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A Mixed-Integer Conic Formulation for Optimal Placement and Dimensioning of DGs in DC Distribution Networks

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Abstract: The problem of the optimal placement and dimensioning of constant power sources (i.e., distributed generators) in electrical direct current (DC) distribution networks has been addressed in this research from the point of view of convex optimization. The original mixed-integer nonlinear programming (MINLP) model has been transformed into a mixed-integer conic equivalent via second-order cone programming, which produces a MI-SOCP approximation. The main advantage of the proposed MI-SOCP model is the possibility of ensuring global optimum finding using a combination of the branch and bound method to address the integer part of the problem (i.e., the location of the power sources) and the interior-point method to solve the dimensioning problem. Numerical results in the 21- and 69-node test feeders demonstrated its efficiency and robustness compared to an exact MINLP method available in GAMS: in the case of the 69-node test feeders, the exact MINLP solvers are stuck in local optimal solutions, while the proposed MI-SOCP model enables the finding of the global optimal solution. Additional simulations with daily load curves and photovoltaic sources confirmed the effectiveness of the proposed MI-SOCP methodology in locating and sizing distributed generators in DC grids; it also had low processing times since the location of three photovoltaic sources only requires 233.16 s, which is 3.7 times faster than the time required by the SOCP model in the absence of power sources.

Keywords: second-order cone programming; power losses minimization; optimal power flow model; convex optimization; power sources; photovoltaic generation

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1. Introduction

Direct current (DC) distribution networks have attracted much attention in recent years in specialized literature [1], since these networks have better voltage profiles [2] and low energy losses [3]. They are easily controllable since frequency or reactive power are nonexistent concepts in these electrical networks [4]. In the literature, multiple approaches regarding DC networks have been described, such as control methodologies for power electronic converters that interface renewable energies [5] and batteries [6], optimization approaches associated with power losses minimization [2] and analysis of convergence algorithms for power flow analysis [7].

Here, we explore the optimization approaches recently presented in scientific literature to analyze DC distribution networks under steady-state conditions regarding energy losses minimization. The authors in [3] performed a semidefinite programming relaxation for the optimal locating and dimensioning of distributed generators (DGs) in DC grids, combined with a heuristic method based on random hyperplanes. This approach demonstrated the possibility of finding better solutions than the different metaheuristic approaches. A similar approach was presented in [8] with a quadratic approximation of the optimal power flow problem; this was combined with a heuristic search based on hyperplanes to determine the best set of nodes for locating constant power sources. The authors of [9] had proposed the complete MINLP model for optimal placement and sizing of DGs in DC grids, taking into consideration the different levels of power penetration. This model was solved by using the GAMS software and comparing the different solvers available on it for addressing MINLP models. The authors of [10] proposed a mixed-integer quadratic transformation of the original MINLP model by solving both models in the GAMS software; they observed that the quadratic approximation was easier to solve and took tens of seconds, while the exact MINLP model took hundreds of seconds. The former also had the best numerical results for the minimization of the total grid losses. In [11] was proposed a multi-period MINLP model for the location and sizing of photovoltaic sources in DC distribution networks located in rural areas with the objective of minimizing the total greenhouse emissions (i.e., carbon dioxide CO_2) caused by diesel generators. The solution of the MINLP model was reached with the GAMS optimization package. The authors of [12] proposed multiple combinations of combinatorial optimization algorithms for optimal the placement and dimensioning of DGs in DC networks via sequential quadratic programming [13]. The optimal placement problem of generators was addressed using the population-based incremental learning (PBIL) optimization algorithm. Simultaneously, the sizing problem was solved using continuous metaheuristic algorithms such as particle swarm optimizer (PSO), constant genetic algorithm, and black hole optimizer. Numerical results showed that the best trade-off is found using the PBIL-PSO method. However, this approach has not yet been compared with exact techniques.

The DC distribution optimization problems were presented in [14,15], in which the issue of the optimal reconfiguration of DC feeders was addressed using convex approximations and exact MINLP models. The authors of [16] proposed a reformulation of the exact nonlinear programming (NLP) model for optimal power flow studies in DC grids using a conic representation of the power balance equations. This reformulation was compared with the exact one in different test feeders, and it was confirmed that convex approximation can find the global optimum, which coincides with the solution of the NLP model. The author of [17] proposed a convexification of the exact NLP model for optimal operation of battery energy storage systems using a semidefinite programming model, which was solved using the CVX package on the MATLAB environment. The semidefinite programming model ensured the global optimum finding due to the convex properties of the semidefinite matrices; in addition, this work has been further improved through the reduction of solution space by proposal of a second-order cone programming equivalent, as was presented in [18], that also ensures global optimum finding. The main advantage of this was the reduction of the processing times by more than 50% compared to the semidefinite programming approach.

Additional developments in the current literature regarding the inclusion of stochastic behaviors in expansion planning problems for distribution networks, transmission grids and generation sources have been proposed in [19–21], respectively. These works demonstrated the importance of having adequate representations of the electrical networks, including the related uncertainties in renewable generation, to make their expansion effective using efficient techniques that involve exact and metaheuristic optimization methods depending on the complexity of the problem under study.

Based on the importance of DC distribution networks in the literature, in this paper, we propose an exact mathematical optimization based on second-order cone programming (SOCP) optimization for optimal power flow analysis [16]. At the same time, integer optimization based on the branch and bound (B&B) method is used to solve the problem of the optimal placement and sizing of constant power sources in DC grids. This combination generates a mixed-integer second-order cone programming (MI-SOCP) reformulation of the exact MINLP model presented in [22].

The most important contributions of this paper can be condensed as follows:

✓ The guarantee of reaching the global optimum for the problem of the optimal siting and dimensioning of constant power sources in DC grids via an MI-SOCP model solved through B&B and interior-point methods.

✓ The verification of the complexity that the original MINLP model presents in finding the optimal solution using powerful optimization tools available in GAMS. Numerical results in the 21-bus test feeder are optimal based on the GAMS solutions. However, in the 69-bus test feeder case, most of the solvers available in GAMS are stuck in local optimum solutions. This is not the case with the proposed MI-SOCP model, in which global solutions are reached for both test systems.

The remainder of this document takes the following structure: Section 2 describes the exact MINLP formulation for the problem of the optimal siting and dimensioning of constant power sources in DC grids by remarking the main non-convexities of this model. Section 3 offers the reformulation of optimal power flow problems via SOCP. Additionally, the SOCP is combined with the model's binary nature to produce an MI-SOCP reformulation for the exact MINLP optimization model. Section 4 describes the main properties of the MI-SOCP model and its solution via B&B and interior-point methods. Section 5 presents the main characteristics of the test feeders, which are composed of 21 and 69 nodes and have radial topology and the possibility of locating three distributed power sources. Section 6 shows the numerical performance of the proposed MI-SCOP and its comparison with multiple nonlinear solvers available in the GAMS optimization package. Finally, Section 7 lists the leading conclusions obtained from the proposed study as well as some possible future developments.

2. Exact MINLP Model

The optimal siting and dimensioning of power sources in DC distribution networks is a complex optimization task in electrical engineering since it combines integer and continuous variables [10]. The continuous variables are related to power generations and voltage profiles, whereas the integer ones are associated with the possibility of placing (or not) in a particular node a distributed generator [9]. The exact MINLP formulation that describes the problem under study is shown below:

$$\text{Objective function:} \quad \min p_{\text{loss}} = \sum_{\{km\}\in\mathcal{L}} g_{km}(v_k - v_m)^2, \tag{1}$$

$$\text{Subject to:} \quad p_k^{ss} + p_k^{dg} - p_k^{d} = v_k \sum_{m\in\mathcal{N}} G_{km} v_m, \ \forall k \in \mathcal{N} \tag{2}$$

$$p_{km} = g_{km}\left(v_k^2 - v_k v_m\right), \ \forall \{km\} \in \mathcal{L} \tag{3}$$

$$p_{km}^{\min} \leq p_{km} \leq p_{km}^{\max}, \ \forall \{km\} \in \mathcal{L} \tag{4}$$

$$x_k p_k^{gd,\min} \leq p_k^{dg} \leq x_k p_k^{dg,\max}, \ \forall k \in \mathcal{N} \tag{5}$$

$$v^{\min} \leq v_k \leq v^{\max}, \ \forall k \in \mathcal{N} \tag{6}$$

$$\sum_{k\in\mathcal{N}} p_k^{dg} \leq \alpha \sum_{k\in\mathcal{N}} p_k^{d}, \tag{7}$$

$$\sum_{k\in\mathcal{N}} x_k \leq N_{gd}^{ava}, \tag{8}$$

$$x_k \in \{0,1\}, \ \forall k \in \mathcal{N}, \tag{9}$$

where p_{loss} is the objective function value associated with the total grid power losses; v_k and v_m are the values of the voltage variables associated with nodes k and m, respectively; g_{km} is the conductance parameter of the branch that connects nodes k and m; G_{km} corresponds to the element of the matrix of conductances that is associated with nodes k and m; p_k^{ss}, p_k^{dg}, and p_k^d are the power generation in slack nodes (voltage-controlled sources) and distributed generators, and power consumptions in load nodes, respectively; p_{km} is the flow of power in the line that connects nodes k and m; $p_{km}^{\min}$ and $p_{km}^{\max}$ are the lower and upper power flow bounds permitted at the branch km; $p_k^{gd,\min}$ and $p_k^{gd,\max}$ are the lower and upper power generation limits associated with a distributed generator connected at node k; x_k is the binary variable associated with the placement ($x_k = 1$) or non-placement ($x_k = 0$) of a constant power source at node k; $v^{\min}$ and $v^{\max}$ are the minimum and maximum voltage regulation bounds allowed at each node; α is a positive constant between 0 and 1 that is related to the amount of distributed generation that is permitted in the DC grid; and N_{gd}^{ava} refers to the number of power sources available for installation.

The complete interpretation of the general MINLP formulation for the problem of the optimal placement and sizing of DGs in DC grids defined from (1) to (9) is as follows: Equation (1) corresponds to the objective function of the optimization problem, which is associated with the total grid losses in all the branches of the network. This objective function is formulated using the voltage drop at each line and its conductance. The Equation (2) corresponds to the power balance constraint at each bus of the systems, which defines the hyperbolic relation between the voltages and powers generated or consumed. Please note that this set of equations defines the nonlinear quadratic constraints that make the continuous part of the problem non-convex [16]. Expression (3) determines the amount of power flow sent from node k to node m as a function of the voltage drop in the line, its conductance and the sending voltage. The inequality constraint (4) limits the amount of power flow that can be transported through the distribution line that connects nodes k and m. Please note that this restriction corresponds to the maximum thermal bound supported by the conduction written as a function of its power flow. The inequality constraint (5) determines the maximum amount of power that can be injected by a distributed generator connected at node k if the binary variable x_k is activated, i.e., $x_k = 1$, if the DG is not connected at node k, then $x_k = 0$, which makes the amount of power injected be zero. Inequality expression (6) is known in the current literature as the voltage regulation constraint, since this limits the admissible voltage profiles in all the nodes of the distribution network. The upper and lower voltage bounds are defined by regulatory policies, and the operative consigns are defined by the grid operator. The inequality constraint (7) determines the maximum amount of power that can be injected via the distributed generators into the distribution network, which is also defined by regulatory policies in the electric distribution sector [10]. The inequality constraint (8) limits the maximum number of DGs that can be installed in the distribution network. This limitation is typically assigned by the grid operator as a function of the investment budget available. Finally, the binary nature of the decision variable, i.e., x_k, is defined through (9), which defines the placement (or non-placement) of a constant power source in the DC grid.

Remark 1. *The expression (1), related to the objective function, is a quadratic function associated with the sums of the products between voltage variables; this is convex due to the properties associated with the matrix of conductance, G which is positive semidefinite [23,24].*

Remark 2. *The set of nonlinear constraints associated with the balance of active power at each node and the power flow at each line, i.e., (2) and (3), are not convex as these generate quadratic equality constraints that are non-affine and non-convex [25]. However, as the unique non-linearity is the product between voltage variables, i.e., $v_k v_m$, these equations can be convexified using an SOCP approximation [18]. This has been described in the next section.*

3. MI-SOCP Reformulation

The conic programming approach corresponds to a part of the convex optimization [18], which is a field of exact mathematical optimization that has attracted great attention in electrical engineering. The main advantage of it is that it can solve convex problems, guaranteeing a unique solution (global optimum) in a reliable and efficient manner [26].

The SOCP model works with a linear function over a convex region with the goal of minimizing it. In summary, the SOCP is an intersection of an affine linear space and conic constraints [27].

Here, we employ the hyperbolic equivalent of the product with two variables for rewriting Equations (1)–(3) using a conic representation [16]. Let us define the product between voltage variables as follows:

$$y_{km} = v_k v_m, \tag{10}$$

in addition, if we pre-multiply (10) by $v_k v_m$ on both sides, the following result is reached:

$$\|y_{km}\|^2 = \|v_k\|^2 \|v_m\|^2. \tag{11}$$

Now, let us define the two auxiliary variables $u_k = \|v_k\|^2$ and $u_m = \|v_m\|^2$; they imply that (11) can be rewritten as follows:

$$\|y_{km}\|^2 = u_k u_m. \tag{12}$$

Please note that the product between the auxiliary variables u_k and u_m can be redefined by using its hyperbolic representation, i.e.,

$$\begin{aligned} \|y_{km}\|^2 &= u_k u_m, \\ &= \frac{1}{4}(u_k + u_m)^2 - \frac{1}{4}(u_k - u_m)^2, \\ 4\|y_{km}\|^2 + \|u_k - u_m\|^2 &= \|u_k + u_m\|^2, \\ \left\| \begin{matrix} 2y_{km} \\ u_k - u_m \end{matrix} \right\| &= u_k + u_m. \end{aligned} \tag{13}$$

Remark 3. *Observe that Expression (13) continues being non-convex due to the equality restriction; however, to obtain a conic convex relaxation, the equality symbol can be replaced with the lower-equal symbol, as suggested in [28], which transforms this constraint into a second-order one.*

Relaxing the equality constraint (13) allowed us to recover the original variables as $y_{kk} = u_k$ and $y_{mm} = u_m$, which implies that (13) takes the following form:

$$\left\| \begin{matrix} 2y_{km} \\ y_{kk} - y_{mm} \end{matrix} \right\| \leq y_{kk} + y_{mm}. \tag{14}$$

According to the SOCP relaxation, as mentioned above, the exact MINLP formulation (1)–(9) can be rewritten as (15), which corresponds to an MI-SOCP reformulation for the problem of siting and dimensioning constant power sources in DC grids.

$$
\begin{aligned}
\text{Objective function:} \quad & \min p_{\text{loss}} = \sum_{\{km\} \in \mathcal{L}} g_{km}(y_{kk} - 2y_{km} + y_{mm}), \\
\text{Subject to:} \quad & p_k^{ss} + p_k^{dg} - p_k^d = \sum_{m \in \mathcal{N}} G_{km} y_{km}, \ \forall k \in \mathcal{N} \\
& p_{km} = g_{km}(y_{kk} - y_{km}), \ \forall \{km\} \in \mathcal{L} \\
& p_{km}^{\min} \leq p_{km} \leq p_{km}^{\max}, \ \forall \{km\} \in \mathcal{L} \\
& x_k p_k^{gd,\min} \leq p_k^{dg} \leq x_k p_k^{dg,\max}, \ \forall k \in \mathcal{N} \\
& \left(v^{\min}\right)^2 \leq y_{kk} \leq (v^{\max})^2, \ \forall k \in \mathcal{N} \\
& \sum_{k \in \mathcal{N}} p_k^{dg} \leq \alpha \sum_{k \in \mathcal{N}} p_k^d, \\
& \sum_{k \in \mathcal{N}} x_k \leq N_{gd}^{ava}, \\
& x_k \in \{0, 1\}, \ \forall k \in \mathcal{N}
\end{aligned}
\tag{15}
$$

Remark 4. *The main characteristic of the proposed MI-SOCP formulation defined in (15) is that it guarantees the reaching of the global optimum if it is solved through the B&B method since each explored node is convex, i.e., it has a unique solution. This has been briefly explained in the next section.*

4. Strategy of Solution

As mentioned in the previous section, the SOCP is a sub-field of the convex optimization that works with the optimization models composed of linear affine and conic constraints [27,29]. Multiple algorithms are available for efficiently solving these types of large SOCP models in only microseconds. To exemplify the structure of a conic constraint, let us define a general second-order cone as follows:

$$\|x\| \leq z, \tag{16}$$

where $x \in \mathbb{R}^n$ and $z \in \mathbb{R}$, and $\|x\|$ are the Euclidean norm, i.e., norm-2, of the vector x. Figure 1 shows a second-order cone in $\mathbb{R}^3$, clearly a convex set [30]. For further details about optimization convex, see references [31,32].

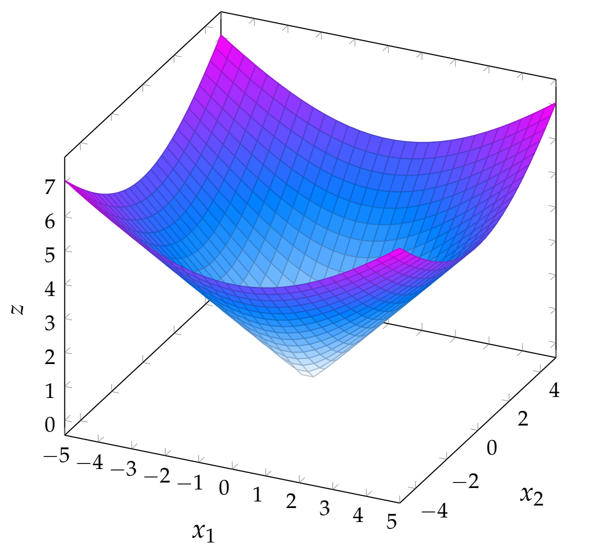

Figure 1. Schematic representation of an SOCP constraint $\Omega = \{\|x\| \leq z\}$. with $x \in \mathbb{R}^2$ and $z \in \mathbb{R}$ [30].

The problem of the optimal siting and dimensioning of power sources in DC grids can be addressed with a combination of the SCOP formulation and the B&B algorithm. In general terms, an MI-SOCP takes the following structure:

$$\|A_i x + b_i\| \leq \alpha_i^\top x + \beta_i^\top z_i + \theta_i, \tag{17}$$

where decision variables are composed by a combination of continuous and binary ones (i.e., x and z); A_i are real matrices; b_i, α_i and β_i are real vectors; and θ_i refers to the constants for each constraint i.

The solution of the MI-SOCP with the general structure (18) can be reached as follows: (*i*) each iteration of the B&B method defines the combination of the binary variables associated with the power sources' placement (exploration node); (*ii*) for each one of the explored nodes is solved the SOCP relaxation, which deals with the problem of power sources' sizing. This method benefits from the properties of the SOCP problems related to convexity and the fast convergence of the interior-point methods [33], and guarantees the finding of the optimal global solution at each node. Figure 2 illustrates the schematic solution of an MI-SOCP problem with two binary variables.

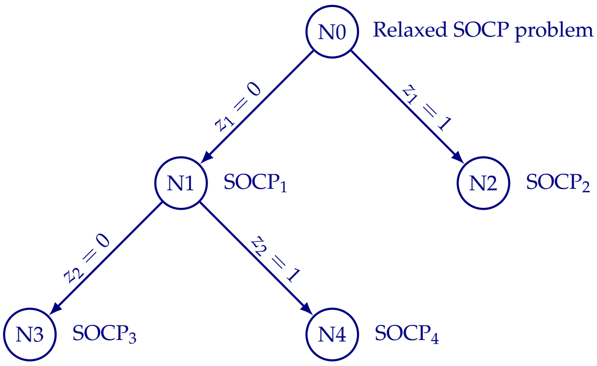

Figure 2. Exploration of the solution space with a branch and bound method to solve an MI-SOCP formulation with two binary variables.

Please note that SOCPs have been widely studied in the literature, for example, the authors of [34] reported a general introduction of SOCP and an extensive list of applications. The authors of [27] presented an overview of the general properties, the duality theory and the interior-point methods applied to SOCP problems. Interior-point techniques have been the most popular methods for solving this class of problems, offering good theoretical convergence properties [35] and efficient computational performance in numerical validations, such as SeDuMi [36], and SDPT3 [37].

Remark 5. *To solve the MI-SOCP formulation reported in (15) to locate and size constant power sources in DC grids, the CVX optimization package available for MATLAB and the MOSEK solver has been selected here [38].*

It is worth mentioning that in the case of the optimal power flow problems, we can ensure that the optimal solution of the original NLP model is the same one reached by conic approximations, as demonstrated in [39]; however, there exist multiple simple NLP programming with few variables, as in the case of the parametric estimation in photovoltaic modules [40], single-phase transformers [41] or induction motors [42], in which it is not possible to obtain conic equivalents. In this sense, the main limitation of the conic programming is its application to NLP models that only contain products between continuous variables.

5. Test Feeders

The computational testing of the proposed MI-SOCP reformulation was done in two conventional DC test feeders. These had 21 and 69 buses. These test feeders have been reported in the literature for addressing the studied problem with MINLP models [9] and heuristic methods [3]. The information regarding these distribution grids has been presented below.

5.1. 21-Bus Test Feeder

This is an electrical DC distribution grid conformed by 20 lines and 21 nodes in a radial connection (see Figure 3). The slack node (i.e., voltage-controlled source) is located at node 1 and supports a voltage magnitude of 1.00 pu [3]. The electrical parameters of this test system, i.e., line and load parameters, have been listed in Table 1 [43].

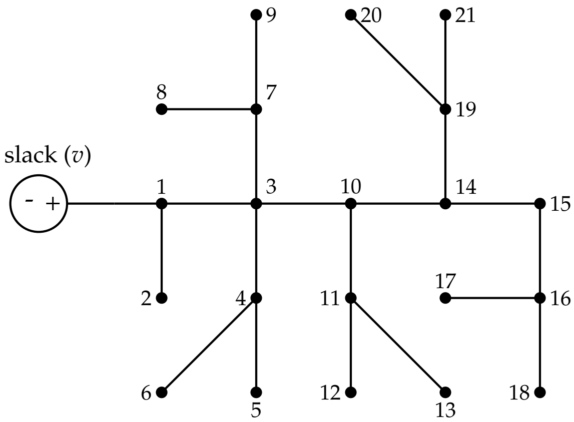

Figure 3. Grid configuration of the 21-bus test system.

Table 1. Branch and load information of the 21-bus test system.

Bus i	Bus j	R_{ij} (pu)	P_j (pu)	Bus i (pu)	Bus j (pu)	R_{ij} (pu)	P_j (pu)
1	2	0.0053	0.70	11	12	0.0079	0.68
1	3	0.0054	0.00	11	13	0.0078	0.10
3	4	0.0054	0.36	10	14	0.0083	0.00
4	5	0.0063	0.04	14	15	0.0065	0.22
4	6	0.0051	0.36	15	16	0.0064	0.23
3	7	0.0037	0.00	16	17	0.0074	0.43
7	8	0.0079	0.32	16	18	0.0081	0.34
7	9	0.0072	0.80	14	19	0.0078	0.09
3	10	0.0053	0.00	19	20	0.0084	0.21
10	11	0.0038	0.45	19	21	0.0082	0.21

It is worth mentioning that the information reported in Table 1 was calculated with a voltage base of 1.0 kV, and a power base of 100 kW. Please note that with these values we calculated the base of the resistance to be 10 Ω.

5.2. 69-Bus Test Feeder

The 69-bus test system is an electrical distribution grid conformed by 68 lines and 69 nodes connected in a radial form and operated with 12.66 kV in the root node (see Figure 4) [44]. Here, we consider the DC adaptation of this test feeder as informed in [3,8,43]. The data about nodal and branch parameters has been listed in Table 2. Mark that the information reported in Table 1 was calculated with a voltage base of 12.66 kV and a power base of 100 kVA.

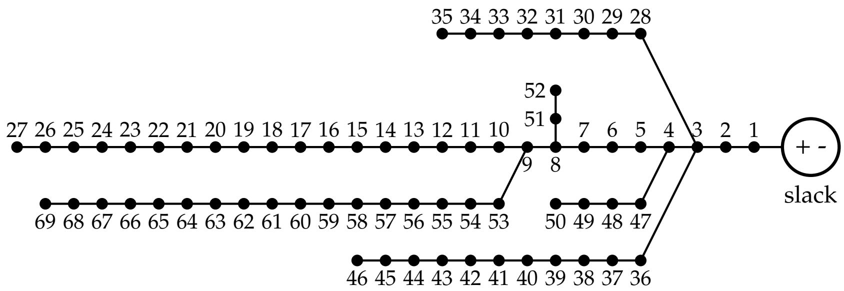

Figure 4. Grid configuration of the 69-bus test system.

Table 2. Branch and load information of the 69-bus test system.

Bus i	Bus j	R_{ij} (pu)	P_j (pu)	Bus i (pu)	Bus j (pu)	R_{ij} (pu)	P_j (pu)
1	2	0.0005	0	3	36	0.0044	26
2	3	0.0005	0	36	37	0.0640	26
3	4	0.0015	0	37	38	0.1053	0
4	5	0.0215	0	38	39	0.0304	24
5	6	0.3660	2.6	39	40	0.0018	24
6	7	0.3810	40.4	40	41	0.7283	102
7	8	0.0922	75	41	42	0.3100	0
8	9	0.0493	30	42	43	0.0410	6
9	10	0.8190	28	43	44	0.0092	0
10	11	0.1872	145	44	45	0.1089	39.22
11	12	0.7114	145	45	46	0.0009	39.22
12	13	1.0300	8	4	47	0.0034	0
13	14	1.0440	8	47	48	0.0851	79
14	15	1.0580	0	48	49	0.2898	384.7
15	16	0.1966	45	49	50	0.0822	384.7
16	17	0.3744	60	8	51	0.0928	40.5
17	18	0.0047	60	51	52	0.3319	3.6
18	19	0.3276	0	9	53	0.1740	4.35
19	20	0.2106	1	53	54	0.2030	26.4
20	21	0.3416	114	54	55	0.2842	24
21	22	0.0140	5	55	56	0.2813	0
22	23	0.1591	0	56	57	1.5900	0
23	24	0.3463	28	57	58	0.7837	0
24	25	0.7488	0	58	59	0.3042	100
25	26	0.3089	14	59	60	0.3861	0
26	27	0.1732	14	60	61	0.5075	1244
3	28	0.0044	26	61	62	0.0974	32
28	29	0.0640	26	62	63	0.1450	0
29	30	0.3978	0	63	64	0.7105	227
30	31	0.0702	0	64	65	1.0410	59
31	32	0.3510	0	65	66	0.2012	18
32	33	0.8390	10	66	67	0.0047	18
33	34	1.7080	14	67	68	0.7394	28
34	35	1.4740	4	68	69	0.0047	28

5.3. Implementation Characteristics of the Test Feeders

For the numerical implementation of both test feeders, the following characteristics were taken into account: (i) the possibility of installing three DGs in both test feeders, with single capacities of 1.5 pu for the 21-bus test system and 12 pu for the 69-bus test feeder; (ii) the admissible active power injection via distributed generation was fixed as 60% in both test feeders, where the percentage of penetration was related to the total power consumption of each test feeder.

6. Computational Implementation

The developed optimization model based on MI-SOCP formulation was programmed in the MATLAB 2020*a* with the help of the CVX package and the MOSEK solver in a personal computer with an INTEL(R) Core(TM) i7-7700 2.8-GHz processor and 16.0 GB of RAM, running on a 64-bit version of Microsoft Windows 10 Single language. To validate our mixed-integer convex reformulation's robustness and effectiveness for the optimal siting and dimensioning of constant power sources in DC grids, we compared our model with multiple MINLP solvers available in the GAMS optimization tool [10].

6.1. Solution under the Peak Load Condition

Table 3 reports the optimal placement and sizing of the constant power sources in both DC distribution grids.

Table 3. Comparative results between the proposed MI-SOCP approach and the GAMS package.

21-Bus Test Feeder			
Solver	**Location (Nodes)**	**Size (pu)**	p_{loss} **(pu)**
ALPHAECP	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
ANTIGONE	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
BARON	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
BONMIN	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
DICOPT	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
SBB	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
MI-SOCP	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
69-Bus Test Feeder			
Solver	**Location (Nodes)**	**Size (pu)**	p_{loss} **(pu)**
ALPHAECP	$\{16, 61, 64\}$	$\{5.0537, 12.0000, 5.7763\}$	0.0425
BARON	$\{11, 61, 69\}$	$\{8.0194, 12.0000, 4.2392\}$	0.0812
BONMIN	$\{17, 61, 64\}$	$\{4.9245, 12.0000, 5.7944\}$	0.0414
COUENNE	$\{52, 60, 61\}$	$\{5.8706, 6.2416, 12.0000\}$	0.1216
DICOPT	$\{17, 61, 67\}$	$\{5.0311, 12.0000, 5.0797\}$	0.0524
SBB	$\{18, 61, 63\}$	$\{4.8851, 12.0000, 6.0630\}$	0.0458
MI-SOCP	$\{17, 61, 64\}$	$\{4.9245, 12.0000, 5.7944\}$	0.0414

The results reported in Table 3 yield the following observations:

- ✓ All the comparative solvers in the 21-bus test system reach the same numerical solution reported by the proposed MI-SOCP model, which corresponds to the final power losses of about 3.06 kW. This confirms that nodes 9, 12 and 16 are the ones nodes associated with the global solution of the problem of siting and dimensioning of DGs in DC grids.
- ✓ For the 69-bus test feeder, we can highlight that the proposed MI-SOCP reformulation finds the global solution with 4.14 kW of final power losses. The BONMIN solver is the only solver that reaches this solution since the other optimization tools in GAMS are trapped in local solutions. The best places for locating constant power sources in this test feeder correspond to the nodes 17, 61 and 64, with a total power injection of about 2294.8 kW.
- ✓ As for the power losses improvement in the 21-bus test system regarding the base case (0.2760 pu), this was about 88.91%, while in the 69-bus test feeder (base case with 1.5385 pu) this reduction was about 97.31%.
- ✓ Even though the ANTIGONE solver reaches the global solution for the 21-bus test system, it fails in the 69-bus test system because it cannot find a combination of three nodes to minimize power losses; it only identifies one, leaving the other two options free. Therefore, this solver was not reported in the second test feeder.

Figure 5 shows the power losses enhancement found by each one of the solvers tested in GAMS and our proposed MI-SOCP for the 69-bus test system.

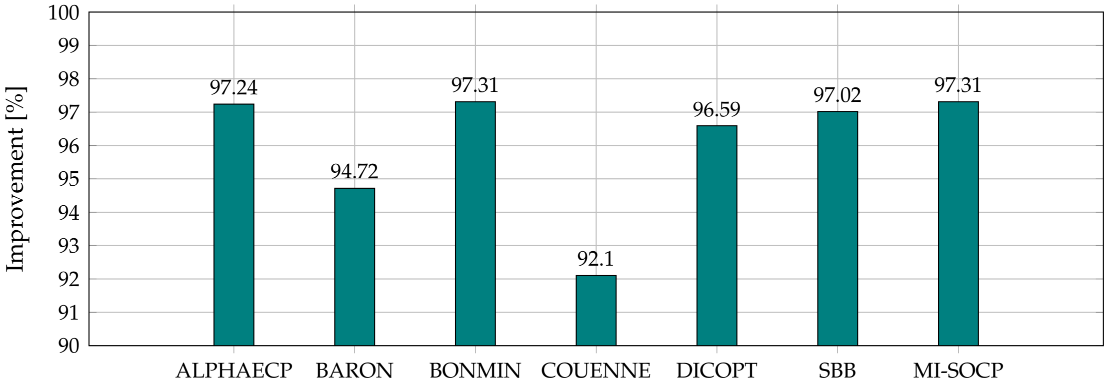

Figure 5. Improvement of the power losses using GAMS solvers and the proposed MI-SOCP approach.

Please note that all the solution techniques presented in Figure 5 allow for improvements of higher than 90% in power losses. Nevertheless, the best solution was found by the BONMIN and the MI-SOCP proposal, and the COUENNE solver had the worse result.

Remark 6. *The solutions reported by the GAMS solvers, as shown in Table 3, demonstrate that the MINLP model presented from (1) to (9) is complex to solve, and that it is impossible to ensure the global optimum achievement by using the GAMS optimization tools since these can be trapped in local solutions. However, the proposed MI-SOCP allows for the reaching of the global optimum by combining the B&B method and SOCP optimization, as presented in Table 3, for the 21- and 69-bus test feeders.*

It is worth mentioning that in this research, we decided not to compare the proposed MI-SOCP model with metaheuristic optimization techniques as these are highly dependent on the programmer. These also require the tuning of multiple parameters, while this is not the case with the proposed model, which has exact mathematical methods. In addition, validation of metaheuristic methods requires statistical tests since their random procedures result in different solutions at each running. Thus, these are not recommended in problems where convex optimization is applied, as the one studied in this article.

Remark 7. *As for the comparison of the proposed MI-SOCP with the results reported in [3] and [8], where convex optimization methods based on semidefinite and sequential quadratic programming models were combined with a heuristic algorithm based on hyperplanes, we can affirm that our MI-SOCP approach allows for the reaching of the global optimum. It has better performance as regards the metaheuristics presented in those papers.*

To demonstrate that the solutions found by the MI-SOCP formulation corresponded to the global optima, we evaluated all the possible combinations of three constant power sources in the 21- and 69-bus test feeder, which yielded about 1140 and 50,116 options, respectively. These exhaustive searches demonstrated that the nodes 9, 12 and 16 are the best places for the power sources in the 21-bus test feeder, and nodes 17, 61 and 64 are the best possible solution in 69-bus test feeder.

6.2. Solution Considering the Installation of Photovoltaic Generators

In this section, we discuss the ability of the proposed MI-SOCP formulation to site and size renewable distributed generators in DC grids, taking into consideration the daily load variations in the 69-bus test system (see mathematical model (16)). The curves analyzed have been presented in Figure 6. In this simulation, we considered one to three distributed generators available for installation without the limitations associated with the renewable energy penetration.

$$
\begin{aligned}
\text{Objective function:} \quad & \min E_{\text{loss}} = \sum_{t \in \mathcal{T}} \sum_{\{km\} \in \mathcal{L}} g_{km}(y_{kk,t} - 2y_{km,t} + y_{mm,t}), \\
\text{Subject to:} \quad & p_{k,t}^{ss} + p_{k,t}^{dg} - p_{k,t}^{d} = \sum_{m \in \mathcal{N}} G_{km} y_{km,t}, \ \forall \{k \in \mathcal{N},\ t \in \mathcal{T}\} \\
& p_{km,t} = g_{km}(y_{kk,t} - y_{km,t}), \ \forall \{\{km\} \in \mathcal{L},\ t \in \mathcal{T}\} \\
& p_{km}^{\min} \leq p_{km,t} \leq p_{km}^{\max}, \ \forall \{\{km\} \in \mathcal{L},\ t \in \mathcal{T}\} \\
& x_k p_{k,t}^{gd,\min} \leq p_{k,t}^{dg} \leq x_k p_{k,t}^{dg,\max}, \ \forall \{k \in \mathcal{N},\ t \in \mathcal{T}\} \\
& \left(v^{\min}\right)^2 \leq y_{kk,t} \leq (v^{\max})^2, \ \forall \{k \in \mathcal{N},\ t \in \mathcal{T}\} \\
& \sum_{k \in \mathcal{N}} \max_{t \in \mathcal{T}} \left\{ p_{k,t}^{dg} \right\} \leq \alpha \sum_{k \in \mathcal{N}} \max_{t \in \mathcal{T}} \left\{ p_{k,t}^{d} \right\} \\
& \sum_{k \in \mathcal{N}} x_k \leq N_{gd}^{ava}, \\
& x_k \in \{0, 1\}, \ \forall k \in \mathcal{N}
\end{aligned}
\tag{18}
$$

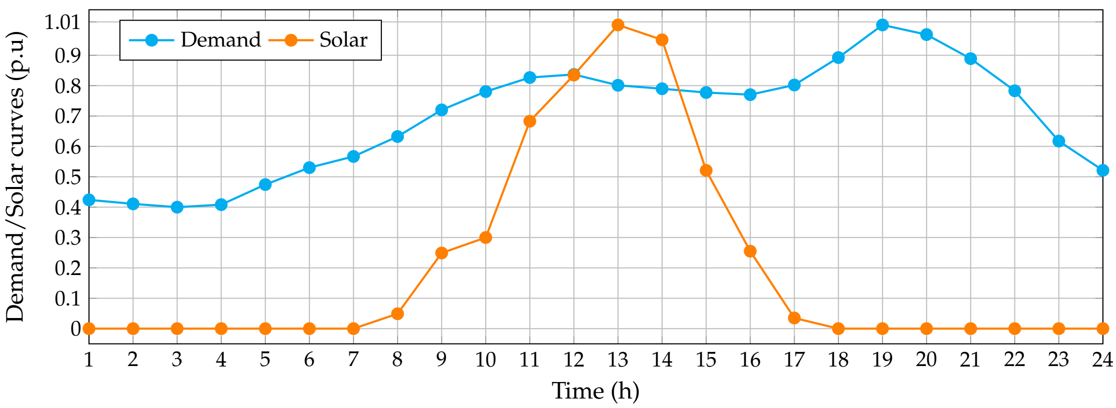

Figure 6. Daily behavior of the load and renewable generation.

Table 4 presents the numerical results associated with the installation of PV sources in DC grids using the proposed MI-SOCP reformulation.

Table 4. Siting and dimensioning of PV sources in DC grids using the proposed MI-SOCP.

No. of PVs	Location (Nodes)	Size (kW)	En. Losses (kWh/Day)	Proc. Time (s)
0	$\{-\}$	$\{-\}$	1838.5218	63.73
1	$\{61\}$	$\{2119.4980\}$	1255.0827	75.70
2	$\{17,\ 61\}$	$\{535.8994,\ 2025.81399\}$	1215.2164	95.22
3	$\{18,\ 49,\ 61\}$	$\{535.3449,\ 864.2630,\ 2025.4377\}$	1208.7995	233.16

The numerical results presented in Table 4 show the following: (i) in all the combinations of PV sources, node 61 is identified in the set of optimal solutions with power injections higher than 2000 kW; (ii) an injection of 2119.4980 kW at node 61 allows for the reduction of the daily energy losses by about 31.73% with only one PV source; however, three PV sources in nodes 18, 49 and 61 inject about 3425.0456 , leading to a daily energy losses reduction of about 34.25%. This difference corresponds to 46.2832 kWh/Day which no justifies an increment about 1000 kW in the PV capacity; (iii) the required processing time in the solution of the multi-period MI-SOCP increases as a function of the number of PV sources that will be located, since the dimension of the discrete space increases rapidly. In the case of one PV source, the dimension of the solution space is 68 nodal options, while in the case of three PVs the size of the solution space is 50,116 nodal combinations.

Please note that the numerical results reported in Table 4 confirm the robustness and effectiveness of the proposed MI-SOCP formulation in locating and sizing power sources in DC grids in one or over multiple periods of analysis; on the other hand, the software GAMS is trapped in local optimal solutions.

7. Conclusions and Future Works

This research paper proposed an MI-SOCP reformulation for the optimal siting and dimensioning of constant power sources in DC grids that allows for the reaching of global optimal solutions, which is not possible with the exact MINLP model. This solution of the proposed MI-SOCP model was reached by combining the classical branch and bound method with interior-point algorithms for SOCP problems. This combination guarantees the global optimum since each node explored by the branch and bound algorithm is, in fact, convex, i.e., it has a unique global solution.

Computational validations in the 21- and 69-bus test feeders showed that our proposed (i.e., the MI-SOCP approach) model has the best numerical performance when compared to multiple MINLP solvers in GAMS optimization since these reached the global solution only in the case of the 21-bus test feeder; they were stuck in locally optimal solutions in the 69-bus test feeder case due to the non-convex properties of the exact MINLP model.

The main result obtained after applying the proposed MI-SOCP model for optimal siting and dimensioning of constant power sources in the DC networks was the global optimization properties of this method. This demonstrated that this kind of MINLP problem can be addressed using exact mathematical optimization without going back to metaheuristic optimization techniques that cannot guarantee global optimal solution finding at every running. In addition, simulations considering daily load curves and renewable generation demonstrated this model's applicability to distribution grids with high penetration of PV sources; another benefit of this model is that it requires low computational effort.

Future research can explore the following points: (*i*) the application of the proposed MI-SOCP model to solve the problem of the optimal location of capacitor banks and distributed generators in AC grids; (*ii*) the formulation of the economic dispatch problem, taking into consideration batteries and renewable generation in AC distribution networks, via SOCP representations; (*iii*) the study of the optimal location of batteries and renewables in DC grids with an MI-SOCP representation.

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Conflicts of Interest: The authors declare no conflicts of interest.

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