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MARKET VALUE CALCULATION AND THE SOLUTION OF CIRCULARITY BETWEEN VALUE AND THE WEIGHTED AVERAGE COST OF CAPITAL WACC

CÁLCULO DO VALOR DE MERCADO E A SOLUÇÃO DA CIRCULARIDADE ENTRE VALOR E CUSTO MÉDIO PONDERADO DE CAPITAL CMEPC

IGNACIO VÉLEZ-PAREJA
Mestre em Engenharia Industrial pelo Department of Industrial Engineering, Missouri University, Estados Unidos.
Professor associado de Finanças no Programa de Finanças e Negócios Internacionais, Universidad Tecnológica de Bolívar, Colômbia.
Calle del Bouquet Cra.21, 25-92, Manga – Cartagena de Indias – Colombia
E-mail: *ivelez@unitecnologica.edu.co*

JOSEPH THAM
Doutor em Educação pela Harvard Graduate School of Education, Harvard University, Estados Unidos.
Professor assistente visitante no Duke Center for International Development, Duke University, Estados Unidos.
Duke Center for International Development – Sanford Institute of Public Policy – Box 90237
Durham – NC 27708-0237 – USA
E-mail: *thamjx@duke.edu*

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ABSTRACT

Most finance textbooks present the Weighted Average Cost of Capital (WACC) calculation as: WACC = Kd×(1-T)×D% + Ke×E%, where Kd is the cost of debt before taxes, T is the tax rate, D% is the percentage of debt on total value, Ke is the cost of equity and E% is the percentage of equity on total value. All of them precise (but not with enough emphasis) that the values to calculate D% y E% are market values. Although they devote special space and thought to calculate Kd and Ke, little effort is made to the correct calculation of market values. This means that there are several points that are not sufficiently dealt with: Market values, location in time, occurrence of tax payments, WACC changes in time and the circularity in calculating WACC. The purpose of this note is to clear up these ideas, solve the circularity problem and emphasize in some ideas that usually are looked over. Also, some suggestions are presented on how to calculate, or estimate, the equity cost of capital.

KEYWORDS

Weighted average cost of capital; Firm valuation; Capital budgeting; Equity cost of capital; Market value.

RESUMO

A maioria dos livros apresenta o cálculo do custo médio ponderado de capital como: CMePC = Kd×(1-T)×D% + Ke×E%, em que Kd é o custo de capital de terceiros antes dos tributos; T, alíquota de imposto; D%, o porcentual dos empréstimos em relação ao ativo total; Ke, o custo do capital próprio; e E%, o porcentual do patrimônio líquido sobre o ativo total. Todos eles necessitam (mas não com suficiente ênfase) que os valores para calcular D% e E% sejam valores de mercado. Muito embora se dedique atenção especial para calcular Kd e Ke, pouco esforço é destinado ao cálculo correto de seus valores de mercado. Isso significa que há inúmeros pontos que não são suficientemente esclarecidos: valores de mercado, localização no tempo, ocorrência do pagamento de tributos, a variação do CMePC ao longo do tempo e circularidade no cálculo do CMePC. O objetivo deste artigo é esclarecer esses conceitos, resolver o problema da circularidade e discutir alguns ideias geralmente ainda obscuras. Igualmente, algumas sugestões são apresentadas para o cálculo ou estimação do custo do capital próprio.

PALAVRAS-CHAVE

Custo médio ponderado de capital; Avaliação da empresa; Orçamento de capital; Custo do capital próprio; Valor de mercado.

1 INTRODUCTION

Most finance textbooks (BENNINGA; SARIG, 1997; BREALEY; MYERS; MARCUS, 2004; COPELAND; KOLLER; MURRIN, 1995; DAMODARAN, 1996; GALLAGHER; ANDREW JR., 2000; VAN HORNE, 1998; WESTON; COPELAND, 1992) present the Weighted Average Cost of Capital (WACC) calculation as:

$$WACC = Kd \times (1\text{-}T) \times D\% + Ke \times E\% \qquad (1)^{1}$$

Where Kd is the cost of debt before taxes, T is the tax rate, D% is the percentage of debt on total value, Ke is the cost of equity and E% is the percentage of equity on total value. All of them precise (but not with enough emphasis) that the values to calculate D% y E% are market values. Although they devote special space and thought to calculate Kd and Ke, little effort is made to the correct calculation of market values. This means that there are several points that are not sufficiently dealt with:

1. Market values are calculated period by period and they are the present value at WACC of the future cash flows.
2. These values to calculate D% and E% are located at the beginning of period t, where the WACC belongs. From here on, the right notation will be used.
3. Kd × (1-T), the after tax cost of debt, implies that the tax payments coincides in time with the tax accrual. (Some firms could present this payment behavior, but it is not the rule. Only those that are subject to tax withheld from their customers, pay taxes as soon as they invoice their goods or services.)

[1] This formula is derived in Appendix A.

4. Because of 1, 2 and the existence of changing macroeconomic environment (say, inflation rates), WACC changes from period to period.
5. That there exists circularity when calculating WACC. In order to know the firm value it is necessary to know the WACC, but to calculate WACC, the firm value and the financing profile are needed.
6. That we obtain full advantage of the tax savings in the same year as taxes are paid. This means that earnings before interest and taxes (Ebit) are greater than or equal to the interest charges.
7. There are no losses carried forward.
8. That (1) implies a definition for Ke, the cost of equity, in most cases they use;

$$Ke_t = Ku_t + (Ku_t - Kd) \times (1\text{-}T) \times D\%_{t-1}/E\%_{t-1} \qquad (2)$$

This formula is derived in Appendix B. This is the typical formulation of Ke, but it has to be said, it only applies to perpetuities and not to finite periods.

In this expression, Ke_t is the levered cost of equity, Ku_t is the cost of unlevered equity, Kd is the cost of debt, T is the tax rate, $D\%_{t-1}$ is the proportion of debt on the total market value for the firm, at t-1 and $E\%_{t-1}$ is the proportion of equity on the total market value for the firm, at t-1. It can be shown that equation 2 results from the assumption that the discount rate for the tax savings. In this case that rate is Kd and expression 2 is valid only for perpetuities. When working with n finite it can be shown that the expression for Ke changes for every period (THAM; VÉLEZ-PAREJA, 2001c). The assumption behind Kd as the discount rate is that the tax savings are a non-risky cash flow.

9. The only source of tax shields is the interest expenses.

The purpose of this work is to clear up these ideas, solve the circularity problem and emphasize in some ideas that usually are looked over.

2 THE MODIGLIANI-MILLER PROPOSAL

The basic idea is that under a scenario of no taxes, the firm value does not depend on how the stakeholders finance it. This is the stockholders (equity) and creditors (liabilities to banks, bondholders, etc.) The reader should examine this idea in an intuitive manner and she will find it is reasonable. Because of this idea, Franco Modigliani and Merton Miller (MM from here on) were awarded the Nobel Prize in Economics. They proposed that with perfect market conditions, (perfect and complete information, no taxes, etc.) the capital structure does not affect the value of the firm because the equity holder can borrow and lend and thus determine the optimal amount of leverage. The capital structure of the firm is the combination of debt and equity in it.

That is, V^L the value of the levered firm is equal to V^{UL} the value of the unlevered firm.

$$V^L = V^{UL} \tag{3}$$

And in turn, the value of the levered firm is equal to V^{Equity} the value of the equity plus V^{Debt} the value of the debt.

$$V^L = V^{Equity} + V^{Debt} \tag{4}$$

What does it imply regarding the Weighted Average Cost of Capital (WACC)? Simple. If the firm has a given cash flow, the present value of it at WACC (the firm total value) does not change if the capital structure changes. If this is true, it implies that the WACC will remain constant no matter how the capital structure changes. This situation happens when no taxes exist. To maintain the equality of the unlevered and levered firms, the return to the equity holder (levered) must change with the amount of leverage (assuming that the cost of debt is constant)

One of the major market imperfections are taxes. When corporate taxes exist (and no personal taxes), the situation posited by MM is different. They proposed that when taxes exist the total value of the firm does change. This occurs because no matter how well managed is the firm, if it pays taxes, there exists what economists call an externality. When the firm deducts any expense, the government pays a subsidy for the expense. It is reflected in less tax. In particular, this is true for interest payments. The value of the subsidy (the tax saving) is $T \times Kd \times D$, where the variables have been defined above.

Hence the value of the firm is increased by the present value of the tax savings or tax shield.

$$V^L = V^{UL} + V^{TS} = V^D + V^E \tag{5a}$$

Associated to equations (4) and (5a) there exists correlated cash flows, as follows:

$$FCF + TS = CFD + CFE \tag{5b}$$

Where FCF is free cash flow, TS is tax savings, CFD is cash flow to debt and CFE is cash flow to equity.

When a firm has debt there exists some other contingent or hidden costs associated to the fact to the possibility that the firm goes to bankruptcy. Then, there are some expected costs that could reduce the value of the firm. The existence of these costs deters the firm to take leverage up to 100%. One of the key issues is the appropriate discount rate for the tax shield. In this note, we assert that the correct discount rate for the tax shield is Ku, the return to unlevered equity, and the choice of Ku is appropriate whether the percentage of debt is constant or varying over the life of the project.

In this work the effects of taxes on the WACC will be studied. When calculating WACC two situations can be found: with or without taxes. In the first case, as said above, the WACC is constant, no matter how the firm value be split between creditors and stockholders. (The assumption is that if inflation is kept constant, otherwise, the WACC should change accordingly.) When inflation is not constant, WACC changes, but due to the inflationary component and not due to the capital structure. In this situation, WACC is the cost of the assets, K_A, or the cost of the firm, Ku and at the same time is the cost of equity when unlevered. This means,

$$Ku_t = Kd \times D_{t-1}\% + Ke \times E_{t-1}\% \qquad (6)$$

This Ku is defined as the return to unlevered equity. The WACC is defined as the weighted average cost of debt and the cost of levered equity. In a MM world Ku is equal to WACC without taxes. When taxes exist, the WACC calculation will change taking into account the tax savings.

If it is true that the cost Ku, is constant, Ke, the cost of equity changes according to the leverage. Here for simplicity we assume that the Ku is constant, but this assumption is not necessary. If the Ku is changing then in each period, the WACC will change as well, not only for the eventual change in the financing profile, but for the change in Ku. In any case, Ke has to change in order to keep Ku constant or in order to be consistent with the changing Ku.

The cost of equity, Ke is:

$$Ke_t = (Ku_t - Kd \times D\%_{t-1})/E\%_{t-1} = Ku_t + (Ku_t - Kd) \times D\%_{t-1}/E\%_{t-1} \qquad (7)^2$$

This equation is proposed by Harris and Pringle (1985) and is part of their definition of WACC[3]. A complete derivation for Ke and WACC can be found in Tham and Vélez-Pareja (2002, 2004b). The Ke is derived under different assumptions for the discount rate for the tax savings and for perpetuities and finite periods. Note the absence of the (1-T) factor.

2 This formula is derived in Appendix B.

3 This was the original proposal by MM in a seminal paper published in 1958, but corrected in 1963.

As before, it can be shown that equation 7 results from the assumption that the discount rate for the tax savings is Ku and it can be shown that Ke, defined in equation 7, is the same for finite periods and for perpetuities, see Tham and Vélez-Pareja (2004a, 2004b). The assumption behind Ku as the discount rate is that the tax savings are a strictly correlated to the free cash flow.

What is the meaning of equation 7? Since Ku and Kd are constant, we see that the return to levered equity Ke is a linear function of the debt-equity ratio. It should be no surprise that there is a positive relationship between Ke, the return to levered equity and the debt-equity ratio. Since the debt holder has a prior claim on the expected cash flow generated by the firm, relative to the debt holder, the risk to the equity holder is higher and the equity holder demands a higher return to compensate for the higher risk. The higher the amount of debt, given a constant total value, the higher is the risk to the equity holder, who is the residual claimant.

Equation 7 shows the relationship between the Ke, the return to levered equity and the debt-equity ratio. Table 1 shows the relationship between D, the amount of debt, the debt-equity ratio, E, the amount of equity and Ke, the return to levered equity.

TABLE 1

RELATIONSHIP BETWEEN D, THE AMOUNT OF DEBT, THE DEBT-EQUITY RATIO AND KE, THE RETURN TO LEVERED EQUITY FOR KU = 15.1% AND KD = 11.2%

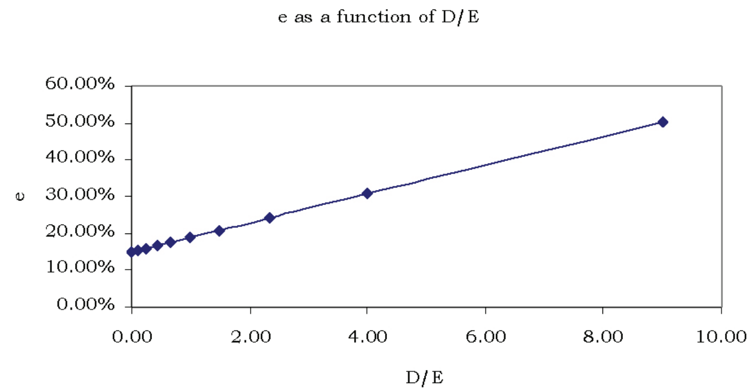
Debt, D	Equity, E	D/E Ratio	Ke
0	1,000	0.00	15.10%
100	900	0.11	15.53%
200	800	0.25	16.08%
300	700	0.43	16.77%
400	600	0.67	17.70%
500	500	1.00	19.00%
600	400	1.50	20.95%
700	300	2.33	24.20%
800	200	4.00	30.70%
900	100	9.00	50.20%

Source: Elaborated by authors.

If the amount of debt is \$ 100, the debt-equity ratio is 0.11 and the return to levered equity is 15.53%. As presented in Figure 1, note that there is a linear relationship between Ke, the return to levered equity and the debt-equity ratio.

FIGURE 1

KE AS A FUNCTION OF D/KE

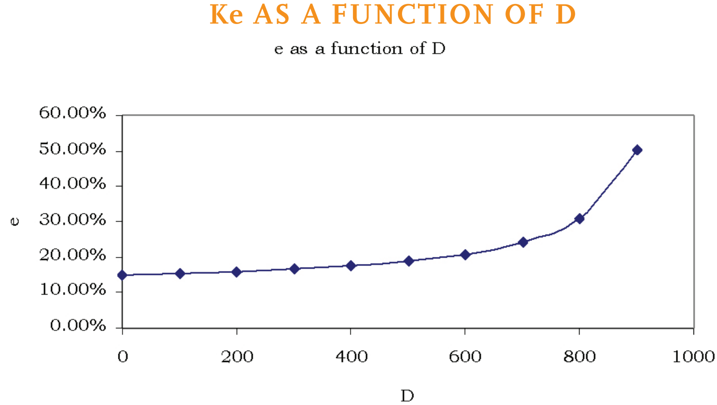


Source: Elaborated by authors.

If the amount of debt increases from 100 to 200, the return to levered equity increases by 0.43 percentage points, from 15.1% to 15.53%. However, the relationship between Ke, the return to levered equity and the amount of debt D is non-linear (remember that E = Total value – D and D/(V – D)). If the amount of debt increases from 500 to 600, the return to levered equity increases by 1.95 percentage points, from 19% to 20.95%.

FIGURE 2

KE AS A FUNCTION OF D



Source: Elaborated by authors.

As can be seen in Appendix A, WACC after taxes can be calculated as:

$$WACC_t = Kd_t \times (1\text{-}T) \times D\%_{t-1} + Ke_t \times E\%_{t-1} \quad (8)$$

The values for D% and E% have to be calculated on the total value of the firm for the beginning of each period. This is the well known expression for the weighted average cost of capital.

It can be shown that under the assumption of the discount rate of tax savings is Ku, the WACC for the FCF can be expressed as (THAM; VÉLEZ-PAREJA, 2002, 2004a):

$$WACC_t = Ku_t - TS_t/TV_{t-1} \quad (9)$$

Where TS means tax savings and TV is the total levered value of the firm. This means that Kd × T × D% is the same as Kd × T × D/TV and in general, we call TS to the tax savings –Kd × D × T. However, it must be said that the tax savings are equal to Kd × D × T only when taxes are paid in the same year as accrued. The implicit assumption in (9) is that we consider the actual tax savings earned and when they occur. This new version of WACC has the property to give the same results as (8) and what is most important, as TS is the actual tax savings earned, it takes into account the losses carried forward (LCF), when they occur. This problem has been studied by Vélez-Pareja and Tham (2001a, 2001b) and Tham and Vélez-Pareja (2004b).

If the Capital Asset Pricing Model (CAPM) is used, it can be demonstrated that there is a relationship between the betas of the components (debt and equity) in such a way that

$$\beta_{t\ firm} = \beta_{t\ debt}\, D_{t-1}\% + \beta_{t\ stock}\, Ke t_{-1}\% \quad (10)$$

If $\beta_{t\ stock}$, $\beta_{t\ debt}$, $D_{t-1}\%$ and $E_{t-1}\%$ are known, then Ku can be calculated as

$$Ku = R_f + \beta_{t\ firm}\,(R_m - R_f) \quad (11)$$

Where R_f is the risk free rate of return and R_m is the market return and $(R_m - R_f)$ is the market or equity risk premium. And this means the Ku can be calculated for any period.

3 CALCULATIONS FOR Ke AND Ku

The secret is to calculate Ke or Ku. If Ke is known for a given period, the initial period, for instance, Ku can be calculated. On the contrary, if Ku is known

Ke can be calculated. For this reason several options to calculate Ke and Ku are presented. In order to calculate Ke, we have several alternatives:

1. With the Capital Asset Pricing Model (CAPM). This is the case of a firm that is traded at the stock exchange, it is traded on a regularly basis and we think the CAPM works well. However, it has to be said that if we know the value of the equity (it is traded at the stock exchange) it is not necessary to discount the cash flows to calculate the value.

2. With the Capital Asset Pricing Model (CAPM) adjusting the betas. This is the case for a firm that is not listed at the stock exchange or if registered, is not frequently traded and we believe the model works well. It is necessary to pick a stock or industry similar to the one we are studying (from the same industrial sector, about the same size and about the same leverage). This is called the proxy firm.

• Example:

The beta adjustment is done with[4]:

$$\beta_{nt} = \beta_{proxy} \frac{\left[1+\frac{D_{nt}}{E_{nt}}(1-T)\right]}{\left[1+\frac{D_{proxy}}{E_{proxy}}(1-T)\right]} \qquad (12)$$

Where, β_{nt} is the beta for the stock not registered at the stock exchange; D_{nt} is the market value of debt, E_{anb} is the equity for the stock not registered in the exchange; D_{proxy} is the market value of debt for the proxy firm, E_{proxy} is the market value of equity for the proxy firm.

For instance, if you have a stock traded at the stock exchange and the beta is β_{proxy} of 1.3, a debt D_{proxy} of 80, E_{proxy} worth 100, and we desire to estimate the beta for a stock not listed in the stock exchange. This non-traded stock has a debt D_{nt} of 70 and equity of E_{nt} of 145 and a tax rate of 35%, and then beta for the non-traded stock can be adjusted as

$$\beta_{nt} = \beta_{proxy} \frac{\left[1+\frac{D_{nt}}{E_{nt}}(1-T)\right]}{\left[1+\frac{D_{proxy}}{P_{proxy}}(1-T)\right]} = 1.3 \frac{\left[1+\frac{70}{145}(1-35\%)\right]}{\left[1+\frac{80}{100}(1-35\%)\right]} = 1.12$$

4 Based on Hamada (1969). This assumes Kd as the discount rate for the TS and perpetuities.

This is easier said than done. Although we have illustrated the use of the formula, we have to recall that the market value of equity for the non traded firm is not known. That value is what we are looking for. Hence, there will be a circularity when using this approach.

3. Subjectively and assisted by a methodology such as the Analytical Hierarchy Process developed by Tom Saaty and presented by Cotner and Fletcher (2000) applied to the owner of the firm. With this approach the owner given a leverage level estimates the perceived risk. This risk premium is added to the risk free rate and the result would be an estimate for Ke.

4. Subjectively as 3, but direct. This is, asking the owner, for a given value level of debt and a given cost of debt, what is the required return to equity?

5. An estimate based on book value (given that these values are adjusted either by inflation adjustments or asset revaluation, so the book value is a good proxy to the market value).

An example: assume a privately held firm. Tax rate is 35%.

TABLE 2

FINANCIAL INFORMATION OF HYPOTHETICAL FIRM

YEAR	ADJUSTED BOOK VALUE FOR EQUITY E	DIVIDENDS PAID D	RETURN $R_t = ((E_t + D_t)/E_{t-1} - 1$
1990	$ 1,159	$ 63	
1991	$ 1,341	$ 72	21.92%
1992	$ 2,095	$ 79	62.12%
1993	$ 1,979	$ 91	-1.19%
1994	$ 3,481	$ 104	81.15%
1995	$ 4,046	$ 126	19.85%
1996	$ 3,456	$ 176	-10.23%
1997	$ 3,732	$ 201	13.80%
1998	$ 4,712	$ 232	32.48%
1999	$ 4,144	$ 264	-6.45%
2000	$ 5,950	$ 270	50.10%

Source: Elaborated by authors.

TABLE 3

ADDITIONAL MACROECONOMIC INFORMATION

YEAR	NOMINAL RISK FREE RATE OF INTEREST[5] R_f	INFLATION RATE	REAL INTEREST RATE $i_r = (1+R_f)/(1+i_f)-1$	RETURN TO EQUITY $Ke_t=((D_t+E_t)/E_{t-1})-1$	RISK PREMIUM $i_\theta = Ket - R_{ft} \times (1-T)$
1990	36.3%				
1991	30.6%	26.8%	3.0%	21.92%	2.0%
1992	28.9%	25.1%	3.0%	62.12%	43.3%
1993	26.3%	22.6%	3.0%	-1.19%	-18.3%
1994	26.3%	22.6%	3.0%	81.15%	64.1%
1995	15.8%	19.5%	-3.1%	19.85%	9.6%
1996	16.3%	21.6%	-4.4%	-10.23%	-20.8%
1997	21.2%	17.7%	3.0%	13.80%	0.0%
1998	51.7%	16.7%	30.0%	32.48%	-1.1%
1999	16.4%	13.0%	3.0%	-6.45%	-17.1%
2000	12.9%	9.6%	3.0%	50.10%	41.7%
2001		Expected 10%	Average 4.4%		Average 10.3%

Source: Elaborated by authors.

Estimated risk free rate for 2001:

$R_{f\,2001} = ((1 + i_{f\,est.})(1 + i_{r\,avg.}) - 1) \times (1-T) = ((1 + 10\%)(1 + 4.4\%) - 1) \times (1 - 0.35) = 9{,}61\%$

Cost of equity $Ke = R_{f\,2001} + i_{\theta\,average} = 9{,}61\% + 10{,}30\% = 20{,}0\%$

6. Calculate the market risk premium as the average of $R_m - R_f$, where R_m is the return of the market based upon the stock exchange index and R_f is the risk free rate (say, the return of treasury bills or similar). Then, subjectively, the owner could estimate if he prefers, in terms of risk, to stay in the actual business or to buy the stock exchange index basket. If the actual business is preferred, then one could say that the beta of the actual business is lower than 1, the market beta, and

5 This information is based on actual data for nominal risk free rates in the Colombian bond market.

the risk perceived is lower than the market risk premium, $R_m - R_f$. This is an upper limit for the risk premium of the owner. This upper limit could be compared with zero risk premium, the risk free rate risk premium which is the lower limit for the risk perceived by the equity owner.

If the owner prefers to buy the stock exchange index basket, we could say that the actual business is riskier than the market. Then, the beta should be greater than 1 and the perceived risk for the actual business should be greater that $R_m - R_f$.

In the first case, the owner could be confronted with different combinations – from 0% to 100% – of the stock exchange index basket and the risk free investment and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk, or simply, the market risk premium ($R_m - R_f$) times the proportion of the stock exchange index basket accepted. In fact what has been found is the beta for the equity holders in the actual business.

In the second case one must choose the highest beta found in the stock exchange index basket. This beta should be used to multiply the market risk premium $R_m - R_f$, and the result would be an estimate of the risk premium for the riskiest stock in the index. This might be an upper limit for the risk perceived by the owner. In case this risk is lower that the perceived risk by the owner, it might be considered as the lower limit. In case that the riskier stock is considered riskier than the actual business, then the lower limit is the market risk premium, $R_m - R_f$. In this second case, the owner could be confronted with different combinations – from 0% to 100% – of the stock exchange index basket and the riskiest stock and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk. That is, the market risk premium ($R_m - R_f$) times the proportion of the stock exchange index basket accepted plus the risk premium for the riskiest stock in the index (its beta times the market risk premium, $R_m - R_f$) times the proportion accepted for that stock.

In both cases the result might be an estimation of the risk premium for the actual business. This risk premium could be added to the risk free rate and this might be a rough estimate of Ke.

If Ke, D% and E% are known, then Ku is calculated with (6). As it is necessary to know the market values that are the result of discounting the future cash flows at WACC, then circularity is found, but it is possible to solve it with a spreadsheet.

Another option is to calculate Ku directly. One of the following alternatives could be used:

1. Using the CAPM and unlevering the beta and using equation (13), which is derived from (12).

$$\beta_{nt} = \frac{\beta_{proxy}}{\left[1 + \frac{D_{proxy}}{E_{proxy}}(1 - T)\right]} \tag{13}$$

With this beta we apply CAPM to obtain Ku.

2. According to MM, the WACC before taxes (Ku) is constant and independent from the capital structure of the firm. Then we could ask the owner for an estimate on how much she is willing to earn assuming no debt. A hint for this value of Ke could be found looking how much she could earn in a risk free security when bought in the "secondary" market. On top of this, a risk premium, subjectively calculated must be included.

3. Another way to estimate Ku is assessing subjectively *the risk for the firm* and this risk could be used to calculate Ku using CAPM with the risk free rate. Cotner and Fletcher (2000) present a methodology to calculate the risk of a firm not publicly held[6]. This methodology might be applied to the managers and other executives of the firm. This would give the risk premium for the firm. As this risk component would be added to the risk free rate, the result is Ku calculated in a subjective manner. A hint that could help in the process is to establish minimum or maximum levels for this Ku the minimum could be the cost of debt before taxes. The maximum could be the opportunity cost of owners, if it is perceptible this is, if it has been "told" by them or if, by observation, it is known observing were they are investing (other investments made by them).

This Ku is in accordance to the actual level of debt. It has to be remembered that Ku is, according to MM, constant and independent from the capital structure of the firm. This Ku is named in other texts as K_A cost of the assets or the firm, for instance, Ruback (2002), or Ku cost of unlevered equity, for instance, Fernández (1999a, 1999b).

If Ku is estimated directly and we wish to estimate the WACC (or the Ke), then circularities will be present. However, as will be shown below, the total value of the firm can be calculated with Ku using the Capital Cash Flow, CCF, and no circularities will be present and there is no need to calculate the leverage ratio for every period.

[6] In fact, in the article the authors say that the methodology is to calculate the risk of the cost of capital, although at the end they say it is to define the risk for the equity cost. The way the methodology is presented allows thinking that it is the firm risk that is dealt with and this risk is added to the risk free rate. With this, the cost of capital before taxes for the firm is found. This would be Ku.

4 AN EXAMPLE FOR CALCULATING WACC AND THE FIRM VALUE

For a better understanding of these ideas, an example is presented. This example is done assuming that the discount rate for TS is Ku. In this example it is assumed that Ku is the correct discount rate for tax savings.

Assume a firm with the following information:

- The cost of the unlevered equity Ku 15.1%
- Cost of Debt, Kd 11.2%
- Tax rate 35.0%

The information about the initial investment, free cash flows, debt balances and initial equity is presented in Table 4.

TABLE 4

FREE CASH FLOW AND INITIAL INVESTMENT

YEAR	0	1	2	3	4
Free cash flow FCF[7]		170,625.00	195,750.00	220,875.00	253,399.45
Debt at end of period, D	375,000.00	243,750.00	75,000.00	37,500.00	
Initial equity investment	125,000.00				
Total initial investment	500,000.00				

Source: Elaborated by authors.

The WACC calculations are made estimating the debt and equity participation in the total value of the firm for each period and calculating the contribution of each to the WACC after taxes. As a first step, we will not add up these components to find the value of WACC and we will calculate the total firm value with the WACC set at 0. We will construct each table, step by step, assuming that WACC is zero. Remember that $D_{t-1}\% = D_{t-1}/V_{t-1}$, where D is market value of debt, and V is the total firm value.

7 In the FCF at year 4 we assume there is a terminal value, that takes into account the value added by the firm from year 5 to infinity. This is a very important issue in firm valuation because experience shows that more than 50% of the firm value might be provided by terminal value. The subject is not addressed in detail because it is beyond the scope of this paper. It is a complex issue and the purpose of this text is to illustrate how to involve market values in the calculation of WACC. The interested reader can read several papers on this at http://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=145648.

As said, the first step is to calculate the value with an arbitrary value for WACC, for instance, zero. See this in the next table. Our table for WACC and Total Value will appear as presented in Table 5.

TABLE 5

WACC CALCULATIONS

YEAR	0	1	2	3	4
WACC after taxes (Debt + equity contributions)					
Total value TV, at t-1 and WACC = 0	840,649.45	670,024.45	474,274.45	253,399.45	

Source: Elaborated by authors.

We use a well known formulation in finance:

$$V_t = \frac{CF_{t+1}+V_{t+1}}{1+WACC_{t+1}} \quad (14)$$

Where CF is cash flow, V is market value and WACC is the weighted average cost of capital.

- Example: firm value at end of year 3 is (253,399.45+0)/(1+0%) = 253,399.45.

For year 2 it will be (253,399.45 + 220,875.00)/(1+0%) = 474,274.45 and so on for the other years.

We do this to avoid a division by zero. Done this we can calculate temporary values for D%, E% and Ke. The contribution of debt to WACC (temporary results) is presented in Table 6.

TABLE 6

WACC CALCULATION – CONTRIBUTION OF DEBT TO WACC (TEMPORARY RESULTS)

YEAR	0	1	2	3	4
Debt					
Relative weight of debt D% (Debt balance at t-1)/Total value of firm at t-1)	44.61%	36.38%	15.81%	14.80%	
Cost of debt after taxes Kd×(1-T)		7.28%	7.28%	7.28%	7.28%
Contribution of debt to WACC, Kd×(1-T)×D_{t-1}%		3.25%	2.65%	1.15%	1.08%

Source: Elaborated by authors.

The same procedure is used to estimate the contribution of equity to WACC, as presented in Table 7.

TABLE 7

WACC CALCULATION – CONTRIBUTION OF EQUITY TO WACC (TEMPORARY RESULTS)

YEAR	0	1	2	3	4
Equity					
Relative weight of equity E% = (1-D%)	55.39%	63.62%	84.19%	85.20%	
Cost of equity Ke = $(Ku_t - Kd) \times D\%_{t-1} / E\%_{t-1}$		18.24%	17.33%	15.83%	15.78%
Contribution of equity to WACC = $E\%_{t-1} \times Ke$		10.10%	11.03%	13.33%	13.44%

Source: Elaborated by authors.

It is recommended that the last arithmetic operation be the WACC calculation as the sum of the debt and equity contribution to the cost of capital.

At this point we recommend to set the spreadsheet to handle circularities following these instructions:

1. Select the Office Button at the top left and select Excel Options (down to the right) in Excel (2007).
2. Select Formula.
3. Enable Iterations.
4. Click Ok.

This procedure can be done before starting the work in the spreadsheet or when Excel declares the presence of circularity. After these instructions are done, then, the WACC can be calculated as the sum of the debt and equity contribution to the cost of capital.

Now we can proceed to formulate the WACC as the sum of the two components: debt contribution and equity contribution. When the WACC is calculated, previous tables will be shown as in Table 8.

TABLE 8

WACC CALCULATION – CONTRIBUTION OF DEBT TO WACC (FINAL)

YEAR	0	1	2	3	4
Debt					
Relative weight of debt D% (Debt balance$_{t-1}$/Total value$_{t-1}$)	61.68%	47.38%	19.39%	16.94%	
Cost of debt after taxes Kd×(1-T)		7.28%	7.28%	7.28%	7.28%
Contribution of debt to WACC Kd×(1-T)×D$_{t-1}$%		4.49%	3.45%	1.41%	1.23%

Source: Elaborated by authors.

The same procedure is used to estimate the contribution of equity to WACC, as shown in Table 9.

TABLE 9

WACC CALCULATION – CONTRIBUTION OF EQUITY TO WACC (FINAL)

YEAR	0	1	2	3	4
Equity					
Relative weight of equity E% = (1-D%)	38.32%	52.62%	80.61%	83.06%	
Cost of equity $Ke_t = Ku_t + (Ku_t – Kd)× D\%_{t-1}/E\%_{t-1}$		21.38%	18.61%	16.04%	15.90%
Contribution of equity to WACC = E%×Ke		8.19%	9.79%	12.93%	13.20%

Source: Elaborated by authors.

Note that the cost of equity –Ke– is larger than Ku as expected, because Ku is the cost of the stockholder, as if the firm were unlevered[8]. When there is debt

[8] As MM say that Ku is constant and independent from the capital structure, it will be equal to Ku when debt is zero. This Ku is WACC before taxes. And this is the condition for the validity of the first proposition of MM.

– Ke calculation – necessarily Ke ends up being greater than Ku, because of leverage. With these values it is possible to calculate the firm value for each period.

If Ke_1 is known, as it was said above, Ku is found with (6). Excel solves the circularity that is found and the same values result.

Now we have our final table with WACC and value obtained simultaneously as follows in Table 10.

TABLE 10

WACC CALCULATIONS (FINAL)

YEAR	0	1	2	3	4
WACC after taxes (Debt + equity contributions)		12.7%	13.2%	14.3%	14.4%
Firm value a end of t	607,978.04	514,457.73	386,835.85	221,433.06	

Source: Elaborated by authors.

Notice that WACC results in a lower value than Ku. WACC is after taxes.

Using (14) and from tables 14 and 10, we have that the firm value at end of year 3 is (253,399.45+0)/(1+14.4%) = 221,433.06.

For year 2 it will be (221,433.06 + 220,875.00)/(1+14.3%) = 386,835.85 and so on for the other years.

The reader has to realize that the values 14.4% and 14.3%, etc. are not calculated from the beginning because they depend on the firm value that is going to be calculated with the WACC. In this case circularity is generated. This is solved allowing the spreadsheet to make enough iteration until it finds the final numbers.

With the WACC values for each period the present value of future cash flows and the NPV are calculated (Table 11).

TABLE 11

NPV CALCULATIONS

YEAR	0
Present value of cash flows	607,978.04
Initial total investment	500,000.00
NPV	107,978.04

Source: Elaborated by authors.

If the initial investment is 500,000, then, NPV is 107,978.04.

The same result can be reached calculating the present value for the free cash flow assuming no debt and discount it a Ku, or what is the same, at WACC before taxes and add up the present value of tax savings at the same rate of discount, Ku. Myers proposed this in 1974 and it is known as Adjusted Present Value APV. Myers and all the finance textbooks teach that the discount rate for the TS should be the cost of debt. However, the tax savings depend on the firm profits. Hence, the risk associated to the tax savings is the same as the risk of the cash flows of the firm rather than the value of the debt. Hence, the discount rate should be Ku. For this reason the tax savings are also discounted at Ku. This way, the present value for the free cash flows discounted at WACC after taxes coincides with the present value of the free cash flow assuming no debt discounted at Ku and added to the present value of the tax savings discounted at the same Ku.

The use of Ku to discount the tax savings has been proposed by Tham (1999, 2000) and Ruback (2000). Tham proposes to add to the unlevered value of the firm (the present value of the FCF at Ku), the present value of the tax savings discounted at Ku. Ruback presents the Capital Cash Flow and discount it at Ku. The CCF is simply the FCF plus the tax savings so:

$$\text{CCF} = \text{FCF} + \text{Tax savings} \quad (15)$$

$$\text{PV(FCF at WACC after taxes)} = \text{PV(FCF without debt at Ku)} + \text{PV(Tax savings at Ku)} = \text{PV(CCF at Ku)} \quad (16)$$

TABLE 12

CALCULATION OF VALUE AND APV WITH Ku

YEAR	0	1	2	3	4
Interest payments		42,000.00	27,300.00	8,400.00	4,200.00
Tax savings TS = T×I (T=35%)		14,700.00	9,555.00	2,940.00	1,470.00
Free cash flow FCF		170,625.00	195,750.00	220,875.00	253,399.45
Capital Cash Flow (CCF) = FCF + Tax savings		185,325.00	205,305.00	223,815.00	254,869.45
Ku		15.10%	15.10%	15.10%	15.10%
PV (CCF) at Ku		607,978.04			

(continue)

Table 12 (Continuation)

CALCULATION OF VALUE AND APV WITH Ku

YEAR	0	1	2	3	4
Adjusted NPV (APV)					
PV(FCF at Ku)		585,228.51			
PV(TS at Ku)		22,749.53			
PV(FCF at Ku) + PV(TS at Ku)		607,978.04			
NPV		107,978.04			

Source: Elaborated by authors.

Notice that the same result is reached with the three methods. At this time, the reader can test that equation (9) matches with WACC from Table 10. For instance, for year 1, 15.1% – 14,700/607,978.04 = 15.1% – 2.42% = 12.68%.

From the point of view of equity valuation, the value is calculated with the present value of the free cash flow discounted at WACC minus the debt at 0. This value also can be reached with the equity cash flow (CFE) and it is equal to

$$CFE = FCF + TS - \text{Cash flow to debt before taxes } CFD \qquad (17)$$

Table 13

CALCULATING THE VALUE OF EQUITY WITH CFE

YEAR	0	1	2	3	4
Free cash flow FCF		170,625.00	95,750.00	20,875.00	53,399.45
Tax savings TS = T × I (T = 35%)		14,700.00	9,555.00	2,940.00	1,470.00
CFD = Interest + principal payment		173,250.00	196,050.00	45,900.00	41,700.00
CFE		12,075.00	9,255.00	177,915.00	213,169.45
Ke		21.38%	18.61%	16.04%	15.90%
PV(CFE at Ke)	232,978.04	9,948.31	6,428.52	106,499.41	110,101.80

Source: Elaborated by authors.

FCF is taken from Table 4, TS comes from Table 12 and CFD comes from tables 4 and 12.

When the present value of CFE at Ke, is calculated the same result is obtained. This is, 607,978.04 – 375,000 = 232,978.04. This means that the right discount rate to discount the CFE is Ke, and its discounted value is consistent with the value calculated with the FCF.

In table 13 we calculated the market value of equity using the market value calculated before. However, this is not an independent method when we use the values from other method. In order to calculate the market value of equity in an independent way we will use the same procedure utilized for the calculation with WACC. The difference is that we will calculate again the value of Ke. The first table with Ke equal to zero is Table 14.

TABLE 14

INITIAL TABLE TO CALCULATE THE MARKET EQUITY VALUE (TEMPORARY)

YEAR	0	1	2	3	4
Free Cash Flow FCF		170,625.00	195,750.00	220,875.00	253,399.45
Interest charges		42,000.00	27,300.00	8,400.00	4,200.00
Debt payment		131,250.00	168,750.00	37,500.00	37,500.00
CFD		173,250.00	196,050.00	45,900.00	41,700.00
Tax savings TS		14,700.00	9,555.00	2,940.00	1,470.00
CFE = FCF – CFD + TS		12,075.00	9,255.00	177,915.00	213,169.45
Relative weigh to debt D%	47.6%	37.8%	16.1%	15.0%	
Relative weigh to equity E%	52.4%	62.2%	83.9%	85.0%	
$Ke_t = Ku_t + (Ku_t - Kd) \times D\%_{t-1}/E\%_{t-1}$					
Debt at end of period	375,000.00	243,750.00	75,000.00	37,500.00	-
Market value of equity	412,414.45	400,339.45	391,084.45	213,169.45	
Total value	787,414.45	644,089.45	466,084.45	250,669.45	-

Source: Elaborated by authors.

The final table for this calculation is as follows,

TABLE 15

INDEPENDENT CALCULATION OF MARKET EQUITY VALUE (FINAL)

YEAR	0	1	2	3	4
Free Cash Flow FCF		170,625.00	195,750.00	220,875.00	253,399.45
Interest charges		42,000.00	27,300.00	8,400.00	4,200.00
Debt payment		131,250.00	168,750.00	37,500.00	37,500.00
CFD		173,250.00	196,050.00	45,900.00	41,700.00
Tax savings TS		14,700.00	9,555.00	2,940.00	1,470.00
CFE = FCF – CFD + TS		12,075.00	9,255.00	177,915.00	213,169.45
Relative weight of debt D%	61.7%	47.4%	19.4%	16.9%	
Relative weight of equity E%	38.3%	52.6%	80.6%	83.1%	
$Ke_t = Ku_t + (Ku_t - Kd) \times D\%_{t-1}/E\%_{t-1}$		21.4%	18.6%	16.0%	15.9%
Debt at end of period	375,000.00	243,750.00	75,000.00	37,500.00	-
Market value of equity	232,978.04	270,707.73	311,835.85	183,933.06	
Total value	607,978.04	514,457.73	386,835.85	221,433.06	-

Source: Elaborated by authors.

Observe that working independently we reach the same values for equity, total value and Ke. Observe as well that the NPV for the equity holder is the same as the NPV for the project (firm).

TABLE 16

NPV CALCULATIONS FROM EQUITY INVESTMENT

YEAR	0
Present value of cash flows	232,978.04
Initial equity investment	125,000.00
NPV	107,978.04

Source: Elaborated by authors.

The investment from the equity holders was 125,000 and hence NPV for them is 107,978.04. There is no surprise that both NPVs are identical. The very definition of NPV says that NPV for the firm (project) is the same as the NPV for the equity holder.

Summarizing, the different methodologies presented to calculate the total value of the firm are[9]:

Total Value for the firm V = PV(FCF at WACC)

Total Value for the firm V = PV(FCF at Ku) + PV(TS at Ku)

Total Value for the firm V = PV(CCF at Ku).

Market value of equity E_{mv} = TV – D

Market value of equity E_{mv} = PV(CFE at Ke).

All these calculations give identical results.

In this example, it is shown in Table 17.

TABLE 17

A COMPARISON OF VALUES BY DIFFERENT APPROACHES

METHOD	VALUE	EQUITY VALUE = VALUE - DEBT	NPV EQUITY = NPV FIRM
PV(FCF at $WACC_{t.}$) Table 10	607,978.04	232,978.04	107,978.04
PV(FCF at Ku) + PV(TS at Ku)	607,978.04	232,978.04	107,978.04
PV(FCF+TS at Ku) Table 12	607,978.04	232,978.04	107,978.04
PV(CFE at Ke) Table 15		232,978.04	107,978.04

Source: Elaborated by authors.

The value of equity is the price that the owners would sell their participation in the firm and this is higher than the initial equity contribution of 125,000.

When using Kd as the discount rate for the TS, we find a higher value and full consistency as we did with the assumption the discount rate for the TS is Ku (in this example). In short, ALL methods if properly calculated yield the same firm and equity value and identical NPV for the firm and for the equity holder. See Tham and Vélez-Pareja (2004b) and Vélez-Pareja and Burbano-Pérez (2008).

5 CONCLUSIONS

The misuse of WACC might be due to several reasons. Traditionally there have not been computing tools to solve the circularity problem in WACC calcula-

9 There exist other methodologies, but they do not coincide among them. See Taggart Jr. (1991).

tions. Now it is possible and easy with the existence of spreadsheets. Not having these computing resources in the previous years it was necessary to use simplifications such as calculating just one single discount rate or in the best of cases to use the book values in order to calculate the WACC.

Here a detailed (but known) methodology to calculate the WACC has been presented taken into account the market values in order to weigh the cost of debt and the cost of equity. By the same token a methodology based on the WACC before taxes Ku, constant (assuming stable macroeconomic variables, such as inflation) that does not depend on the capital structure of the firm has been presented.

The most difficult task is the estimation of Ku, or alternatively, the estimation of Ke. Here, a methodology to estimate those parameters is suggested. If it is possible to estimate Ku from the beginning, it will be possible to calculate the total and equity value independently from the capital structure of the firm, using the CCF approach or the Adjusted Present Value approach and discounting the tax savings at Ku.

In summary, the different methodologies presented to calculate the total value of the firm are consistent and yield identical values, as presented in Table 18.

TABLE 18

SUMMARY

METHOD	TOTAL VALUE	EQUITY VALUE
PV(FCF at $WACC_{t.}$)	607,978.04	232,978.04
PV(FCF at Ku) + PV(Tax savings at Ku)	607,978.04	232,978.04
PV(FCF + TS at Ku)	607,978.04	232,978.04
PV(CFE at Ke)		232,978.04

Source: Elaborated by authors.

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APPENDIX A

TRADITIONAL WACC FOR A FINITE STREAM OF FREE CASH FLOW (FCF)

In this appendix, we derive the traditional WACC for a finite stream of free cash flow. Consider a finite stream of cash flows where FCF_i is the free cash flow in year i. Similarly, CFE_i is the cash flow to equity in year i, CFD_i is the cash flow to debt in year i, and TS_i is the tax shield in year i, based on the value of the debt at the end of the previous year i-1.

In any year i, the capital cash flow (CCF) is equal to the sum of the free cash flow and the tax shield.

$$CCF_i = FCF_i + TS_i \tag{A1}$$

Also, in any year i, the capital cash flow is equal to the sum of the cash flow to equity and the cash flow to debt.

$$CCF_i = CFE_i + CFD_i \tag{A2}$$

Combining equation A1 and equation A2, we obtain,

$$FCF_i + TS_i = CFE_i + CFD_i \tag{A3}$$

Returns and taxes

The return to unlevered equity in year i is Ku_i, the return to levered equity in year i is Ke_i, the cost of debt in year i is Kd_i and the discount rate for the tax shield in year i is ψ_i. We assume only corporate tax τ. Furthermore, the corporate tax rate is constant. If the debt is risk-free, then the cost of debt is equal to the risk-free rate r_f.

M & M world

The unlevered value in year i is V^{Un}_i, the levered value in year i is V^L_i, the (levered) equity value in year i is E^L_i, the value of debt in year i is D_i and the value of the tax shield in year i is V^{TS}_i.

With perfect capital markets in an M & M world, we make the following assumptions. In any year i, the levered value is equal to the sum of the unlevered value and the value of the tax shield.

$$V^L_i = V^{Un}_i + V^{TS}_i \qquad (A4)$$

Also, in any year i, the levered value is equal to the sum of the value of (levered) equity and value of debt.

$$V^L_i = E^L_i + D_i \qquad (A5)$$

Combining equation A4 and equation A5, we obtain,

$$V^{Un}_i + V^{TS}_i = E^L_i + D_i \qquad (A6)$$

The expressions for the unlevered value, the (levered) equity value, the value of debt and the value of the tax shield are shown below. In any year i-1, the value is equal to the cash flow discounted by the appropriate discount rate.

$$V^{Un}_{t-1} = \frac{FCF_i}{1+Ku_i} \qquad (A7.1)$$

$$E_{i-1} = \frac{CFE_i}{1+Ke_i} \qquad (A7.2)$$

$$D_{i-1} = \frac{CFD_i}{1+Kd_i} \qquad (A7.3)$$

$$V^{TS}_{i-1} = \frac{TS_i}{1+\psi} \qquad (A7.4)$$

Substituting equation A7.1 to equation A7.4 in equation A3, we obtain:

$$(1 + Ku_i)\times V^{Un}_{i-1} + (1 + \psi_i)\times V^{TS}_{i-1} = (1 + Ke_i)\times E^L_{i-1} + (1 + Kd_i)\times D_{i-1} \qquad (A8.1)$$

Substituting equation A6 into equation A8.1 and simplifying, we obtain,

$$Ku_i\times V^{Un}_{i-1} + \psi_i\times V^{TS}_{i-1} = Ke_i\times E^L_{i-1} + Kd_i\times D_{i-1} \qquad (A8.2)$$

The weighted average cost of capital with the FCF

Let W_i be the WACC in year i based on the FCF_i. Then in year i-1, the levered value is equal to the FCF in year i discounted by W_i.

$$V^L_{i-1} = \frac{FCF_i}{1+W_i} \tag{A9.1}$$

Rewriting equation A9.1, we obtain that

$$FCF_i = (1 + W_i) \times V^L_{i-1} \tag{A9.2}$$

From equation A3, we know that

$$FCF_i = CFE_i + CFD_i - TS_i \tag{A10}$$

Substituting equation A9.2, and equation A7.2 to equation A7.4 into equation A10, we obtain:

$$(1 + W_i) \times V^L_{i-1} = (1 + Ke_i) \times E^L_{i-1} + (1 + Kd_i) \times D_{i-1} - (1 + \psi_i) \times V^{TS}_{i-1} \tag{A11}$$

Simplifying equation A11.1 we obtain:

$$V^L_{i-1} + W_i \times V^L_{i-1} = Ke_i \times E^L_{i-1} + Kd_i \times D_{i-1} - (1 + \psi_i) \times V^{TS}_{i-1} + E^L_{i-1} + D_{i-1} \tag{A12.1}$$

$$W_i \times V^L_{i-1} = Ke_i \times E^L_{i-1} + Kd_i \times D_{i-1} - (1 + \psi_i) \times V^{TS}_{i-1} \tag{A12.2}$$

We know that the tax shield in year i is equal to the tax rate τ times the cost of debt times the value of debt at the end of the previous year i-1.

$$TS_i = \tau \times Kd_i \times D_{i-1} \tag{A13}$$

Substituting equation A7.4 and equation A13 into equation A12.2, we obtain the traditional formulation of the WACC.

$$W_i \times V^L_{i-1} = Ke_i \times E^L_{i-1} + Kdi \times D_{i-1} - \tau \times d_i \times D_{i-1} \tag{A14.1}$$

$$W_i = \frac{E_i}{V^L_{i-1}} \times Ke_i + \frac{D_{i-1}}{V^L_{i-1}} + Kd_i \times (1 - \tau) \tag{A14.2}$$

The WACC is a weighted average of the cost of equity and the cost of debt, where the cost of debt is adjusted by the coefficient $(1 - \tau)$ and the weights are the market value of equity and market value of debt, as percentages of the levered market value. Equation B14.2 is equation 1 in the text.

APPENDIX B

DERIVING KE FOR A PERPETUITY

List of symbols

Ku	The cost of the unlevered equity
Kd	The cost of debt (assumed constant)
D	Market value of debt
Ke_n	Levered cost of equity at year n
E^L	Market value of levered equity
ψ_n	Appropriate discount rate for tax savings at year n
V^{TS}	Value of TS
V^{ND}	Value of unlevered firm
V^L	Value of levered firm

$$V^{TS} = \tau \times Kd \times D/\psi \quad \text{(B1a)}$$
$$\psi \times V^{TS} = \tau \times Kd \times D \quad \text{(B1b)}$$
$$V^{ND} = FCF/Ku \quad \text{(B2a)}$$
$$V^{ND} \times Ku = FCF \quad \text{(B2b)}$$
$$E^L = Z/Ke \quad \text{(B3a)}$$
$$E^L \times Ke = Z = FCF - Kd \times D + \tau \times Kd \times D \quad \text{(B3b)}$$
$$E^L \times Ke = V^{ND} \times Ku - Kd \times D + \psi \times V^{TS} \quad \text{(B4a)}$$
$$E^L \times Ke = [V^L - V^{TS}]Ku - Kd \times D + \psi \times V^{TS} \quad \text{(B4b)}$$
$$Ke \times E^L = Ku \times E^L + (Ku - Kd) \times D - (Ku - \psi) \times V^{TS} \quad \text{(B4c)}$$
$$Ke = Ku + (Ku - Kd) \times D/ E^L - (Ku - \psi) \times V^{TS}/E^L \quad \text{(B4d)}$$

This is the most general formulation for Ke, the cost of levered equity.

- Case 1

Assume ψ = Kd and perpetuities

$$Ke = Ku + (Ku - Kd) \times D/ E^L - (Ku - Kd) \times \tau \times D/ E^L \quad \text{(B4e)}$$

Reorganizing

$$Ke = Ku + (Ku - Kd) \times (1 - \tau) \times D/E^L \quad \text{(B4f)}$$

This equation C4f is equation (2) in the text. For a finite horizon we have to use B4d.

- Case 2

Assume $\psi = Ku$

$$Ke = Ku + (Ku - Kd) \times D/E^{D} \tag{B4g}$$

This is equation (7) in the text. This formula is valid for finite horizons and perpetuities.