(\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} y_{ij}^{*} v_i v_j^{*}\right), \tag{1}$$

where p_{loss} represents the objective function value regarding the value of power losses; v_i and v_j are complex variables, i.e., phasors, related with voltages in nodes i and j, respectively; y_{ij} is the admittance parameter that relates nodes i and j, respectively; $\mathcal{N}$ is a set that contain all the nodes of the grid.

2.2. Set of Constraints

The set of restrictions listed below corresponds to the power balance equations, voltage regulation bounds, and nominal capabilities in grid devices.

$$\left(\frac{s_i + s_i^{\text{cap}} - d_i}{v_i}\right)^{*} = \sum_{j \in \mathcal{N}} y_{ij} v_j, \; \forall i \in \mathcal{N}, \tag{2}$$

where s_i is the apparent power generation at node i, s_i^{cap} is the apparent power injection by the capacitor bank connected at node i, and d_i is the apparent power demand at node i.

The voltage in the slack node (voltage-controlled source) is assigned to be $v_1 = 1\angle 0^{\circ}$; the rest of the voltages are limited as follows:

$$v^{\text{nom}} - \gamma \leq \|v_k\| \leq v^{\text{nom}} + \gamma, \; \forall\, k \in \mathcal{N}, \tag{3}$$

where γ is the maximum deviation defined by the grid code (typically $\gamma = 0.05$ pu to $\gamma = 0.10$ pu). Note that Equation (3) represents the classical voltage regulation bound constraint rewritten by using a l_2-norm [31].

The power flow in each line is bounded as given in (4).

$$\|s_{ij}\| \leq s_{ij}^{\max}, \; \forall \{i,j\} \in \mathcal{E}, \tag{4}$$

where s_{ij} is the power flow through line ij, and $s_{ij}^{\max}$ is the maximum apparent power flow permitted at this line.

The capacity of existing generators and new capacitor banks is defined as follows:

$$s_i^{\min} \leq s_i \leq s_i^{\max},\ \forall i \in \mathcal{N}, \tag{5}$$

$$s_i^{\text{cap}} = \sum_{k \in \mathcal{C}} z_{ik} s_k^{\text{nom}},\ \forall i \in \mathcal{N}, \tag{6}$$

where z_{ik} is a binary variable that determines whether the capacitor bank type k with capacity s_k^{nom} is located at node i ($z_{ik} = 1$) or not ($z_{ik} = 0$). Note that $\mathcal{C}$ is the set that contains all the capacitor types available for location in the distribution network. The number of capacitor banks available for location can be upper bounded as presented in (7).

$$\sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{C}} z_{ik} \leq n_{\text{cap}}, \tag{7}$$

where n_{cap} is the number of capacitors available for location in the distribution network.

Figure 1 presents the characterization of the optimization model, which represents the problem of the optimal selection and location of capacitor banks in distribution networks for power loss minimization. Note that six of the seven equations are convex, and the only one complication in the model is the power balance constraint defined in (2), which allows the model (1)–(7) to be classified as an MINLP optimization problem.

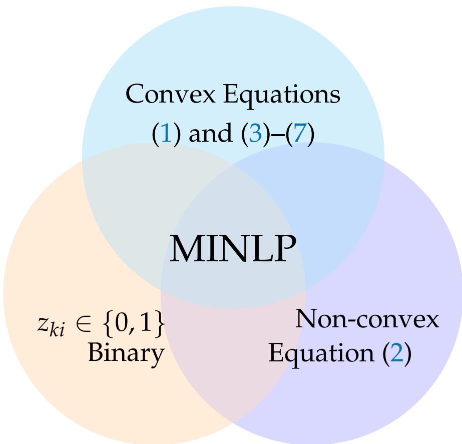

Figure 1. Model characterization.

Note that the solution of the MINLP model presented in Figure 1 via MI-SOCP optimization requires addressing the discrete part of the problem, i.e., the binary nature of the capacitor banks' location problem, via the B&B method. Simultaneously, it is ensured that the continuous problem, i.e., the optimal dimensioning of the capacitor banks, has a second-order cone programming (SOCP) structure. The complete MI-SOCP model is presented in the next section.

3. MI-SOCP Relaxation

The optimization approach based on SOCP corresponds to a sub-area of the convex optimization that works with optimization models composed of linear affine and conic constraints [17]. The conic models can be solved efficiently with tailored algorithms taking a few milliseconds. In general, a cone is a convex set as presented below [32]:

$$\|x\| \leq z \tag{8}$$

where $x \in \mathbb{R}^n$ and $z \in \mathbb{R}$. $\|x\|$ is the l_2-norm of the vector x. Figure 2 shows a second order cone in $\mathbb{R}^3$, clearly a convex set. For further details about the optimization convex, see [33].

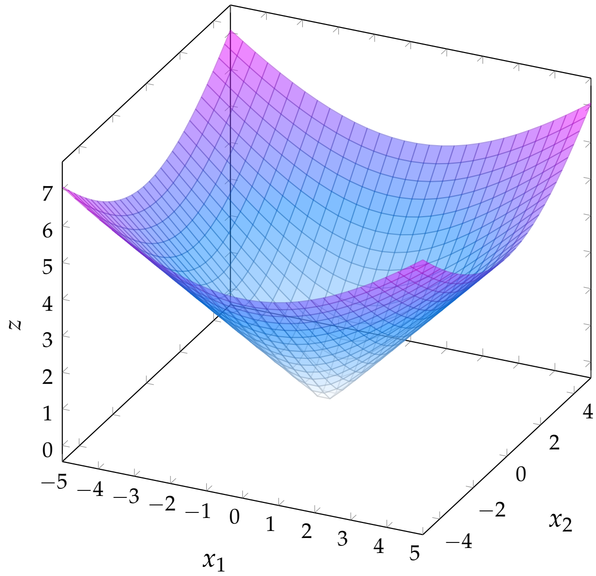

Figure 2. Representation of the second order cone $\Omega = \{\|x\| \leq z\}$. with $x \in \mathbb{R}^2$ and $z \in \mathbb{R}$.

SOCP Approximation for the Power Flow Equations

The SOCP approach transforms the continuous part of the mathematical model (1)–(7) into a convex formulation that ensures the global optimum for each possible combination of binary variables provided by the B&B method. To transform the general MINLP model into an MI-SOCP model, let us define a matrix of complex variables $X = [x_{ij}] \in \mathbb{C}^{n \times n}$, where x_{ij} takes the following form:

$$x_{ij} = v_i^* v_j, \tag{9}$$

where x_{ij} can be split in its real and imaginary parts as $x_{ij} = x_{ij}^{\text{real}} + j x_{ij}^{\text{imag}}$.

Note that in the objective function, if we substitute x_{ij}, then this can be rewritten as follows:

$$\begin{aligned} \min p_{\text{loss}} &= \text{real}\left(\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} y_{ij}^* x_{ij}^*\right), \\ &= \text{real}\left(\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \left(y_{ij}^{\text{real}} - j y_{ij}^{\text{imag}}\right)\left(x_{ij}^{\text{real}} - j x_{ij}^{\text{imag}}\right)\right), \\ &= \left(\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \left(y_{ij}^{\text{real}} x_{ij}^{\text{real}} - y_{ij}^{\text{imag}} x_{ij}^{\text{imag}}\right)\right), \end{aligned} \tag{10}$$

now, the objective function is convex, as it is a linear combination of the real and imaginary parts of the new variable x_{ij}.

Now, if we replace the new variable x_{ij} in the non-convex constraint related with the complex power balance equations, i.e., in Equation (2), then we reach a convex affine relaxation of this as follows:

$$s_i^* + s_i^{\text{cap}*} - d_i^* = \sum_{j \in \mathcal{N}} y_{ij} x_{ij}, \; \forall i \in \mathcal{N}, \tag{11}$$

Nevertheless, the non-convexity remains in Equation (9), which implies that this needs to be relaxed as described below:

$$||x_{ij}||^2 = ||v_i||^2 ||v_j||^2, \tag{12}$$

now, if we define a new real variable ω_i as $||v_i||^2$, then, expression (12) can be rewritten as follows:

$$
\begin{aligned}
||x_{ij}||^2 &= \omega_i \omega_j,\\
||x_{ij}||^2 &= \frac{1}{4}(\omega_i+\omega_j)^2 - \frac{1}{4}(\omega_i-\omega_j)^2,\\
||x_{ij}||^2 + \frac{1}{4}(\omega_i-\omega_j)^2 &= \frac{1}{4}(\omega_i+\omega_j)^2,\\
\left|\left|\begin{matrix} 2x_{ij} \\ \omega_i-\omega_j \end{matrix}\right|\right| &= \omega_i+\omega_j. \qquad (13)
\end{aligned}
$$

Note that Equation (13) is still a non-convex equality constraint; however, as recommended in [29], this can be relaxed as a second-order constraint by replacing the equality symbol with an inequality one as presented below:

$$
\left|\left|\begin{matrix} 2x_{ij} \\ \omega_i-\omega_j \end{matrix}\right|\right| \leq \ \omega_i+\omega_j. \qquad (14)
$$

At this point, the problem of the optimal location and sizing of capacitor banks in an electrical distribution system is convexly treatable via MI-SOCP, whose mathematical characteristics are summarized in Figure 3.

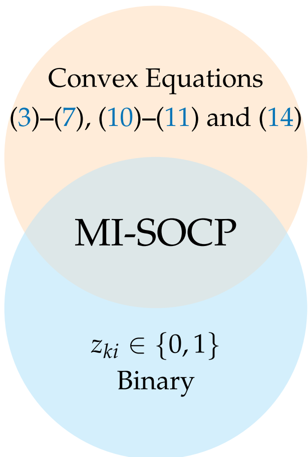

Figure 3. Equivalent MI-SOCP model for the problem of the optimal location and sizing of capacitor banks in distribution networks.

Note that the SOCP approximation is presented as a function of x_{ij} and ω_i instead of the voltages v_i. Notwithstanding, it is possible to recover the original voltages by the following two-step procedure: First, the voltage magnitudes are computed as $v_i = \sqrt{\omega_i}$. This value exists, and it is real since $\omega_i \geq 0$; Second, the angle of the voltages is calculated from $\theta_{ij} = \text{ang}(x_{ij})$ in a forward iteration, starting from $\theta_1 = 0$. Therefore, a power flow calculation is not required after the optimization problem is solved.

4. Solution Methodology

To address the optimal solution of the problem of the optimal location and sizing of capacitor banks in power distribution networks, the classical B&B method is combined with the SOCP relaxation of the power flow problem as depicted in Figure 3. Note that an MI-SOCP problem has the following general structure:

$$
||A_i x + b_i|| \leq \alpha_i^\top x + \beta_i^\top z_i + \gamma_i, \qquad (15)
$$

where decision variables contain continuous x and binary ones z; A_i are real matrices; b_i, α_i and β_i are real vectors; and γ_i are constants for each constraint i.

Observe that most of the integer programming models and the MI-SOCP model may be solved using a modified version of the B&B method, as depicted in Figure 4. At each iteration, this computes an SOCP problem that uses an interior point method specially designed for this type of problem [34]. The method benefits from the properties of the SOCP problems related to convexity and the fast convergence of the interior point methods [17], with a guarantee of finding the optimal global solution at each node.

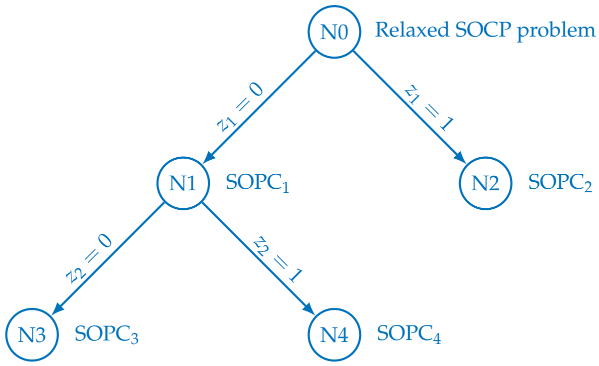

Figure 4. Schematic representation of the B&B method for addressing MI-SOCP problems.

Note that the literature reports different versions of the interior point method for convex optimization problems. Generally, these methods can be classified into two groups: primal and primal-dual methods. Primal methods are those based on the work reported in [35], which can be easily extended to SOCP problems. Primal-dual requires more effort in application. However, numerical validations have demonstrated the advantages of this in terms of convergence [36].

5. Test System and Simulation Cases

To validate the proposed MI-SOCP approach, two classical radial distribution networks composed of 33- and 69-nodes operated at 12.66 kV are employed. The main characteristics of these test feeders are summarized below.

5.1. IEEE 33-Node System

The system consists of 33 nodes, 32 branches, and a unique slack node located at node 1. The feeder configuration is depicted in Figure 5a, and all its parameters are reported in Table 1. Its initial active power losses are equal to 210.9876 kW. This system's total active and reactive power demands are 3715 kW and 2300 kvar, respectively. The voltage and power base values considered are 12.66 kV and 1 MW, respectively.

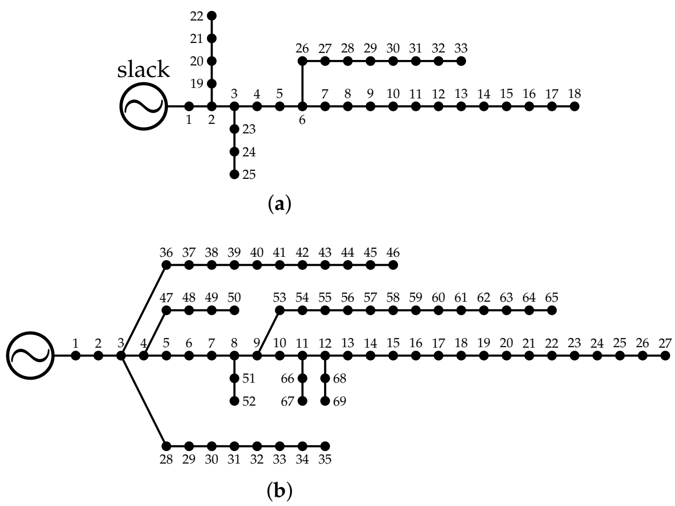

Figure 5. Electrical configuration of the test systems: (**a**) IEEE 33-node system and (**b**) IEEE 69-node system.

Table 1. Electrical parameters of the IEEE 33-node test feeder.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0477	100	60	17	18	0.7320	0.5740	90	40
2	3	0.4930	0.2511	90	40	2	19	0.1640	0.1565	90	40
3	4	0.3660	0.1864	120	80	19	20	1.5042	1.3554	90	40
4	5	0.3811	0.1941	60	30	20	21	0.4095	0.4784	90	40
5	6	0.8190	0.7070	60	20	21	22	0.7089	0.9373	90	40
6	7	0.1872	0.6188	200	100	3	23	0.4512	0.3083	90	50
7	8	1.7114	1.2351	200	100	23	24	0.8980	0.7091	420	200
8	9	1.0300	0.7400	60	20	24	25	0.8960	0.7011	420	200
9	10	1.0400	0.7400	60	20	6	26	0.2030	0.1034	60	25
10	11	0.1966	0.0650	45	30	26	27	0.2842	0.1447	60	25
11	12	0.3744	0.1238	60	35	27	28	1.0590	0.9337	60	20
12	13	1.4680	1.1550	60	35	28	29	0.8042	0.7006	120	70
13	14	0.5416	0.7129	120	80	29	30	0.5075	0.2585	200	600
14	15	0.5910	0.5260	60	10	30	31	0.9744	0.9630	150	70
15	16	0.7463	0.5450	60	20	31	32	0.3105	0.3619	210	100
16	17	1.2860	1.7210	60	20	32	33	0.3410	0.5302	60	40

5.2. 69-Bus System

This test system is composed of 69 nodes, 68 branches, and a unique slack node located at node 1. The configuration of this feeder is illustrated in Figure 5b, and all its parameters are listed in Table 2. Its initial active power losses are equal to 225.0720 kW. This system's total active and reactive power demands are 3890.7 kW and 2693.6 kvar, respectively. The voltage and power base values chosen are 12.66 kV and 1 MW, respectively.

Table 2. Electrical parameters of the IEEE 69-node test feeder.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0005	0.0012	0	0	3	36	0.0044	0.0108	26	18.55
2	3	0.0005	0.0012	0	0	36	37	0.0640	0.1565	26	18.55
3	4	0.0015	0.0036	0	0	37	38	0.1053	0.1230	0	0
4	5	0.0251	0.0294	0	0	38	39	0.0304	0.0355	24	17
5	6	0.3660	0.1864	2.6	2.2	39	40	0.0018	0.0021	24	17
6	7	0.3810	0.1941	40.4	30	40	41	0.7283	0.8509	1.2	1
7	8	0.0922	0.0470	75	54	41	42	0.3100	0.3623	0	0
8	9	0.0493	0.0251	30	22	42	43	0.0410	0.0475	6	4.3
9	10	0.8190	0.2707	28	19	43	44	0.0092	0.0116	0	0
10	11	0.1872	0.0619	145	104	44	45	0.1089	0.1373	39.22	26.3
11	12	0.7114	0.2351	145	104	45	46	0.0009	0.0012	39.22	26.3
12	13	1.0300	0.3400	8	5	4	47	0.0034	0.0084	0	0
13	14	1.0440	0.3450	8	5.5	47	48	0.0851	0.2083	79	56.4
14	15	1.0580	0.3496	0	0	48	49	0.2898	0.7091	384.7	274.5
15	16	0.1966	0.0650	45.5	30	49	50	0.0822	0.2011	384.7	274.5
16	17	0.3744	0.1238	60	35	8	51	0.0928	0.0473	40.5	28.3
17	18	0.0047	0.0016	60	35	51	52	0.3319	0.1114	3.6	2.7
18	19	0.3276	0.1083	0	0	9	53	0.1740	0.0886	4.35	3.5
19	20	0.2106	0.0690	1	0.6	53	54	0.2030	0.1034	26.4	19
20	21	0.3416	0.1129	114	81	54	55	0.2842	0.1447	24	17.2
21	22	0.0140	0.0046	5	3.5	55	56	0.2813	0.1433	0	0
22	23	0.1591	0.0526	0	0	56	57	1.5900	0.5337	0	0
23	24	0.3460	0.1145	28	20	57	58	0.7837	0.2630	0	0
24	25	0.7488	0.2475	0	0	58	59	0.3042	0.1006	100	72
25	26	0.3089	0.1021	14	10	59	60	0.3861	0.1172	0	0
26	27	0.1732	0.0572	14	10	60	61	0.5075	0.2585	1244	888

Table 2. *Cont.*

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
3	28	0.0044	0.0108	26	18.6	61	62	0.0974	0.0496	32	23
28	29	0.0640	0.1565	26	18.6	62	63	0.1450	0.0738	0	0
29	30	0.3978	0.1315	0	0	63	64	0.7105	0.3619	227	162
30	31	0.0702	0.0232	0	0	64	65	1.0410	0.5302	59	42
31	32	0.3510	0.1160	0	0	11	66	0.2012	0.0611	18	13
32	33	0.8390	0.2816	14	10	66	67	0.0047	0.0014	18	13
33	34	1.7080	0.5646	19.5	14	12	68	0.7394	0.2444	28	20
34	35	1.4740	0.4873	6	4	68	69	0.0047	0.0016	28	20

To validate the effectiveness and robustness of the proposed MI-SOCP model, we consider the following scenario: capacitor banks' location considering discrete sizes from 0 kvar to 2100 kvar in steps about 150 kvar, as recommended in [13].

6. Computational Validation

The MI-SOCP convex model was solved using CVX and Gurobi [37]. It was implemented in the MATLAB software version 2019*b* on a desktop computer with an INTEL(R) Core(TM) $i7-7700$ 2.8-GHz processor and 16.0 GB of RAM on a 64-bit version of Microsoft Windows 10 Home. The robustness and efficiency of the proposed methodology for placement and sizing capacitor banks in distribution networks were compared with the previously reported solutions in the scientific literature. It is worth mentioning that the solution MI-SOCP model shown in the results is a solution recovery in original variables using the exact power flow model presented in Section 2.

6.1. Power Loss Analyses of the Proposed MI-SOCP Model

Table 3 presents the results reached by the proposed MI-SOCP approach in the IEEE 33-node test feeder. Note that its result is compared with optimization procedures based on metaheuristics reported in the literature: gravitational search algorithm (GSA) [24], two-stage method (TSM) [38], fuzzy-real coded genetic algorithm (FRCGA) [39], and flower pollination algorithm (FPA) [13].

Table 3. Optimal location of capacitor banks in the IEEE 33-node test feeder.

Method	Nodes	Size [kvar]	Total Losses [kW]
Base case	-	-	210.987
GSA [24]	{9, 29, 30}	{450, 800, 900}	171.780
TSM [38]	{7, 29, 30}	{850, 25, 900}	144.040
FRCGA [39]	{28, 6, 29}	{25, 475, 300}	141.240
	{8, 30, 9}	{175, 400, 350}	
FPA [13]	{30, 13 24}	{900, 450, 450}	139.075
MI-SOCP	{12, 24, 30}	{450, 450, 1050}	138.416

From Table 3, we can observe that (*i*) the proposed approach identifies nodes 12, 24, and 30 as the optimal places to locate capacitor banks with 450 kvar, 450 kvar, and 1050 kvar, respectively. One of these nodes differs from the solution reported by the FRCGA in which node 12 is replaced by node 13, and (*ii*) regarding the objective function, it is possible to observe that the proposed MI-SOCP model reduces grid power losses by about 34.40%, followed by the FRCGA and the TSM with 34.08% and 33.06%. These results confirm that our approach ensures the optimal global solution of the IEEE 33-node test feeder problem by improving on the best results reported in the literature.

In the case of the IEEE 69-node test feeder, the results achieved by the proposed MI-SOCP approach are reported in Table 4. Additionally, the teaching-learning-based optimization (TLBO) [40] is also included for comparison.

Table 4. Optimal location of capacitor banks in the IEEE 69-node test feeder.

Method	Nodes	Size [kvar]	Losses [kW]
Base case	-	-	225.072
GSA [24]	{11, 29, 60}	{900, 1050, 450}	163.280
TSM [38]	{19, 62, 63}	{225, 900, 225}	148.910
TBLO [40]	{12, 61, 64}	{600 1050, 150}	146.350
FPA [13]	{11, 61, 22}	{450, 1350 150}	145.860
MI-SOCP	{11, 18, 61}	{300, 300, 1200}	145.397

Note that from Table 4, it is possible to observe that the proposed MI-SOCP approach is the best optimal solution with a total power loss reduction of about 35.40%, followed by the FPA and the TBLO approaches with 35.19% and 34.98%. Additionally, our MI-SOCP approach identifies different nodes for optimal location of the capacitor banks, namely, nodes 11, 18, and 61 with injections of 300 kvar, 300 kvar, and 1200 kvar, respectively. Conversely, in the case of the FPA approach, the nodes identified are 12, 61, and 22 with injections of 600 kvar, 1350 kvar, and 150 kvar, respectively. These results confirm that the proposed MI-SOCP programming finds the global optimum, while the comparative approaches are stuck in local solutions. Note that a local solution is a result that is optimum in its closer area, and it is found typically by metaheuristics or commercial solvers when the solution space is disjunct and nonlinear, i.e., an MINLP model as the case analyzed in this research.

6.2. Economic Assessment of the Proposed MI-SOCP Model

To demonstrate the effectiveness of the proposed MI-SOCP model in solving the problem of the optimal placement and sizing of capacitor banks in distribution networks, in this section, we economically compare the proposed MI-SOCP model and two recent literature reports presented by Riaño et al., in [3], in which the Chu & Beasley genetic algorithm (CBGA) and the exact solution in the GAMS software were applied to the IEEE 33- and IEEE 69 bus systems. The considered costs for the capacitor banks are listed in Table 5.

Table 5. Capacitor options and costs per capacity.

Option	Q_c (kvar)	Cost ($/kvar-Year)	Option	Q_c (kvar)	Cost ($/kvar-Year)
1	150	0.500	8	1200	0.170
2	300	0.350	9	1350	0.207
3	450	0.253	10	1500	0.201
4	600	0.220	11	1650	0.193
5	750	0.276	12	1800	0.870
6	900	0.183	13	1950	0.211
7	1050	0.228	14	2100	0.176

Note that when the costs of the capacitors are considered, the objective function takes the following form:

$$\min A_{\text{cost}} = K_p p_{\text{loss}} + \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{C}} K_c Q_c z_{ik}, \tag{16}$$

where k_p is a factor that quantifies the costs of the energy losses during a year of operation considering the peak load scenario during 8760 h. The k_p factor based on the recommendations in [3] is taken as US$/kW-year 168.

The numerical results of the CBGA, the GAMS software, and the proposed MI-SOCP model for the IEEE 33- and IEEE 69- bus systems are presented in Table 6,

Table 6. Location, sizes, and annual operative costs for both test feeders when fixed-step capacitors are included.

Method	Size (Node) (Mvar)	Losses (kW)	C. Caps. US$	C. Total US$
		IEEE 33-bus system		
GAMS	{0.30(14),0.45(24),1.05(30)}	139.292	458.25	23,859.313
CBGA	{0.45(12),0.45(24),1.05(30)}	138.416	467.10	23,721.108
MI-SOCP	{0.45(12),0.45(24),1.05(30)}	138.416	467.10	23,721.108
		IEEE 69-bus system		
GAMS	{0.45(11),0.15(27),1.20(61)}	145.738	392.85	24,876.910
CBGA	{0.45(12),0.15(22),1.20(61)}	145.521	392.85	24,840.347
MI-SOCP	{0.45(12),0.15(21),1.20(61)}	145.520	392.85	24.840.189

The numerical results in Table 6 show the following:

- ✓ For both test feeders, the CBGA and the proposed MI-SOCP reach the same numerical solution; however, the effectiveness of the CBGA was 5% in the IEEE 33-bus system and 12% for the IEEE 69-bus system when 100 consecutive evaluations were executed. These results imply that there exists less than 88% probability that in only one execution of the CBGA, it effectively reaches the optimal objective function value, while the MI-SOCP, due to the convexity of the solution space, finds the global optimum without recurring in any statistical evaluation.
- ✓ Numerical results with the GAMS solver confirm that the MINLP model that represents the studied problem is hard to solve owing to the non-convexity of the power flow equations. Note that for both test feeders, the GAMS software is stuck in locally optimal solutions.
- ✓ Regarding the total processing times, the MI-SOCP approach takes about 6.85 s to solve the studied problem in the IEEE 33-bus system and 26.99 s in the case of the IEEE 69-bus system, which can be considered fast processing times owing to the large size of the solution space explored (e.g., one million possible solutions), confirming the effectiveness of the proposed mixed-integer conic approximation to find the global optimal solution of complex MINLP models for distribution grids, which is clearly not ensurable with metaheuristic methods.

7. Conclusions and Future Work

This paper focused on the optimal siting and sizing of fixed-step capacitor banks in electrical AC distribution using an MI-SOCP approach. The proposed approach mixed the B&B method and second-order cone relaxation to find the optimal solution to the problem treated in this work. The B&B method is responsible for the location problem of the capacitor banks. At the same time, the SOCP relaxation focuses on the capacitor banks' sizing problem from the optimal power flow solution. The main advantages of the MI-SOCP approach are guaranteeing the global optimum and not requiring parametric adjustments, such as the metaheuristic optimization methods conventionally implemented to solve this problem. Numerical validations demonstrate the superiority of the MI-SOCP approach, in which the global optimal solution is always reached, for both energy losses and economic evaluation. For the IEEE 33-node test feeder, the MI-SOCP approach noted a total power loss minimization of about 34.40%, which is better than the solution reached by the FRCGA, about 34.08%, while for the IEEE 69-node test feeder, a total power loss minimization percentage of about 35.40% is noted. This is better than the solution report by the TBLO approach, which is about 35.19%. In the case of the economic evaluation, the MI-SOCP approach achieved the best solution for both test feeders.

Comparisons with the CBGA show that it has the ability to find the global optimum for the IEEE 33- and IEEE 69-bus system, but the probability of finding this is about 5% for the IEEE 33-bus system and 12% in the case of the IEEE 69-bus system, which implies at least 88% probability that the CBGA stays stuck in the local optimal solution, which does not happen with the proposed MI-SOCP model due to the combination of the B&B and the conic formulation.

In future research, we can suggest using the MI-SOCP model for the optimal location and operation of battery energy storage systems in alternating current (AC) and direct current (DC) grids under an economic dispatch environment and the reformulation of the phase-balancing problem for electrical networks in AC and DC paradigms from the convex point of view to minimize grid power losses with the guarantee of global convergence.

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