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H2 controller design on engine rotor rotation

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Abstract. This article exposes the implementation of the H2 controller, which is rare in the literature because it is a more robust control and not only errors but also disturbances that can affect the dynamics of the control system are considered. The deduction of the equations for the continuous and discrete case is shown, which is little identified in this type of controllers and finally the implementation of said control over the rotation angle of the rotor of a motor is exposed, which is of great utility and importance. in electrical power systems for the generation and service of energy.

1. Introduction

Designing control techniques for a physical model is of great importance in order to have systems that are stable at the plant's entrances and in the event of small disturbances that can be faced while it is in operation, traditionally control techniques address the problem control the inputs and bring them to a predetermined reference, but they do not address the problem in the face of disturbances during their operation, it is for them that other methodologies, such as the optimal H₂ control or the robust H_∞ control, are now mature techniques for which they are efficient mathematical design tools are available, especially in the linear framework [1]. There are also notable applications of these methodologies to practical problems in different fields, ranging from the process industry to applications in the aeronautical industry.

In all of them, the mentioned control techniques have exhibited remarkable results, showing that it is possible to include aspects such as robustness in the face of modeling uncertainties or the robust rejection of disturbances, so important in practical applications [2]. The design of a robust H₂ controller is presented, which is implemented in the prototype plant to control the angle of rotation of a motor, which is essential for the operation of mechanical or electrical systems in a power system. The controller designed is based on linear matrix inequalities (LMI) [3]. Which allow us to introduce an interesting mathematical concept by proposing an order relationship between matrices and to be able to use them in optimization problems such as optimal control in the that you want to find an optimal function and not an optimal point as suggested by the classical calculation.



For this reason, in this document a robust controller is presented using the mathematical tools described above, on the angle of rotation of a motor to provide stability to the electrical power system, which is highly sensitive to load variations, which are very frequent in the day to day.

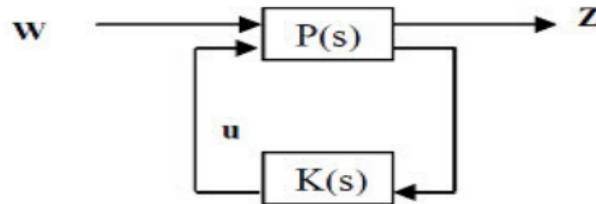


Figure 1. Two port scheme.

Starting from the two-port scheme shown in Figure 1, the model given in the equation shown below is taken into account, which corresponds to the plant model $P(s)$ the equations in state space have the matrices A and B for driver development [4]. The representation using the system transfer function is now as shown in the following Equation (1), Equation (2), and Equation (3).

$$z = p_{zw}w + p_{zu}u, \quad (1)$$

$$y = p_{yw}w + p_{yu}u, \quad (2)$$

$$u = ky. \quad (3)$$

The control system is defined by the equation of states (Equations (4) and Equation (5)) [5].

$$\dot{x} = Ax + Bw + Bu, \quad (4)$$

$$y = Cx + Du. \quad (5)$$

Considering the control signal as shown Equation (6).

$$u = -kx. \quad (6)$$

Substituting the Equation (6) of the control signal in the Equation (4) and Equation (5), gives Equations (7) and Equation (8).

$$\dot{x} = (A - Bk)x + Bw, \quad (7)$$

$$y = (C - Dk)x, \quad (8)$$

where: K is the matrix that will control the plant, (A , B and C) are the matrices of the plant, x are the states, w are the disturbances. The problem is established in terms of matrix inequalities [6], replacing the previous equation in Riccati's algebraic Equation (9) and Equation (10):

$$(A - Bk)P + P(A - Bk)^T + BB^T < 0, \quad (9)$$

$$P > 0. \quad (10)$$

By entering the change of variable $KP = Y$, the equations are reached, which is an LMI in variables P and Y Equation (11) and Equation (12) [7].

$$AP + PA^T - BY - Y^T B^T + BB^T < 0, \quad (11)$$

$$P > 0. \quad (12)$$

The solution to the LMI problem contains difficult-to-solve Riccati equations [8], MATLAB commands are used to solve LMIs and using the matrices obtained for the state space system, the values of K are obtained (Equation 13) [9].

$$K = [-0.6976 \quad -1.2052 \quad 0.6976 \quad 1.2052]. \quad (13)$$

As mentioned above, the development and testing of the controllers is simultaneous, so that to evaluate the operation of the controller in Simulink it is necessary to implement the H_2 controller in discrete [10]. In the case of the H_2 controller, a procedure called digital redesign is done.

2. Discrete linear matrix inequalities controller design

The same matrices A_2 and B_2 described in the continuous part are used, and the transformation is used Bilinear (Tustin's method) to discretize the linear system [11]. The discrete model of the closed loop system in $(K + 1) T$ is given by the system of Equation (14) and Equation (15).

$$X_d(\overline{K + 1T}) = (G - Hk_d)X_d(KT) + HE_dR, \quad (14)$$

$$Y_d(KT) = CX_d(KT), \quad (15)$$

where Equations (16) and Equation (17):

$$G \left(I - \frac{T}{2}A \right)^{-1} = \left(I - \frac{T}{2}A \right), \quad (16)$$

$$H = \frac{T}{2} \left(I - \frac{T}{2}A \right)^{-1} B_2, \quad (17)$$

and the discrete control signal is given by the Equation (18):

$$u_d(KT) = -K_d X_d(KT) + E_d R. \quad (18)$$

The design method to determine the earnings matrix K_d is in accordance with the following theorem: Theorem: If there is a positive definite symmetric matrix G , a matrix F , and a scalar $\alpha > 0$, such that the following generalized problem of own values can be solved. Then the digital control law with $E_d = 0$ satisfies the formulated objective of stabilizing the system if the following LMI is feasible (Equation (19), Equation (20), and Equation (21)) [12].

$$\min_{\Gamma, F} \alpha \text{ s. t.}, \quad (19)$$

$$\begin{bmatrix} -\alpha\Gamma & * \\ G_c\Gamma - G\Gamma + HF & -\alpha I \end{bmatrix} < 0, \quad (20)$$

$$\begin{bmatrix} -\Gamma & * \\ G\Gamma + HF & -I \end{bmatrix} < 0, \quad (21)$$

where $F = K_d G$ and $*$ denotes the transposed element in the symmetrical position, the feedback gain matrix K_d is given by Equation (22).

$$K_d = F\Gamma^{-1}. \quad (22)$$

For the bilinear transformation, the matrix E_d can be obtained by matching the response (output) in stable state of the continuous system, with the response of the discrete system, obtaining Equation (23).

$$E_d = [C_c(I - (G - HK_d))^{-1}H + D_c] + [C_c(I - G_c)^{-1}H_c + D_c]E_c, \quad (23)$$

where E_c is the continuous feedforward gain, and $+$ denotes the pseudo-inverse of Moore - Penrose. From the matrices of Equation (17) to Equation (21) using the MATLAB commands to solve LMIs and the values of K_d are obtained Equation (24).

$$K_d = \begin{bmatrix} -1.5621 & -2.1562 \\ 1.5621 & 2.1566 \end{bmatrix}. \quad (24)$$

To verify the operation of the controller in Simulink, simulations are performed in discrete time, the answer is shown in Figure 2. Figure 2 illustrates the performance of the discrete robust controller H_2 . It can be demonstrated that the chosen control system is capable of rejecting a disturbance. As the control signal is observed for a rotation angle of 45 degrees, it is the expected behavior in which your response tends to establish the system at the same time.

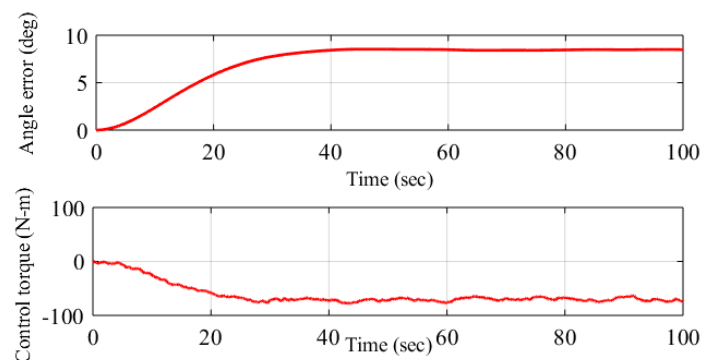


Figure 2. Response of the H_2 controller for motor control.

3. Conclusions

The robust control implemented, is effective in its behavior for the management of the angle of rotation, this has a solid theoretical basis that is characterized by considering uncertainties in the model that are tolerable by the controller. Modern multivariable control theory based on state space models can handle multiple feedback designs, with the added benefit that the design methods derived from it are likely to be computed. The problem of H_2 control is to stabilize the control system while minimizing the H_2 standard of its transfer function. Several solutions to this problem are available. For systems in the form of a state space, an optimal regulator in the form of an observer can be obtained by solving two algebraic Riccati equations.

The suggested control technique based on a robust control is recommended over other classical techniques and more on elements highly sensitive to disturbances such as the electrical power system through the rotor angle. Is to use quality to introduce the system based on linear matrix inequalities since it allows to optimally pose a convex multi-objective problem on which it can be approached without resorting to metaheuristic techniques that relax the restrictions and find low solutions.

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