Applied Mathematics and Computation 229 (2014) 139–150

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items

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ARTICLE INFO

Keywords:
Inventory
Defective
Supply chain

ABSTRACT

The replenishment size/production lot size problem both for perfect and imperfect quality products studied in this paper is motivated by the optimal strategy in a three layer supply chain consisting of multiple suppliers, manufacturers and retailers. In this model, each manufacturer produces each product with a combination of several raw materials which are supplied by each supplier. The defective products at suppliers and manufacturers are sent back to the respective upstream members at lower price than the respective purchasing price. Finally, the expected average profits of suppliers, manufacturers and retailers are formulated by trading off set up costs, purchasing costs, screening costs, production costs, inventory costs and selling prices. The objective of this chain is to compare between the collaborating system and Stakelberg game structure so that the expected average profit of the chain is maximized. In a numerical illustration, the optimal solution of the collaborating system shows a better optimal solution than the approach by Stakelberg.

1. Introduction

The industrial engineers and practitioners have accepted the application of just-in-time (JIT) philosophy in manufacturing systems because it improves product quality and productivity through minimize waste from all operations. They suggested that frequent shipment of purchased raw materials and manufacturing small lots could eliminate waste. Quite often, small lots generate higher productivity through high quality of the products, lower levels of inventory and scrap. The classical economic order quantity/production quantity models are developed for perfect quality items only. In practice, all items are not perfect quality. Inspection/screening process is the method that is used to separate the acceptable/perfect quality of the products from whole lot. At the end of screening process, the imperfect/defective items are sold at a lower price or reworked at cost or returned to the suppliers who are charged the transportation and handling cost or disposal cost. Several researchers have enlightened on this field. Among them, some noteworthy research articles are mentioned as follows. Zhang and Gerchak [31] investigated an EOQ (Economic Order Quantity) model for joint lot sizing and inspection policy with random proportion of defective items. They obtained an optimal order quantity and inspection fraction by considering the case where the defective units could not be used and thus might be replaced by non-defective ones. Liu and Yang [19] developed a single stage production system in which two types of defective items, rework able defects and non-rework able defects, are produced. They found an optimal lot size by maximizing the expected profit over the expected production cycle length. Salameh and Jaber [21] studied an EOQ/EPQ model for imperfect quality products assuming 100% inspection policy. In this

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0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2013.12.006

model, the defective items are sold, as a single batch, to the secondary shop at the end of screening process. Cárdenas-Barrón [1] drew an observation on economic production quantity model both for imperfect and perfect quality products. Goyal and Cárdenas-Barrón [15] found a simple approach to evaluate the EOQ while a random proportion of the whole products were defective. Chan et al. [12] generalized the model of Salameh and Jaber [21], assuming that the defective items could be reworked instantly, could be rejected at a cost or sold at a lower price. Chiu [13] determined an optimal lot size and backlogging for an imperfect production system where random defective items were reworked at a cost. Jamal et al. [16] developed an inventory model dealt with optimum batch size in which rework was done by assuming two different operational policies to minimize the total system costs. The first policy considered that the defective products were reworked within the same production cycle, whereas the defective products were reworked after N-cycle, in the 2nd policy. Papachristos and Konstantaras [20] revisited and extended the papers of Salameh and Jaber [21] and Chan et al. [12] considering the timing of withdrawing the imperfect quality items from stock. Konstantaras et al. [17] proposed a joint lot sizing inventory model for imperfect quality products. In their model, two options of the defective products are proposed: one is to sell them at a price lower than the unit purchasing price, second is to rework them to acceptable quality. Cárdenas-Barrón [2] corrected the solutions of two numerical examples present by Jamal et al. [16]. Cardenas-Barron [4] corrected the solutions of the model of Jamal et al. [16] where main idea and contribution of the pare were not affected. Cardenas-Barron [5] proposed the EPQ (economic production quantity) model with planned backorders for determining the production lot size and backorder size in an imperfect production system while all defective items were reworked in the same cycle. Sarkar [23] investigated an economic manufacturing quantity (EMQ) model for price and advertising sensitive demand in an imperfect production process under the effect of inflation. Konstantaras et al. [18] studied an economic quantity model with shortages for conforming and non-conforming quality products in the light of learning opportunities in logistics and inventory systems. Sarkar and Sarkar [24] proposed an economic manufacturing quantity model for exponential demand with deterioration in a production system over a finite time horizon under the effect of inflation and time value of money.

In recent years, many industries have been involved in various forms of supply chain collaboration in order to survive increasing competition in oligopoly marketing environment. It has been observed that a company can increase its market share by collaborating with other partners of the channel. All steps, from supply raw materials to finished product and end customers, are included into a supply chain. Supply chain coordination ensures better supply chain performance in terms of cost, quantity discount, timely supply, buyback/return policies, quantity flexibility, ordering size and commitment of purchase quantity, etc. Goyal and Gunasekharan [14] discussed a multi-stage production system in order to determine the optimal EPQ (economic production quantity) and EOQ (economic order quantity) for raw materials considering the effect of different marketing policies. Wang and Gerchak [30] developed a collaborating system of two-echelon channel with an initial stock dependent demand. They assumed the case of single manufacturer who offers a product to a retailer at the whole sale price. Zhou et al. [32] investigated the coordination issues of decentralization of two-echelon supply chain, involving stock-dependent demand of the retailer. Sana [22] proposed three layer supply chain involving single supplier, single manufacturer and single retailer who are responsible to satisfy the end customers' demand. Cárdenas-Barrón et al. [10], Cárdenas-Barrón et al. [11] studied an economic manufacturing quantity (EMQ) model with rework and multiple shipments to derive the optimal replenishment lot size and optimal number of shipments jointly. Recently, several researchers [3,6–9,27–29] have enlightened the enormous efforts of supply chain mechanism in order to optimize profit/cost of the supply chain from different perspectives.

In the proposed model, the authors extend a three layer supply chain model for multiple products, consisting of multiple suppliers, multiple manufacturers and multiple retailers as channel members. Each product is manufactured by combination of several raw materials which are delivered by each supplier to each manufacturer. Each manufacturer produces all types of products. Each retailer satisfies the demand of all type of products which are received from each manufacturer at a percent of their requirements. Finally, average profits of all members of the chain are formulated by considering the selling prices, set up costs at different stages, inventory costs, screening costs in order to find out optimal order/production quantity. The centralized and decentralized (Stakelberg approach) are discussed analytically as well as numerically.

The rest of the article is organized as follows. Fundamental assumptions and notations are adopted in Section 2. Section 3 formulates the proposed model. Numerical example is illustrated in Section 4. Section 5 concludes the achievements of the article.

2. Fundamental assumptions and notations

The following assumptions and notations are used to develop the model.

2.1. Assumptions

The following assumptions for suppliers, manufacturers and retailers are considered to analyze the model:

2.1.1. At supplier

s.1: There are multiple suppliers (s).

s.2: The raw material is stored by suppliers.
s.3: Lead time is negligible.
s.4: Suppliers' delivery rates are considered as same as production rates of manufacturers which are assumed as expected value s of their distributions.
s.5: Defective items at each level are returned/bought back at a price to the places from where these were purchased, after completion of whole screening process.
s.6: Setup/installation costs for each member of the chain are different due to different configuration.

2.1.2. At manufacturer

m.1: There are multiple manufacturers and each manufacturer produces every item.
m.2: The manufacturers of the composite products store materials and components and finished products.
m.3: Lead time is negligible.
m.4: Defective items at each level are sold at lower price than the price of good items.
m.5: The unit production cost is a function of the production rate.

2.1.3. At retailer

r.1: There are multiple retailers (r)
r.2: Lead time is neglected.
r.3: Customers' demand rates are assumed to be expected values of the distribution functions.
r.4: All items are good at retailers.
r.5: Setup/installation costs for each member of the chain are different due to different configuration.

2.2. Notation

s – denotes the suppliers, $s \in \{1, 2, 3, \ldots S\}$.
j – denotes the types of items, $j \in \{1, 2, 3, \ldots j\}$.
i – denotes raw material references.
l_j – denotes numbers of raw materials, $l_j \in \{1, 2, 3, \ldots n\}$.
m – denotes the manufacturer, $m \in \{1, 2, 3, \ldots M\}$.
r – denotes the retailers, $r \in \{1, 2, 3, \ldots R\}$.
TS_{jst} – cycle length for the Sth supplier for the ith raw material of jth product.
PS_{jst} – Sth supplier faces the rate of demand from the manufacturer for ith raw material.
α_{jst} – expected percent of defective items at supplier (s) of ith raw material for jth product.
rS_{jst} – screening rate per unit time at supplier s for the ith raw material of jth product.
Cs_{jsi} – screening cost per unit item at supplier s for the ith raw material of jth product.
AS_{jsi} – set up/installation cost of supplier s for the ith raw material of jth product.
hS_{jsi} – holding cost per unit per unit time at supplier s for the ith raw material of jth product.
PC_{jsi} – purchasing cost per unit item of supplier s for the ith raw material of jth product .
WS_{jsi} – selling price per unit good item at supplier s for the ith raw material of jth product.
$\widetilde{WS}_{jst}$ – selling price per unit defective item of supplier (s) for the ith raw material of jth product.
TM_{mj} – cycle length of the mth manufacturer for the jth product.
P_{mj} – production rate of the mth manufacturer for the jth product.
$TPM_{m,j}$ – production run-time of the mth manufacturer for the jth product.
β_{mj} – expected percentage of defective items at manufacturer (m) of jth item.
AM_{mj} – set up/installation cost of manufacturer (m) for the jth product.
hM_{mj} – holding cost per unit per unit time at manufacturer m for the jth product.
L_{mj} – labor/energy cost at mth manufacturer for the jth product.
δ_{mj} – fixed cost per unit finished product at manufacturer m for the jth product.
γ_{mj} – tool/die cost per unit finished product at manufacturer m for the jth product.
$WM_{m,j}$ – selling price per unit good item of manufacturer (m) for the jth product.
$\widetilde{WM}_{mj}$ – selling price per unit defective item of manufacturer (m) for the jth product.
rSM_{mj} – rate of screening of jth product by mth manufacturer.
SM_{mj} – screening cost per unit item at manufacturer (m) for the jth product.
TR_{rj} – cycle length for the rth retailer for the jth product.
TRc_{rj} – time of collecting jth product from manufacturer by the rth retailer.
DR_{rj} – demand rate of retailer r for the jth product.
DC_{rj} – demand rate of the jth product by the customers at the rth retailer.
AR_{rj} – set up/installation cost of retailer (r) for the jth product.
hR_{rj} – holding cost per unit per unit time at retailer (r) for the jth product.

WR_{rj} – selling price per unit good item at retailer (r) of the jth product.
R_{jsi} – replenishment lot size of supplier (s) of the ith raw material of jth product.
$QS_{jsi}(t)$ – on hand inventory at supplier (s) for the ith raw material of jth product.
APS_{jsi} – average profit of supplier (s) for the ith raw material of jth product.
$EAPS$ – expected average profit of supplier s for all raw materials.
$QM_{mj}(t)$ – on hand inventory items at manufacturer (m) for the jth product.
APM_{mj} – average profit of manufacturer (m) for the jth product.
$EAPM$ – expected average profit of manufacturer (m) for the jth product.
$QR_{rj}(t)$ – on hand inventory at retailer (r) for the jth product.
APR_{rj} – average profit of retailer (r) for the jth product.
$EAPR$ – expected average profit of retailer (r) for the jth product.
$EAPC$ – expected average profit of the collaborating system.

3. Formulation of the model

We consider a multi-echelon, multiple products with multiple suppliers, manufacturers and retailers (Fig. 1) who are the members of the supply chain. A percent (α_{jsi}) of the raw materials $i \in \{1, 2, \ldots l_j\}$ of the jth product at supplier $s \in \{1, 2, 3, \ldots S\}$ is perfect (good item) and the rest $(1 - \alpha_{jsi})$ percent of the raw material are imperfect (defective) which are sent back to the places where it was purchased. Each raw material of $j \in \{1, 2, 3, \ldots J\}$ products are supplied by each supplier to each manufacturer at a rate PS_{jsi}. The manufacturer $m \in \{1, 2, 3, \ldots M\}$ produces multiple products at a percentage $(0 \leqslant \varepsilon_{jsi} \leqslant 1)$ combination of materials i.e., the production rate at manufacturer (m) of jth item is

$$PM_{mj} = \sum_{s=1}^{S}\sum_{i=1}^{l_j} \in_{jsi} PS_{jsi}$$

$$\text{such that } \sum_{s=1}^{S}\sum_{i=1}^{l_j} \in_{jsi} = 1\, \forall j \in \{1, 2, 3, \ldots. J\} \text{ and } \in_{jsi} P_{jsi} = \in_{jsk} P_{jsk}\, \forall i \neq k \in l_j. \tag{1}$$

If the demand rate of jth product at the rth retailer $(r \in \{1, 2, 3, \ldots R\})$ is DR_{rj}, then the rate of order size from mth manufacturer is $\theta_{mrj} DR_{rj}$ where $(0 \leqslant \theta_{mrj} \leqslant 1)$ and $\sum_{m=1}^{M} \theta_{mrj} = 1 \forall r \in \{1, 2, 3, \ldots R\}\, \&\, j \in \{1, 2, 3, \ldots J\}$. The expected demand rate of jth product at the retailer (r) in the market is DC_{rj}. Now, we shall formulate the average profit of suppliers, manufacturers and retailers as follows:

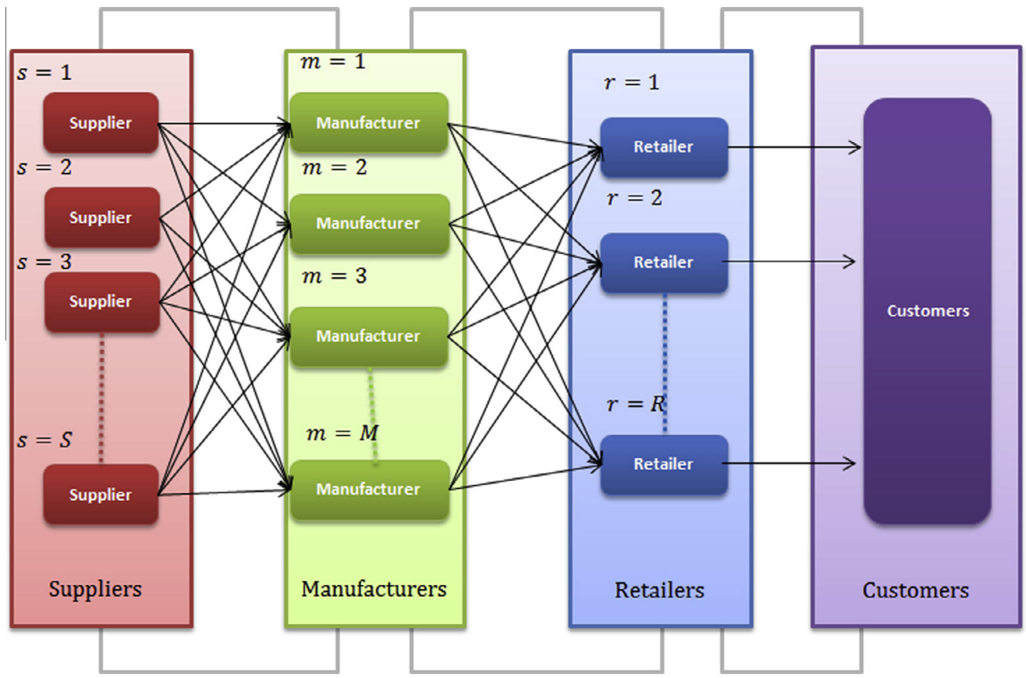

Fig. 1. Diagram of the chain.

3.1. Suppliers' individual average profit

There are S numbered suppliers. Each of them provides various types of materials to each member of the manufacturer. The sth supplier orders quantity R_{jsi} of ith raw material for jth product. Let $QS_{jsi}(t)$ is on hand inventory of good raw materials and TS_{jsi} is cycle length of raw material of jth product at supplier (s). After inspection of the materials, the defective raw materials are sold or sent back to the respective supplier at a price per unit $\tilde{W}S_{jst}$ and good materials are sold by the manufacturer at price WS_{jsi} per unit item. Therefore, the on hand inventory of supplier (s) of ith raw material for jth product is

$$QS_{jsi}(t) = (1-\alpha_{jsi})R_{jsi} - PS_{jsi}t \quad 0 \leqslant t \leqslant TS_{jsi} \, \forall (j,s,i). \tag{2}$$

Using the boundary conditions $QS_{jsi}(TS_{jsi}) = 0$, we have

$$TS_{jsi} = \frac{(1-\alpha_{jsi})R_{jsi}}{PS_{jsi}} \quad \forall (j,s,i) \tag{3}$$

The average inventory cost of good items (ith raw material of jth product at supplier s) is

$$HS_{jsi} = \frac{hS_{jsi}}{TS_{jsi}}\int_0^{TS_{jsi}} QS_{jsi}(t)dt = \frac{hS_{jsi}}{TS_{jsi}}\int_0^{TS_{jsi}} \{(1-\alpha_{jsi})R_{jsi} - P_{jsi}t\}dt = \frac{hS_{jsi}}{2}(1-\alpha_{jsi})R_{jsi}. \tag{4}$$

The average inventory cost of defective raw materials is

$$HDS_{jsi} = \frac{hS_{jsi}}{TS_{jsi}}\alpha_{jsi}R_{jsi}\left(\frac{R_{jsi}}{rS_{jsi}}\right) = hS_{jsi}\left(\frac{\alpha_{jsi}}{1-\alpha_{jsi}}\right)\left(\frac{PS_{jsi}R_{jsi}}{rS_{jsi}}\right). \tag{5}$$

The average screening cost is

$$\frac{Cs_{jsi}}{TS_{jsi}}R_{jsi} = \frac{Cs_{jsi}PS_{jsi}}{1-\alpha_{jsi}}. \tag{6}$$

The average purchasing cost is

$$\frac{PC_{jsi}}{TS_{jsi}}R_{jsi} = \frac{PC_{jsi}PS_{jsi}}{1-\alpha_{jsi}}. \tag{7}$$

The average set up cost is

$$\frac{AS_{jsi}}{TS_{jsi}} = \frac{AS_{jsi}PS_{jsi}}{(1-\alpha_{jsi})R_{jsi}}. \tag{8}$$

The income from selling the good and defective items is

$$\frac{1}{TS_{jsi}}\left[WS_{jsi}(1-\alpha_{jsi})R_{jsi} + \widetilde{W}S_{jsi}\alpha_{jsi}R_{jsi}\right] = WS_{jsi}PS_{jsi} + \widetilde{W}S_{jsi}\left(\frac{\alpha_{jsi}}{1-\alpha_{jsi}}\right)PS_{jsi} \tag{9}$$

Therefore, average profit of supplier (s) from the jth product is

$$APS_{js} = \sum_{i=1}^{l_j} WS_{jsi}PS_{jsi} + \sum_{i=1}^{l_j}\widetilde{W}S_{jsi}\left(\frac{\alpha_{jsi}}{1-\alpha_{jsi}}\right)PS_{jsi} - \sum_{i=1}^{l_j}\frac{PC_{jsi}PS_{jsi}}{1-\alpha_{jsi}} - \sum_{i=1}^{l_j}\frac{hS_{jsi}}{2}(1-\alpha_{jsi})R_{jsi} - \sum_{i=1}^{l_j} hS_{jsi}\left(\frac{\alpha_{jsi}}{1-\alpha_{jsi}}\right)\left(\frac{PS_{jsi}R_{jsi}}{rS_{jsi}}\right) - \sum_{i=1}^{l_j}\frac{Cs_{jsi}PS_{jsi}}{1-\alpha_{jsi}} - \sum_{i=1}^{l_j}\frac{AS_{jsi}PS_{jsi}}{(1-\alpha_{jsi})R_{jsi}}.$$

Using $\in_{jsi}R_{jsi} = \in_{jsk}R_{jsk} \, \forall i \neq k \in l_j$ in the above, we have

$$APS_{js} = \sum_{i=1}^{l_j} WS_{jsi}PS_{jsi} + \sum_{i=1}^{l_j}\tilde{W}S_{jsi}\left(\frac{\alpha_{jsi}}{1-\alpha_{jsi}}\right)PS_{jsi} - \sum_{i=1}^{l_j}\frac{PC_{jsi}PS_{jsi}}{1-\alpha_{jsi}} - \in_{js1}R_{js1}\sum_{i=1}^{l_j}\frac{hS_{jsi}}{2}(1-\alpha_{jsi})\frac{1}{\in_{jsi}} - \in_{js1}R_{js1}\sum_{i=1}^{l_j} hS_{jsi}\left(\frac{\alpha_{jsi}}{1-\alpha_{jsi}}\right)\left(\frac{PS_{jsi}}{rS_{jsi}\in_{jsi}}\right) - \sum_{i=1}^{l_j}\frac{Cs_{jsi}PS_{jsi}}{1-\alpha_{jsi}} - \frac{R_{js1}^{-1}}{\in_{js1}}\sum_{i=1}^{l_j}\frac{AS_{jsi}PS_{jsi}\in_{jsi}}{(1-\alpha_{jsi})}. \tag{10}$$

As a whole, the expected average profit of all suppliers for supplying all raw materials is

$$EAPS = \sum_{j=1}^{J}\sum_{s=1}^{S} APS_{js}. \tag{11}$$

3.2. Manufacturers' individual average profit

In this stage, m manufacturers produce each product separately. The manufacturer (m) produces jth product with a production rate PM_{mj}. Let production-run-time is TPM_{mj} and the inventory cycle length of mth manufacturer for jth product is TM_{mj}. Therefore,

$$TPM_{mj} = \frac{\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}TS_{jsi}}{PM_{mj}} = \frac{\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}(1-\alpha_{jsi})R_{jsi}}{PM_{mj}}\ \forall m,j \text{ with } \in_{jsi}R_{jsi} = \in_{jsk}R_{jsk}\,\forall i \neq k \in l_j.$$

Using $\in_{jsi}R_{jsi} = \in_{jsk}R_{jsk}\,\forall i \neq k \in l_j$ in the above, we have

$$TPM_{mj} = \frac{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{PM_{mj}} \tag{12}$$

The on hand inventory of jth product at mth manufacturer is

$$QM_{mj}(t) = \begin{cases} [(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}]t, & 0 \leqslant t \leqslant TPM_{mj} \\ QM_{mj}(TPM_{mj}) - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}(t-TPM_{mj}), & TPM_{mj} \leqslant t \leqslant TM_{mj} \end{cases} \tag{13}$$

and

$$QM_{m,j}(TPM_{s,i}) = [(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}]TPM_{mj}\forall j. \tag{14}$$

Now $QM_{mj}(TM_{mj}) = 0$ implies

$$\begin{aligned} TM_{mj} &= \frac{(1-\beta_{mj})PM_{mj}TPM_{mj}}{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}} = \frac{(1-\beta_{mj})PM_{mj}}{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{PM_{mj}}\right) \\ &= (1-\beta_{mj})\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}\right) \end{aligned} \tag{15}$$

The average inventory cost of good items of jth product at mth manufacturer is

$$\begin{aligned} HM_{m,j} =& \frac{hM_{mj}}{TM_{mj}}\left[\int_0^{TPM_{mj}}QM_{mj}(t)dt + \int_{TPM_{mj}}^{TM_{mj}}QM_{mj}(t)dt\right] \\ =& \frac{hM_{mj}}{TM_{mj}}\left[\int_0^{TPM_{mj}}\left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]tdt + \int_{TPM_{mj}}^{TM_{mj}}\left\{\left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]TPM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}(t-TPM_{mj})\right\}dt\right] \\ =& \frac{hM_{mj}}{TM_{mj}}\left[\left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]\frac{TPM_{mj}^2}{2} + \left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]TPM_{mj}(TM_{mj}-TPM_{mj}) \right. \\ & \left. -\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{2}(TM_{mj}-TPM_{mj})^2\right] = \frac{hM_{mj}}{2TM_{mj}}\left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]TM_{mj}^2\left[\left\{\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})PM_{mj}}\right\}^2 \right. \\ & \left. + \frac{2\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})PM_{mj}}\left(1-\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})PM_{mj}}\right) - \frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{2}\left(1-\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})PM_{mj}}\right)^2\right] \\ =& \frac{hM_{mj}}{2\{(1-\beta_{mj})PM_{mj}\}^2}\left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]TM_{mj}\left[\left(\sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right)^2 + 2\sum_{r=1}^{R}\theta_{mrj}DR_{rj}\left((1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right)\right. \\ & \left. - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\left((1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right)^2\right] \\ =& \frac{hM_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{2\{(1-\beta_{mj})PM_{mj}\}^2}\left[(1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]TM_{mj}\left[-\sum_{r=1}^{R}\theta_{mrj}DR_{rj} + 2(1-\beta_{mj})PM_{mj} - \left((1-\beta_{mj})PM_{mj} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right)^2\right] \\ =& \frac{hM_{mj}(1-\beta_{mj})\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{2\{(1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}\}^2}\left[(1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]\left[-\sum_{r=1}^{R}\theta_{mrj}DR_{rj} + 2(1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}\right. \\ & \left. - \left((1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi} - \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right)^2\right] \end{aligned} \tag{16}$$

The inventory cost of defective products of jth item at mth manufacturer is

$$\begin{aligned}HDM_{mj} &= \frac{hM_{mj}}{TM_{mj}}\left[\left(\int_0^{TPM_{mj}}\beta_{mj}PM_{mj}(TPM_{mj}-t)dt\right)+\beta_{mj}PM_{mj}TPM_{mj}\left(\frac{PM_{mj}}{r_{mj}}\right)\right]\\ &= \frac{hM_{mj}}{TM_{mj}}\left[\beta_{mj}PM_{mj}\frac{TPM_{mj}^2}{2}+\beta_{mj}PM_{mj}TPM_{mj}\left(\frac{PM_{mj}}{rSM_{mj}}\right)\right]\\ &= \frac{hM_{mj}TPM_{mj}}{TM_{mj}}\left[\beta_{mj}PM_{mj}\left(\frac{TPM_{mj}}{2}\right)+\beta_{mj}PM_{mj}\left(\frac{P_{Mmj}}{rSM_{mj}}\right)\right]\\ &= \frac{hM_{mj}\beta_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}\left[\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{2\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}}\right)+\left(\frac{\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}}{rSM_{mj}}\right)\right]\end{aligned} \quad (17)$$

The average set up cost for products of jth item at mth manufacturer is

$$\frac{AM_{mj}}{TM_{mj}}=\frac{AM_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}. \quad (18)$$

The production cost per unit item of jth product at mth manufacturer is

$$cpM_{mj}=cost\ of\ raw\ materials\ per\ unit+\delta M_{mj}+\frac{L_{mj}}{PM_{mj}TPM_{mj}}+\gamma_{mj}PM_{mj}TPM_{mj} \quad (19)$$

where δM_{mj} is packaging cost per unit of jth product at mth manufacturer, L_{mj} is labor/energy cost per unit of jth product at mth manufacturer which is equally distributed over the production lot size $(PM_{mj}TPM_{mj})$ and γ_{mj} is tool/die costs which is proportional to the production lot size $(PM_{mj}TPM_{mj})$. Therefore, the average production cost of jth product at mth manufacturer is

$$\begin{aligned}\frac{cpM_{mj}}{TM_{mj}}PM_{mj}TPM_{mj} &= \frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}\sum_{i=1}^{l_j}WS_{jsi}(1-\alpha_{jsi})\frac{1}{\in_{jsi}}}{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}+\delta M_{mj}+\frac{L_{mj}}{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}\right.\\ &\quad\left.+\gamma_{mj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})\right)=\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}\sum_{i=1}^{l_j}WS_{jsi}(1-\alpha_{jsi})\frac{1}{\in_{jsi}}}{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}+\delta M_{mj}\right.\\ &\quad\left.+\frac{L_{mj}}{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}+\gamma_{mj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})\right)\end{aligned} \quad (20)$$

The average screening cost of jth product at mth manufacturer is

$$\frac{sM_{mj}}{TM_{mj}}PM_{mj}TPM_{mj}=\frac{sM_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})} \quad (21)$$

The income from selling of good and defective items of jth product from R retailers by mth manufacturer is

$$\begin{aligned}&\frac{1}{TM_{mj}}\left[(1-\beta_{mj})WM_{mj}PM_{mj}TPM_{mj}+\tilde{W}M_{mj}\beta_{mj}PM_{mj}TPM_{mj}\right]=\frac{PM_{mj}TPM_{mj}}{TM_{mj}}\left[(1-\beta_{mj})WM_{mj}+\tilde{W}M_{mj}\beta_{mj}\right]\\ &=\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}\left[(1-\beta_{mj})WM_{mj}+\tilde{W}M_{mj}\beta_{mj}\right]=\sum_{r=1}^{R}\theta_{mrj}DR_{rj}\left[WM_{mj}+\frac{\beta_{mj}\tilde{W}M_{mj}}{(1-\beta_{mj})}\right]\end{aligned} \quad (22)$$

The average profit from jth product by mth manufacturer is

$$\begin{aligned}APM_{mj} &= \sum_{r=1}^{R}\theta_{mrj}DR_{rj}\left[WM_{mj}+\frac{\beta_{mj}\tilde{W}M_{mj}}{(1-\beta_{mj})}\right]-\frac{\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}\sum_{i=1}^{l_j}WS_{jsi}(1-\alpha_{jsi})\frac{1}{\in_{jsi}}}{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}+\delta M_{mj}\right.\\ &\quad\left.+\frac{L_{mj}}{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}+\gamma_{mj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})\right)\\ &\quad-\frac{sM_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}-\frac{hM_{mj}(1-\beta_{mj})\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{2\left\{(1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}\right\}^2}\left[(1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}-\sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right]\\ &\quad\times\left[-\sum_{r=1}^{R}\theta_{mrj}DR_{rj}+2(1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}-\left((1-\beta_{mj})\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}-\sum_{r=1}^{R}\theta_{mrj}DR_{rj}\right)^2\right]\\ &\quad-\frac{hM_{mj}\beta_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})}\left[\left(\frac{\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{2\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}}\right)+\left(\frac{\sum_{s=1}^{S}\sum_{i=1}^{l_j}\in_{jsi}PS_{jsi}}{rSM_{mj}}\right)\right]-\frac{AM_{mj}\sum_{r=1}^{R}\theta_{mrj}DR_{rj}}{(1-\beta_{mj})\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}\end{aligned} \quad (23)$$

Therefore, the expected average profit of J products by M manufacturers is

$$EAPM = \sum_{m=1}^{M}\sum_{j=1}^{J} APM_{mj} \tag{24}$$

3.3. Retailers' individual average profit

The rth retailer receives jth product from manufacturers at a rate DR_{rj} up to time TRc_{rj}. The inventory at rth retailer piles up to time TRc_{rj} after adjusting the demand rate DC_{rj} of the customers of jth product. The inventory level reaches zero at time TR_{rj}. Let $QR_{rj}(t)$ is on hand inventory of jth product of rth retailer, then

$$QR_{rj}(t) = \left\{ \begin{array}{ll} (DR_{rj} - DC_{rj})t, & 0 \leqslant t \leqslant TRC_{rj} \\ (DR_{rj} - DC_{rj})TRC_{rj} - DC_{rj}(t - TRC_{rj}), & TRC_{rj} \leqslant t \leqslant TR_{rj} \end{array} \right\} \tag{25}$$

$$\text{Now, } QR_{rj}(0) = 0 \text{ implies } TR_{rj} = \frac{DR_{rj}TRC_{rj}}{DC_{rj}} \tag{26}$$

Also, we have
$DR_{rj}TRC_{rj} = \sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}PM_{mj}TPM_{mj}$ that provides us

$$TRC_{rj} = \frac{\sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{DR_{rj}} \tag{27}$$

Using the above in Eq. (26), we have

$$TR_{rj} = \frac{\sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}{DC_{rj}}$$

The average holding cost of jth product at rth retailer is

$$\begin{aligned}
HR_{r,j} &= \frac{hR_{rj}}{TR_{rj}}\left[\int_0^{TRC_{rj}}(DR_{rj}-DC_{rj})tdt + \int_{TRC_{rj}}^{TR_{rj}}\{(DR_{rj}-DC_{rj})TRC_{rj} - DC_{rj}(t-TRC_{rj})\}dt\right] \\
&= \frac{hR_{rj}}{TR_{rj}}\left[(DR_{rj}-DC_{rj})\frac{DC_{rj}^2}{2} + (DR_{rj}-DC_{rj})TRC_{rj}(TR_{rj}-TRC_{rj}) - \frac{DC_{rj}}{2}(TR_{rj}-TRC_{rj})^2\right] \\
&= \frac{hR_{rj}TR_{rj}^2}{TR_{rj}}\left[(DR_{rj}-DC_{rj})\frac{DC_{rj}^2}{2DR_{rj}^2} + (DR_{rj}-DC_{rj})\frac{DC_{rj}}{DR_{rj}^2}(DR_{rj}-DC_{rj}) - \frac{DC_{rj}}{2DR_{rj}^2}(DR_{rj}-DC_{rj})^2\right] \\
&= \frac{hR_{rj}TR_{rj}}{2DR_{rj}^2}(DR_{rj}-DC_{rj})DC_{rj}\left[DC_{rj} + 2(DR_{rj}-DC_{rj}) - (DR_{rj}-DC_{rj})\right] \\
&= \frac{hR_{rj}}{2DR_{rj}}(DR_{rj}-DC_{rj})\sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_i}(1-\alpha_{jsi})
\end{aligned} \tag{28}$$

The average set up cost of rth retailer for jth product is

$$\frac{AR_{rj}}{TR_{rj}} = \frac{AR_{rj}DC_{rj}}{\sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})} \tag{29}$$

The average purchasing cost is $WM_{mj}DC_{rj}$ (30)

The average income of rth retailer from selling of jth product is $WR_{rj}DC_{rj}$ (31)

The average profit of rth retailer for jth product is

$$\begin{aligned}
APR_{rj} &= WR_{rj}DC_{rj} - \sum_{m=1}^{M}WM_{mj}\theta_{mrj}DC_{rj} - \frac{hR_{rj}}{2DR_{rj}}(DR_{rj}-DC_{rj})\sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi}) \\
&\quad - \frac{AR_{rj}DC_{rj}}{\sum_{m=1}^{M}(1-\beta_{mj})\theta_{mrj}\sum_{s=1}^{S}\in_{js1}R_{js1}l_j\sum_{i=1}^{l_j}(1-\alpha_{jsi})}
\end{aligned} \tag{32}$$

Now, the expected average profit of R retailers from J products is

$$EAPR = \sum_{r=1}^{R}\sum_{j=1}^{J}APR_{rj} \tag{33}$$

3.4. Stakelberg approach

In Stakelberg approach, a member of the chain is a decision maker who is the controller of the chain and other members are the followers of him. Upstream or downstream members of him follow the optimal strategies taken by him. If manufacturers are the decision makers of the chain, then the suppliers of raw material and retailers are the followers of the manufacturers. In this case optimal solution is obtained by maximizing the expected average profit function $EAPM = \sum_{m=1}^{M}\sum_{j=1}^{J} APM_{mj}$. For optimum values of ordering quantity of raw material R_{js1}, $\frac{\partial EAPM}{\partial R_{js1}} = 0$. If the solution of the system of equations $\left(\frac{\partial EAPM}{\partial R_{js1}} = 0\right)$ are all positive real numbers and the Hessian matrix at the solution $\left(\frac{\partial^2 EAPM}{\partial R_{js1}\partial R_{kp1}}\right)$ is negative definite (i.e., all eigen values are negative), then this solution is the required optimal solution, otherwise this problem may be solved by and random search techniques (Genetic Algorithm).

3.5. Collaborating approach

In collaborating system, all members of the chain have an equal role for taking optimal decision. This system is also called centralized supply chain. In this situation, the expected average profit of the whole system is maximized and the profits of individual members depend on the common optimal strategies.

Therefore, the expected average profit of the whole chain is

$$EAPC = \sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{i=1}^{I_j} APS_{jsi} + \sum_{m=1}^{M}\sum_{j=1}^{J} APM_{mj} + \sum_{r=1}^{R}\sum_{j=1}^{J} APR_{rj} \quad (34)$$

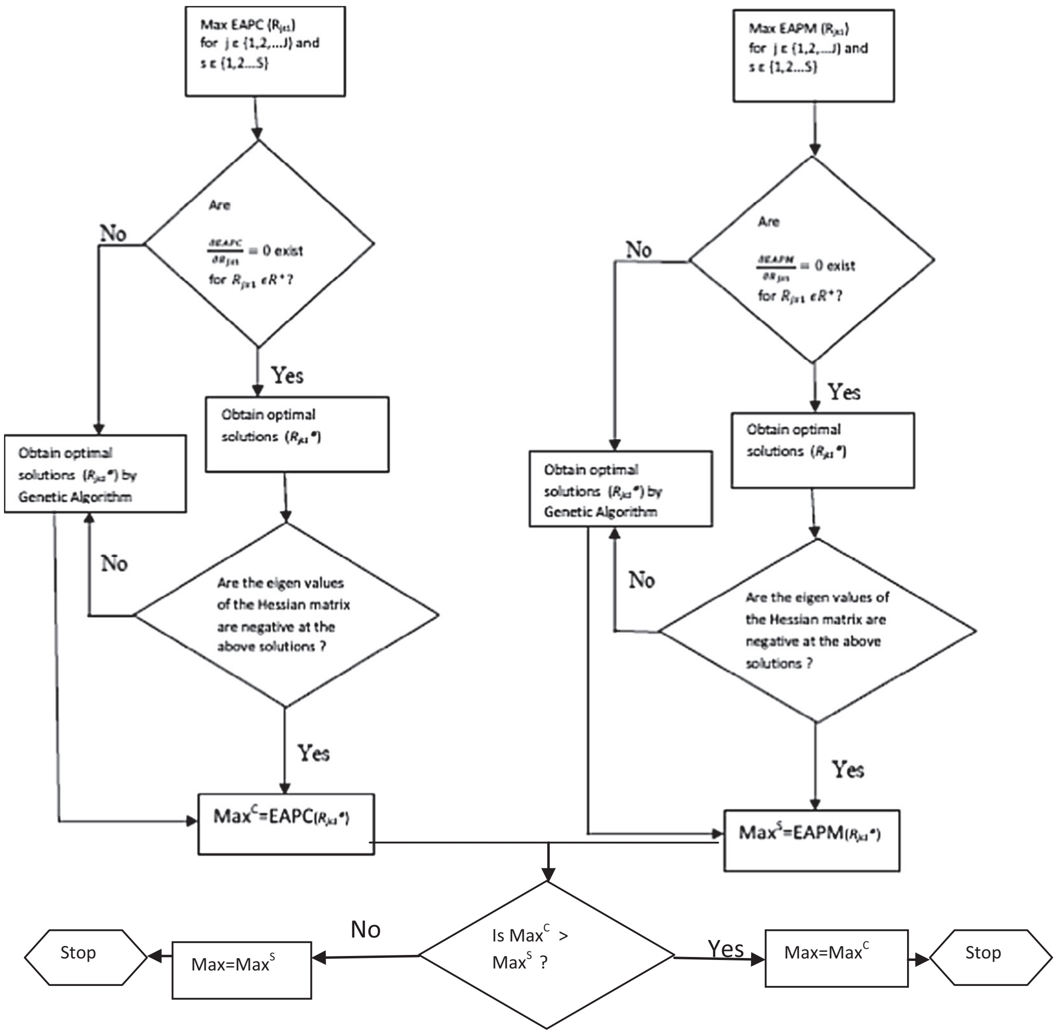

Fig. 2. Flowchart of the solution procedure.

Table 1
Values of parameters of supplier 1 and 2 for product 1 and 2.

Values of parameters	Supplier 1	Supplier 2
Product 1	$\in_{111} = 1/8, \in_{112} = 1/4, PS_{111} = 40$, $PS_{112} = 20$ units, $rS_{111} = 1000$ units, $rS_{112} = 1000$ units, $Cs_{111} = \$0.1$, $Cs_{112} = \$0.1$, $AS_{111} = \$10$, $AS_{112} = \$10$, $hS_{111} = \$0.5, hS_{112} = \$0.6$, $\alpha_{111} = 80\%$, $\alpha_{112} = 75\%$, $PC_{111} = \$1.0$, $PC_{112} = \$1.5$, $WS_{111} = \$5.0$, $WS_{112} = \$6.0$, $\widetilde{W}S_{111} = \$1.0$, $\widetilde{W}S_{112} = \$1.5$	$\in_{121} = 1/8, \in_{122} = 1/2, PS_{121} = 45$ units, $PS_{122} = 11.25$ units, $rS_{121} = 1200$ units, $rS_{122} = 1200$ units, $Cs_{121} = \$0.15$, $Cs_{122} = \$0.2$, $AS_{121} = \$12$, $AS_{122} = \$12$, $hS_{121} = \$0.4, hS_{122} = \$0.3$, $\alpha_{121} = 90\%$, $\alpha_{122} = 95\%, PC_{121} = \$1.2$, $PC_{122} = \$1.5, WS_{121} = \$5.0$, $WS_{122} = \$7.0, \widetilde{W}S_{121} = \$1.2$, $\widetilde{W}S_{122} = \$1.5$
Product 2	$\in_{211} = 1/12 \in_{212} = 1/12 \in_{213} = 1/6$, $PS_{211} = 50$ units, $PS_{212} = 50$units,$PS_{213} = 25$units, $rS_{211} = 800$unitsunits, $rS_{212} = 800$ units, $rS_{213} = 800$ units, $Cs_{211} = \$0.1$, $Cs_{212} = \$0.2$, $Cs_{213} = \$0.2$, $AS_{211} = \$13$, $AS_{212} = \$130$, $AS_{213} = \$13$, $hS_{211} = \$0.2$, $hS_{212} = \$0.2$, $hS_{213} = \$0.25$, $\alpha_{211} = 85\%$, $\alpha_{212} = 80\%$, $\alpha_{213} = 75\%$, $PC_{211} = \$1.5$, $PC_{212} = \$1.0$, $PC_{213} = \$1.0$, $WS_{211} = \$6.0$, $WS_{212} = \$6.5$, $WS_{213} = \$5.5$, $\widetilde{W}S_{211} = \$1.5$, $\widetilde{W}S_{212} = \$2.0$, $\widetilde{W}S_{213} = \$1.0$	$\in_{221} = 1/6$, $\in_{222} = 1/6 \in_{223} = 1/6$, $PS_{221} = 45$ units,$PS_{222} = 45$ units,$PS_{223} = 45$ units, $rS_{221} = 900$ units, $rS_{222} = 900$ units, $rS_{223} = 900$ units, $Cs_{221} = \$0.05$, $Cs_{222} = \$0.05$, $Cs_{223} = \$0.05$, $AS_{221} = \$80$, $AS_{222} = \$80, AS_{223} = \$80$, $hS_{221} = \$0.3$, $hS_{222} = \$0.35$, $hS_{223} = \$0.4$, $\alpha_{221} = 90\%$, $\alpha_{222} = 95\%$, $\alpha_{223} = 85\%$, $PC_{221} = \$1.5$, $PC_{222} = \$1.2$, $PC_{223} = \$1.4, WS_{221} = \$7.5$, $WS_{222} = \$6.0, WS_{223} = \$6.0$, $\widetilde{W}S_{221} = \$1.5$, $\widetilde{W}S_{222} = \$1.2$, $\widetilde{W}S_{223} = \$1.4$

subject to the conditions

$$\sum_{s=1}^{S}\sum_{i=1}^{l_j} \in_{jsi} = 1 \, \forall j \in \{1,2,3,\ldots J\} \text{ and } \in_{jsi}P_{jsi} = \in_{jsk}P_{jsk} \,\&\, \in_{jsi}R_{jsi} = \in_{jsk}R_{jsk} \, \forall i \neq k \in l_j \tag{35}$$

Using the above conditions in the Eq. (34), we have a maximization function $EAPC(R_{jsi})$. For optimum values of ordering quantity of raw material R_{js1}, $\frac{\partial EAPC}{\partial R_{js1}} = 0$. If the solution of the system of equations $\left(\frac{\partial EAPC}{\partial R_{js1}} = 0\right)$ are all positive real numbers and the Hessian matrix at the solution $\left(\frac{\partial^2 EAPC}{\partial R_{js1}\partial R_{kp1}}\right)$ is negative definite (i.e., all eigen values are negative), then this solution is the required optimal solution. The solution procedure of the whole system is shown in the following flowchart (Fig. 2).

4. Numerical example

The following appropriate values of the parameters for two suppliers, two manufacturers and two retailers having two products where 1st product requires two raw materials and 2nd product requires three raw materials are considered for our proposed mode, which are given in Table 1–3l:

Then, the optimal solution of the system by Stakelberg approach is ($R^*_{111} = 232.729$ units, $R^*_{121} = 45.00$ units, $R^*_{211} = 50.00$ units, $R^*_{221} = 536.068$ units, $R^*_{112} = 116.365$ units, $R^*_{122} = 11.25$ units, $R^*_{212} = 50.00$ units, $R^*_{213} = 25.00$ units, $R^*_{222} = 536.068$ units, $R^*_{223} = 536.068$ units, $EAPS^* = \$436.498$, $EAPM^* = \$6972.420$, $EAPR^* = \$3308.54$, $EAPC^* = \$10717.458$) whereas the optimal solution in collaborating system is ($R^*_{111} = 173.824$ units, $R^*_{121} = 257.457$ units, $R^*_{211} = 53.2173$ units, $R^*_{221} = 552.065$ units, $R^*_{112} = 86.912$ units, $R^*_{122} = 64.3643$ units, $R^*_{212} = 53.2173$ units, $R^*_{213} = 26.6087$ units, $R^*_{222} = 552.065$ units, $R^*_{223} = 552.065$ units,$EAPS^* = \$711.80$, $EAPM^* = \$6926.77$, $EAPR^* = \$3339.78$, $EAPC^* = \$10978.35$). Therefore, the optimal solution of the system by collaborating approach is better than the approach by Stakelberg. Hence, our required optimal solution is

($R^*_{111} = 173.824$ units, $R^*_{121} = 257.457$ units, $R^*_{211} = 53.2173$ units, $R^*_{221} = 552.065$ units, $R^*_{112} = 86.912$ units, $R^*_{122} = 64.3643$ units, $R^*_{212} = 53.2173$ units, $R^*_{213} = 26.6087$ units, $R^*_{222} = 552.065$ units, $R^*_{223} = 552.065$ units,$EAPS^* = \$711.80$, $EAPM^* = \$6926.77$, $EAPR^* = \$3339.78$, $EAPC^* = \$10978.35$).

5. Conclusion

In the emerging business paradigm, the concepts of time, volume and capacity become even more crucial to the managerial decision-making. Customers are more sensitive to delivery times and service quality, information collection and comparison of products as far as both prices and quality concern. The joint economic order quantity model plays a crucial role in supply chain management. The collaboration among the members of the chain helps to the members of the chain to make a cost-effective production and distribution as well as better response to the customers' demand. The authors extend a three layer supply chain model consisting of multi-suppliers, multi-manufacturers and multi-retailers as the channel members, in which unit production cost is considered as a function of production rate. The collaborating system and Stakelberg approach of the channel have been compared. The comparison shows that the collaborating system is better than the Stakelberg approach which relates between theory and practice of supply chain management.

In sum, the proposed model provides a normative guidelines on supply chain issue that has been unexplored in the mathematical modeling in supply chain as well as inventory literature. The authors derive the optimal ordering size and production lotsize of multiple items for the chain consisting of multiple suppliers, manufacturers and retailers so that the expected average profit of the chain is maximized. Moreover, the defective items are buyback to the respective upstream members to

Table 2
Values of parameters of Manufacturer 1 and 2 for product 1 and 2.

Values ofparameters	Product 1	Product 2
Manufacturer 1	$\beta_{11} = 90\%$, $AM_{11} = \$15$, $hM_{11} = \$0.02$, $\delta M_{11} = \$0.01$, $LM_{11} = \$150$, $\gamma M_{11} = \$0.002$, $WM_{11} = \$50$, $\widetilde{W}M_{11} = \$30$, $sM_{11} = \$0.05$, $rM_{11} = 100$ units	$\beta_{12} = 85\%$, $AM_{12} = \$20$, $hM_{12} = \$0.03$, $\delta M_{12} = \$0.015$, $LM_{12} = \$200$, $\gamma M_{12} = \$0.001$, $WM_{12} = \$70$, $\widetilde{W}M_{12} = \$35$, $sM_{12} = \$0.04$,$rM_{12} = 120$ units
Manufacturer 2	$\beta_{21} = 95\%$, $AM_{21} = \$18$, $hM_{21} = \$0.04$, $\delta M_{21} = \$0.05$, $LM_{21} = \$250$, $\gamma M_{21} = \$0.001$, $WM_{21} = \$50$, $\widetilde{W}M_{21} = \$30$, $sM_{21} = \$0.07$, $rM_{21} = 125$ units	$\beta_{22} = 80\%$, $AM_{22} = \$19$, $hM_{22} = \$0.02$, $\delta M_{22} = \$0.02$, $sM_{11} = \$0.05$, $rM_{11} = 100$ units., $\widetilde{W}M_{22} = \$35$, $sM_{22} = \$0.06$, $rM_{22} = 150$ units

Table 3
Values of parameters of retailer 1 and 2 for product 1 and 2.

Values of parameters	Product 1	Product 2
Retailer 1	$\theta_{111} = 1/2$, $\theta_{211} = 1/2$, $DR_{11} = 30$ units, $AR_{11} = \$20$, $hR_{11} = \$0.05$, $WR_{11} = \$100$, $DC_{11} = 30$ units	$\theta_{112} = 1/2$, $\theta_{212} = 1/2$, $DR_{12} = 20$ units, $AR_{12} = \$25$, $hR_{12} = \$0.04$, $WR_{12} = \$120$, $DC_{12} = 15$ units
Retailer 2	$\theta_{121} = 1/2$, $\theta_{221} = 1/2$, $DR_{21} = 15$ units, $AR_{21} = \$18$, $hR_{21} = \$0.06$, $WR_{21} = \$100$, $DC_{21} = 10$ units	$\theta_{122} = 1/2$, $\theta_{222} = 1/2$, $DR_{22} = 40$ units, $AR_{22} = \$19$, $hR_{22} = \$0.05$, $WR_{22} = \$130$, $DC_{22} = 30$ units

strengthen coordination among the members of the chain. The authors hope this model helps a company/firm to determine optimal strategies in order to achieve maximum profits of the members of the chain. As far as the authors' knowledge goes, no such type of model has yet been discussed in supply chain literature. As a whole, the proposed model provides insights on the effect of various aspects (i.e., supplier delivery sizes, manufacturing quantities, retailers' ordering sizes) to achieve the maximum expected profit of the chain.

Extensions to this proposed work could focus on many aspects. First, our assumption of constant rates of deliveries of raw materials restricts the applicability of the model in fluctuating (uncertain) nature of the market. This restriction may be relaxed in future extension of the model. In second extension, random production rate may be allowed due to machine breakdown or out of control state of the production system. A third extension of this research could be to examine the optimal strategies considering the effect of trade-credit financing offered by the upstream to the downstream members.

Acknowledgement

The authors would like to express their gratitude to the reviewers for their helpful suggestions and corrections to enhance the clarity of the article.

References

[1] L.E. Cárdenas-Barrón, Observation on: economic production quantity model for items with imperfect quality, Int. J. Prod. Econ. 67 (2000) 201.
[2] L.E. Cárdenas-Barrón, On optimal manufacturing batch size with rework process at single-stage production system, Comput. Ind. Eng. 53 (2007) 196–198.
[3] L.E. Cárdenas-Barrón, Optimizing inventory decisions in a multi-stage multi-customer supply chain: a note, Transp. Res. Part E Log. Transp. Rev. 43 (2007) 647–654.
[4] L.E. Cárdenas-Barrón, Optimal manufacturing batch size with rework in a single-stage production system – a simple derivation, Comput. Ind. Eng. 55 (2008) 758–765.
[5] L.E. Cárdenas-Barrón, Economic production quantity with rework process at a single stage manufacturing system with planned backorders, Comput. Ind. Eng. 57 (2009) 1105–1113.
[6] L.E. Cárdenas-Barrón, H.M. Wee, M.F. Blos, Solving the vendor–buyer integrated inventory system with arithmetic–geometric inequality, Math. Comput. Modell. 53 (2011) 991–997.
[7] L.E. Cárdenas-Barrón, J.T. Teng, G. Treviño-Garza, H.M. Wee, K.R. Lou, An improved algorithm and solution on an integrated production-inventory model in a three-layer supply chain, Int. J. Prod. Econ. 136 (2012) 384–388.
[8] L.E. Cárdenas-Barrón, G. Treviño-Garza, H.M. Wee, A simple and better algorithm to solve the vendor managed inventory control system of multi-product multi-constraint economic order quantity model, Expert Syst. Appl. 39 (2012) 3888–3895.
[9] L.E. Cárdenas-Barrón, A.A. Taleizadeh, G. Treviño-Garza, An improved solution to replenishment lot size problem with discontinuous issuing policy and rework, and the multi-delivery policy into economic production lot size problem with partial rework, Expert Syst. Appl. 39 (2012) 13540–13546.
[10] L.E. Cárdenas-Barrón, B. Sarkar, G. Treviño-Garza, An improved solution to the replenishment policy for the EMQ model with rework and multiple shipments, Appl. Math. Modell. 37 (2013) 5549–5554.
[11] L.E. Cárdenas-Barrón, B. Sarkar, G. Treviño-Garza, Easy and improved algorithms to joint determination of the replenishment lot size and number of shipments for an EPQ model with rework, Math. Comput. Appl. 18 (2013) 132–138.
[12] W.M. Chan, R.N. Ibrahim, P.B. Lochert, A new EPQ model: integrating lower pricing, rework and reject situations, Prod. Planning Control 14 (2003) 588–595.
[13] Y.P. Chiu, Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging, Eng. Opt. 35 (2003) 427–437.
[14] S.K. Goyal, A. Gunasekharan, An integrated production-inventory-marketing model for deteriorating item, Comput. Ind. Eng. 28 (1995) 755–762.
[15] S.K. Goyal, L.E. Cárdenas-Barrón, Note on: economic production quantity model for items with imperfect quality – a practical approach, Int. J. Prod. Econ. 77 (2002) 85–87.
[16] A.A.M. Jamal, B.R. Sarkar, S. Mondal, Optimal manufacturing batch size with rework process at single-stage production system, Comput. Ind. Eng. 47 (2008) 77–89.

[17] I. Konstantaras, S.K. Goyal, S. Papachristos, Economic ordering policy for an ítem with imperfect quality subject to the in-house inspection, Int. J. Syst. Sci. 38 (2007) 473–482.
[18] I. Konstantaras, K. Skouri, M.Y. Jaber, Inventory models for imperfect quality items with shortages and learning in inspection, Appl. Math. Modell. 36 (2012) 5334–5343.
[19] J.J. Liu, P. Yang, Optimal lot-sizing in an imperfect production system with homogeneous reworkable jobs, Eur. J. Oper. Res. 91 (1996) 517–527.
[20] S. Papachristos, I. Konstantaras, Economic ordering quantity models for items with imperfect quality, Int. J. Prod. Econ. 100 (2006) 148–154.
[21] M.K. Salameh, M.Y. Jaber, Economic production quantity model for items with imperfect quality, Int. J. Prod. Econ. 64 (2000) 59–64.
[22] S.S. Sana, A production-inventory model of imperfect quality products in a three-layer supply chain, Decis. Support Syst. 50 (2011) 539–547.
[23] B. Sarkar, An inventory model with reliability in an imperfect production process, Appl. Math. Comput. 218 (2012) 4881–4891.
[24] B. Sarkar, M. Sarkar, An economic manufacturing quantity model with probabilistic deterioration in a production system, Econ. Modell. 31 (2013) 245–252.
[25] J.T. Teng, On the economic order quantity under conditions of permissible delay in payments, J. Oper. Res. Soc. 53 (2002) 915–918.
[26] J.T. Teng, L.E. Cárdenas-Barrón, K.R. Lou, The economic lot size of the integrated vendor–buyer inventory system derived without derivatives: a simple derivation, Appl. Math. Comput. 217 (2011) 5972–5977.
[27] J.T. Teng, L.E. Cárdenas-Barrón, K.R. Lou, H.M. Wee, Optimal economic order quantity for buyer-distributor-vendor supply chain with backlogging derived without derivatives, Int. J. Syst. Sci. 44 (2013) 986–994.
[28] Y. Wang, Y. Gerchak, Supply chain coordination when demand is self-space dependent, Manuf. Serv. Oper. Manage. 3 (2001) 82–87.
[29] X. Zhang, Y. Gerchak, Joint lot sizing and inspection policy in an EOQ model with random yield, IIE Trans. 22 (1990) 41–47.
[30] Y.W. Zhou, J. Min, S.K. Goyal, Supply-chain coordination under an inventory-level-dependent demand rate, Int. J. Prod. Econ. 113 (2008) 518–527.