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Overall heat transfer coefficient optimization in a spiral-plate heat exchanger

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Abstract. Heat exchangers are widely used in the industry to allow the heat transfer between two fluids. For that reason, correctly sizing said devices poses a design problem in order to guarantee the efficiency and appropriate conditions of the equipment and the processes. In this paper, the geometry of a spiral-plate heat exchanger is optimized by means of a particle swarm optimization algorithm, whose objective function is the maximization of the overall heat transfer coefficient. The process variables considered in the model were channel spacing, spiral length, spiral width, and wall thickness. The mathematical model and the particle swarm optimization were programmed in Matlab®, where the parameters and the constraints were defined, limiting the pressure drop and guaranteeing the heat transfer required for a study case taken from Minton's work. In this study, the overall heat transfer coefficient was increased by 12.73% in comparison with the original design.

1. Introduction

In general, heat exchangers allow the heat transfer from a fluid to another trough heat transfer mechanisms, known as conduction and convection [1]. Nevertheless, since they are widely used in the industry, the geometry and configuration of heat exchangers depend on the fluid, temperature, space, and other factors. Different configurations and requirements define their form and, therefore, their heat transfer rate. In many design methods, all the physical factors of the heat exchanger need to be considered in order to produce efficient designs from technical and thermal standpoints [2].

The physical phenomena are represented with a set of nonlinear equations that find the right geometry, which requires long spans of iterative computing based on trial-and-error methods [3]. For that reason, optimization techniques have been developed to offer accurate solutions to these design problems faster than conventional methods. Such techniques start with the evaluation of the mathematical model using an objective function, which will be minimized or maximized, and a group of constraints that limit the solution space of the problem and avoid incorrect solutions [4].

This is the case of [5], where the authors use a multi-objective genetic algorithm, known as NSGA-II, to minimize the heat transfer area and pumping power of a tube-and-shell heat exchanger in order to reduce its manufacturing and operating costs. With their methodology, they found a fast and precise response that respects the required pressure drop and heat restrictions. In another work [6], a spiral-plate heat exchanger was studied to minimize its operational and construction costs by means of an algorithm known as wind-driven optimization (WDO). In that case, a 19.3% reduction of the total cost was achieved. In 2010, Patel and Rao, implementing a particle swarm optimization (PSO) algorithm, conducted the same study into a shell-and-tube heat exchanger in terms of sizing and respecting the required pressure drop and heat constraints, reducing the total costs by 5% with a low computational cost [7].

In the petrochemical and food industries, it is common to work with viscous fluids required for specific types of heat exchangers, such as spiral plate, which could operate for a long time without being affected by fouling. This paper examines said heat exchanger type and presents the mathematical model that describes its design with two objective functions: to maximize its overall heat transfer coefficient and to minimize its size meeting two constraints (heat duty and pressure drop). The data were taken from the work by Minton in 1970 [8], and a PSO algorithm was implemented as the solution method.

2. Methodology

In order to obtain a solution for the study case, a PSO algorithm was developed in this work. This section explains the mathematical model that describes the design process of spiral plate heat exchangers, followed by the algorithm used here and the parameters for its implementation, including fluid properties. Figure 1 shows a spiral plate heat exchanger.

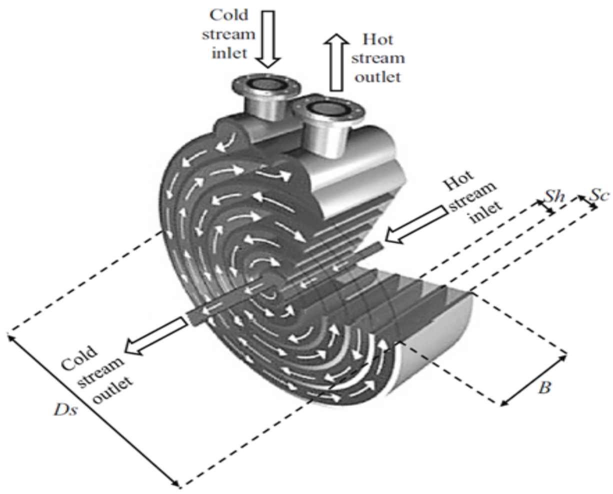

Figure 1. Illustration of a spiral plate heat exchanger taken from [9].

2.1. Mathematical model

The mathematical model developed in this work was based on the equations in [3] and [8] for the design of counter-flow spiral plate heat exchangers. The process starts by computing the heat duty as in Equation (1).

$$Q = \left[\dot{m}C_p(T_o - T_i)\right]_h = \left[\dot{m}C_p(T_i - T_o)\right]_c, \tag{1}$$

where $\dot{m}$ is the mass flux; C_p, the specific heat of each fluid; and T_i and T_o, the inlet and outlet temperatures, respectively. Sub-indices h and c denote hot and cold fluids, respectively. Q can be also expressed by Equation (2).

$$Q = UA\Delta T_{LM}, \tag{2}$$

where U is the overall heat transfer coefficient, which is defined by Equation (3).

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$$U = \left(\frac{1}{h_h} + \frac{t}{k_p} + \frac{1}{h_c} + R_f\right)^{-1}, \quad (3)$$

where h_h and h_c denote the convective heat transfer coefficients of the hot and cold fluids, respectively; k_p, the thermal conductivity of the wall; t, wall thickness; and R_f, the fouling factor, which is added in order to consider how the impurities of the fluid could affect the heat transfer and thermal resistance of the wall. ΔT_{LM} is the logarithmic mean temperature difference, expressed in Equation (4), for a counter-flow configuration.

$$\Delta T_{LM} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln\left(\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}}\right)}. \quad (4)$$

Equation (5) denotes the heat transfer area (A), where L represents the total spiral length and B, the heat exchanger's width.

$$A = 2LB. \quad (5)$$

Nusselt number, hydraulic diameter, and other dimensionless parameters are useful to compute the convective coefficients. The average spiral diameter is expressed in Equation (6), given by the maximum and minimum spiral diameter.

$$R_m = \frac{R_{min} + R_{max}}{2}. \quad (6)$$

The average hydraulic diameter (D_h) is defined in Equation (7), where S is the channel spacing of each channel.

$$D_h = \frac{4(\text{section area})}{(\text{section perimeter})} = \frac{2BS}{B+S}. \quad (7)$$

Reynolds (Re), Prandtl (Pr), and Nusselt (Nu) numbers are dimensionless parameters employed in fluid and heat transfer capacity characterization; they are defined in Equations (8), Equation (9) and Equation (10), respectively.

$$Re = \frac{\rho V L_C}{\mu} = \frac{\dot{m} D_h}{BS\mu}, \quad (8)$$

where ρ is the fluid's density; μ, its dynamic viscosity; L_c, the length that characterizes the cross-section area (which, for this problem, is the hydraulic diameter); and V, the average velocity of the flow through the cross-section area.

$$Pr = \frac{\mu Cp}{k}, \quad (9)$$

where k is the thermal conductivity of the fluid. Equation (10) presents the definition of the experimental correlation proposed by Minton [8] to estimate the Nu in spiral-plate heat exchangers. After the Nusselt number is calculated, the convective heat transfer coefficient is computed in terms of the definition of Nu (Equation (11)).

$$Nu = \frac{hD_h}{k} = 0.0239\left(1 + 5.54\frac{D_h}{R_m}\right) Re^{0.806} Pr^{0.268}, \quad (10)$$

$$h = \frac{kNu}{D_h}. \tag{11}$$

The procedure described above should be applied to both streams, hot and cold. At this point, variables B, S, L, and t (which allow the maximization of U) can be defined. Pressure drop is defined according to the Darcy–Weisbach equation, where f is Darcy's dimensionless friction factor and g is gravitational acceleration. Equation (12) presents the empirical correlation developed in [10]:

$$\Delta P = f\frac{\rho(LV^2)}{2gL_c} = \frac{1.45\rho(LV^2)}{1705}. \tag{12}$$

The outer spiral diameter is determined by Equation (13), where C is the core diameter.

$$D_s = \sqrt{(1.28L(S_h + S_c + 2t) + C^2)}. \tag{13}$$

The set of constraints is given by Equations (14) and Equation (15), which limit the pressure drop based on the value computed in Equation (12) compared to the maximum allowable pressure drop (6.894 kPa) and the required heat duty (186.3 kW) compared to the calculation of Equation (2).

$$\Delta P_{h,c} - \Delta P_{max} = 0, \tag{14}$$

$$Q - UA\Delta T_{LM} = 0. \tag{15}$$

The set of constraints becomes necessary to limit the problem and allow the algorithm to explore the infeasible area, forcing it to reach an adequate solution.

2.2. Particle swarm algorithm
Developed by Eberhart and Kennedy in 1995, PSO algorithms are bio-inspired meta-heuristic methods based on the behavior of fish banks and bird flocks when they explore an area looking for food sources. Each animal is modeled as a particle in order to transform the exploration group into a particle swarm, which is limited by the set of constraints and intervals of the variables associated with each problem [11]. PSO is characterized by the way its particles move over the solution space. At each iteration, both particle information and best particle position are considered, and the weight is controlled by a random component that prevents the algorithm from being trapped in local optima, thus allowing a general and precise exploration of said space [12].

2.2.1. Objective function
Equation (16) presents the objective function as the sum of the value of U found by the algorithm and the penalty associated with the set of constraints (Equation (17)).

$$F = U + Pen, \tag{16}$$

$$Pen = p_1 + p_2, \tag{17}$$

where p_1 is the penalty assigned to pressure drop (Equation (14)), and p_2 expresses the required heat duty (Equation (15)). Such values are determined under the maxima function, which takes the maximum value between 0 (if the constrain is satisfied) and the subtraction shown in Equation (18) and Equation (19).

$$p_1 = \max\{0, \Delta P_{h,c} - \Delta P_{max}\}, \tag{18}$$

$$p_2 = \max\{0, Q - UA\Delta T_{LM}\}. \tag{19}$$

2.2.2. Problem encoding
A 1x4 vector (see Table 1) was implemented in order to represent the group of feasible solutions associated with the solution space. The four columns of said vector contain the values assigned to width (B), length (L), channel spacing ($S_{h,c}$), and thickness (t). It should be emphasized that the variables are limited by the intervals introduced in the section below.

Table 1. Problem encoding created by the authors.

Variable	Width	Length	Channel spacing	Thickness
Name	B	L	$S_{h,c}$	t

2.2.3. Parameters
Table 2 lists the parameters implemented for the application of the algorithm, which defined the solution space and its exploration. In turn, Table 3 details the parameters related to the properties of the stream in the heat exchanger. These values were taken from [9].

Table 2. Parameters of the PSO defined by the authors.

Name	Value
Maximum width	1.5 m
Minimum width	0.5 m
Maximum length	20 m
Minimum length	10 m
Maximum channel spacing	0.032 m
Minimum channel spacing	0.005 m
Maximum wall thickness	0.0079 m
Minimum wall thickness	0.0032 m
Stopping criterion	Convergence
Maximum number of iterations	500
Dimensions of the problem	4
Penalty criterion	1.5

Table 3. Fluid properties [9].

Variable	Nomenclature	Hot stream	Cold stream
Mass flow	m (kg / s)	0.7844	0.7466
Inlet temperature	Ti (K)	473.15	333.15
Outlet temperature	To (K)	393.15	423.55
Density	ρ (kg/m3)	843	843
Specific heat	Cp (J/kg K)	2973	2763
Viscosity	μ (Pa s)	0.00335	0.008
Thermal conductivity	k (W/m K)	0.348	0.322
Fouling resistance	Rf (m2 K / W)	1.0567e-4	1.0567e-4
Thermal conductivity of the surface material	kp (W/m K)	14.53	14.53
Maximum pressure drops	ΔPmax (kPa)	6.894	6.894

3. Results and discussion
This paper presented a PSO algorithm in order to provide an optimal solution to the mathematical model that describes the sizing of a spiral-plate heat exchanger. Such method exhibited quick convergence with a lower computational cost in a time of 0.42 s, thus proving to be a fast and effective tool that offers a solution that meets the set of physical constraints. Table 4 lists the values produced by the PSO and reported by Minton. When the PSO solution was evaluated using the mathematical model, it was found that both constraints, pressure drop and required heat duty, were satisfied. For that reason, such solution was considered good and effective for a low computational cost, representing a more compact heat

exchanger. In Table 4, the value of U given by the PSO algorithm is 12.73% better than that of the original design reported by Minton, producing a higher overall heat transfer coefficient and a size reduction. As a result, the design method proposed in this work is more effective and its construction cost is lower due to its compact size. Furthermore, an analysis of the solution proposed by Minton using our mathematical model revealed that his configuration does not reach the required heat duty, which means it is infeasible.

Table 4. Solution comparison.

Variable	B	L	$S_{h,c}$	t	U
Original design	0.61	12.741	0.0063	0.0032	220
PSO	0.5	13.225	0.005	0.0032	258.084

4. Conclusions

In this article, a particle swarm optimization algorithm was proposed as a solution method for the problem of spiral plate heat exchanger. A single-objective formulation was employed, that considered the maximization of the overall heat transfer coefficient, by using as constraints the heat duty and pressure drop. The results obtained shown that the proposed method offers a quick and efficient way to find a feasible solution that satisfies the physical constraints that represents this type of device, by presenting an improvement of 12.73% of the overall heat transfer coefficient with respect to the conventional design method, with a low computational cost. Finally, it is proven that the optimization techniques represent an efficient tool in the thermal design, reducing the processing times and by considering all the constraints attached to the physics described in the mathematical model. In this way, was possible to provide a reliable and functional solution for the problem of the optimal design of a spiral-plate heat exchanger, which it represents economic savings on the manufacture of this type of devices.

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