VRP Model with Time Window, Multiproduct and Multidepot

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With the increase in the transfer of products in supply chains, the organization of routes requires a complex allocation insofar as different environmental variables are considered, and VRP models are an efficient tool for the solution of routing systems of low, medium and high complexity. In this paper, we developed a vehicle routing model with hard time window, multidepot, multiproduct and heterogeneous fleet for the minimization of the distance travelled. We applied the model to a case study of a company that distributes water bottles and bales in which we made a new distribution of delivery schedules by order applied Pareto analysis. We obtained optimal computational results using exact methods in a very short computational time and minimizing the distance to 35.08 % of the current route.

Keywords: Pareto analysis; mathematical model; vehicle routing; optimization

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1. Introduction

The need to comply with the requirements demanded by customers, has led companies to innovate every day in order to remain competitive in the market, one of the essential aspects being on-time delivery of the products to their final or partial destination, which is affected by the usual transport problems [1]. The transport process can be translated as one of the most important aspects in the supply chains, because your organization can suppose the success of a company, but, at the same time, it is one of the highest logistic costs [2]

This process has several variables that must be considered when defining the best way to carry out the movement of goods at any link in the supply chain, be it supply, internal transport in production or in distribution. Within these variables we have the travel time, the distance traveled, the availability of vehicles, the capacity, the allocated costs, among others [3].

In this sense, the VRP (Vehicle Routing Problem) Models are an efficient tool to provide solutions to transport problems that consider various variables oriented to the real context [4], and which serve as support for decision-making when wanting to obtain answers oriented to optimization [2].

According to this, with the implementation of a VRP Model, competitive advantages are generated in the company, which translate into lower costs and efficiency in the transport process [5]

In this research, a mixed-integer linear programming mathematical model was developed, which, in consideration of the contextualized problem of a distribution company of bottles and water packs, presented the variants of multi depot, multi product, hard time windows and heterogeneous vehicle fleet to optimize the distance traveled.

Due to the complexity of the model, we decided to perform a Pareto analysis to prioritize the set of customers. In addition, we decided to propose a new organization of delivery schedules, in order to obtain optimal answers in low computational times, through exact methods.

The main contributions of this article are as follows: i. On the basis of the traditional vehicle routing problem, we considers many factors such as hard time windows, multiple depots, multiple products and heterogeneous fleet to solve a real scene of product delivery in supply chain. ii. We carried out a Pareto analysis, organizing the customers in an accumulated sum according to the total of products they demand selecting as few customers as possible, but they have a large number of orders. At the same time, we designed the order can be prioritized to be delivered from the amount of minutes assigned to the time window. These two points make the problem more realistic and the calculation time of the problem is shorter.

The remainder of this study is organized as follows. We present a brief literature review in Section 2. In Section 3 provides a mathematical model of VRP multideposit, multiproduct, hard time windows and heterogeneous fleet for the problem identified. The computational experiments are analyzed through a case study conducted at the distribution company of bottles and water packs in Section 4. Section 5 summarizes the conclusions and discussions. Finally, some lines for further research are provided in Section 6.

2. Vehicle Routing Problems

2.1. Overview of the VRP

The first work developed on routing problems was carried out by [6], who solved a fuel distribution problem. However, the work that included the phrase "vehicle routing" for the first time was the one proposed by [7]. Prior to these authors [8] constructed an effective algorithm for the solution of this type of problems, called savings algorithm or Clarke and Wright, considering that they are part of the Np-Hard family, which cannot be resolved in a polynomial time, and increase in complexity as the number of nodes grows, making the exact methods inefficient [9].Basically, the vehicle routing problem consists of the assignment of one or several vehicles to certain routes for the fulfillment of customer demands, which start from a deposit and return to it at the end of the route [10].

Whenever it comes to modeling a real routing problem (i.e., organizations with product distribution processes to meet their customers' requirements), it is necessary to consider a set of variants that have been included, throughout history, in the development of this type of problems. Generally, considerations are addressed to customers, vehicles and depots [11].

In this sense, the main variants of the VRP —as described by [12] could be grouped into two types: homogeneous VRP, where all the nodes handle exactly the same characteristics; and heterogeneous VRP, where these characteristics are different (e.g., type of vehicles, capacity, time windows, stochastic, and depots).

Within these two great classifications, several variants that can be addressed individually or articulated with others emerge, according to the problem that is trying to solve. [10] illustrated different variants that have been worked on throughout the history of VRP. Among them, heterogeneous fleet HFVRP [13], time windows VRPTW [14], multidepot MDVRP [15], capacitated CVRP [16], stochastic SVRP [17], pickups and deliveries VRPPD [18], and new green variants such as the EMVRP (VRP with energy minimization) and the PRP (pollution routing problem) introduced by [19] and [20] respectively.

2.1.1. Solutions Methods

Throughout the literature reviewed, it is evident that, for the solution of this type of problems, exact and approximate methods are used, according to the complexity of the problem. The exact methods start from formulations, such as linear or mixed whole programming, dynamic programming and direct tree search, and conclude in obtaining feasible or integer results [21]. Within the exact algorithms, we can find the branch-and-bound and the branch-and-cut [22]. On the other hand, approximate methods are found, whose purpose is to obtain good solutions in low computational times, but which do not ensure optimality. These are divided into three main categories: heuristics (e.g., constructive, improvement, and relaxation techniques), metaheuristics (e.g., genetic algorithms, variable neighborhoods search, simulated annealing, taboo search, ant colony, particle swarm optimization, among others) and hybrid algorithms [23].

2.2. Variants of the VRP applied to the model

As mentioned above, the VRP has a large number of variants that increase complexity and computational time to obtain solutions. In this article, we developed a VRP model that considers multiple products, multiple depots, hard time windows and heterogeneous and capacitated vehicle fleet.

The multi depot variant (MDVRP) models a situation where the vehicles assigned to the routes can start from different deposits or supply points, to comply with the visits established to several customers located in different points of an area and, in the end, they must return to the warehouse from which they left. Situation that is presented repeatedly in various organizations and logistics operations [24].

Time windows (VRPTW) intend to model a routing problem where it is necessary to consider the service times s_i that the vehicle takes to perform downloads and/or loads in the same node [24]. Likewise, each customer is associated with a time window $[a_i, b_i]$, in which the service must be performed, considering the time of arrival at the node [11]. The Time Windows can be soft or hard. The first ones (VRPSTW) allow the violation of the arrival times, generating a penalty that translates into costs [14]. In the case of hard time windows (VRPHTW), it is established that they cannot be violated [25, 26].

The first types of VRP assumed a fleet of homogeneous vehicles. However, in real-world transport practical problems, companies use different types of vehicles in their fleet, which have different load capacities to meet the demand of their customers [23]. With this, it is necessary to address the problem of routing with heterogeneous fleet (HFVRP), with which it is tried to satisfy the classic restrictions of the VRP by means of the use of a limited or unlimited fleet of vehicles with capacities different [13]. The organizations aim to diversify their products to increase their market share and competitiveness [27]. Therefore, it is normal that, in the transport process, the capacity of the vehicles must be configured in order to order various types of products. For this type of problems, the multi-product route variation is applied, which has been very little addressed in the literature reviews from the main divisions of the VRP. The multiproduct routing problem model a system in which the delivery of several products to a set of customers must be satisfied, so the capacity of the vehicle must be configured according to the specifications of each product, to meet the demands [28, 29].

3. Description of the Mathematical Model

The model can be represented through a directed graph $G = (N, A)$, where the set of vertices N would be divided into two subsets that would be $N_c = \{n_1, n_2, n_3, ..., n_c\}$ the customers and $N_E = \{n_{c+1}, n_{c+2}, n_{c+3}, ..., n_{c+e}\}$ the headquarters of the company. Fig. 1.

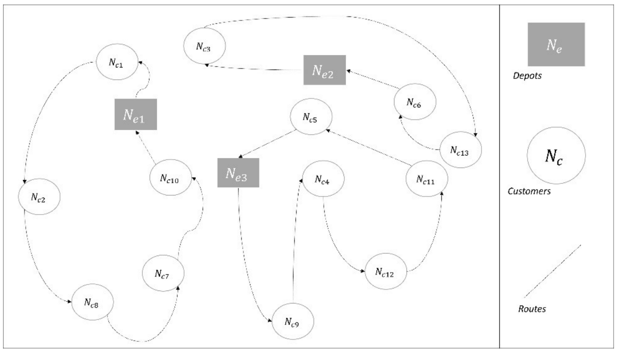

Fig. 1. Graphic representation of multiproduct and multi-depot VRP

The model considers a heterogeneous fleet denoted by $k = \{1, 2, 3, .., k\}$, which has a limited capacity denoted by Q_{km}, These vehicles must perform the service to each customer i, in a service time S_i, to collect a demand d_{im}, of products m, in a hard time window $[a_i, b_i]$. Similarly, each edge $(i, j)\epsilon A$, where $i, j\epsilon N$ and $i \neq j$, is related to a travel time TT_{ij} and a travel distance C_{ij}. In the case of companies, no collection service is being offered, so for the set of N_e, we have $S_i = 0$ and at the same time $d_i = 0$.

The main objective is to minimize the distances traveled by the vehicles, considering that any route made by a vehicle must start and finish at the same company. The service must be performed within the time window established for each customer. Otherwise, the vehicle must visit another customer where it can perform the service. The maximum capacity of the vehicles assigned to the routes must be respected. A vehicle is assigned to a single company.

Next, the notation of the constructed mathematical model is detailed.

Indexes and Sets

$i = j = h$: Node index

m: Product index

k: Vehicle index

N_c: Set of customers $\{1, 2, 3..., c\}$

N_e: Set of companies $\{1, 2, 3..., e\}$

K: Fleet of vehicles $\{1, 2, 3..., k\}$

P : Set of products $\{1, 2, 3..., m\}$

Parameters

C_{ij}: Travel distance from node i to node j

d_{im}: Demand from node i for the product m

TT_{ij}: Travel time to go from node i to node j

S_i: Customer service time i

Q_{km}: Vehicle capacity k for product m

Tr_k: Maximum route time for the vehicle k

M : Very large number to avoid negativity and selection of customer-to-customer points

Variables

$$X_{ijk} = \begin{cases} 1 \text{ If the pair of nodes } i \text{ and } j \text{ are in the route of} \\ \text{vehicle k} \\ 0 \quad\quad \text{other} \end{cases}$$

T_i: Time in which the vehicle arrives at the Node i

Y_{ijkm}: Quantity of product m delivered by a vehicle k from i to j

R_k: Calculation of vehicle route time k

3.1. Formulation of mixed integer linear programming

The proposed model has the following characteristics: multiobjective, multiproduct, heterogeneous fleets, capacitated and hard time windows. Where the objective function that aims to minimize the distance traveled by vehicles in the assigned routes.

Objective function

$$MinZ = \sum_i \sum_j \sum_k C_{ij} X_{ijk} \quad (1)$$

The model is subject to the following constrains:

$$\sum_{j\epsilon N:j\neq i}\sum_{k\epsilon K} X_{ijk} = 1; \forall_i \epsilon N_C \qquad (2)$$

The model is subject to the following constrains:

$$\sum_{j\epsilon N:i\neq h} X_{ihk} = \sum_{j\epsilon N:i\neq h} X_{hjk}; \forall k\epsilon K, \forall h\epsilon N \qquad (3)$$

$$\sum_{d\epsilon N_e}\sum_{j\epsilon N_c} X_{djk} \leq 1; \forall k\epsilon K \qquad (4)$$

$$\sum_{i\epsilon N:i\neq j}\sum_{k\epsilon K} Y_{ijkm} - \sum_{i\epsilon N:i\neq j}\sum_{k\epsilon K} Y_{ijkm} = d_{jm}; \forall j\epsilon N_c \qquad (5)$$

$$Y_{jikm} = 0; \qquad \forall i\epsilon N_e, \forall j\epsilon N_c, \forall k\epsilon K, \forall m\epsilon P \qquad (6)$$

$$Y_{jikm} \leq MX_{ijk}; \forall i, j\epsilon N : i \neq j, \forall k\epsilon K, \forall m\epsilon P \qquad (7)$$

$$Y_{ijkm} \leq Q_{km} X_{ijk}; \forall i, j\epsilon : i \neq j, \forall k\epsilon K \qquad (8)$$

$$T_j \geq TT_{ij} X_{ijk}; \forall i\epsilon N_e, \forall i\epsilon N_c, \qquad \forall k\epsilon K \qquad (9)$$

$$T_j \geq T_i + S_i + TT_{ij} - M(1 - X_{ijk}); \forall i, j\epsilon N_c : i \neq j, \forall k\epsilon K \qquad (10)$$

$$R_k \geq T_i + S_i + TT_{ij} - M(1 - X_{ijk}); \forall i\epsilon N_c, \forall j\epsilon N_c, \forall k\epsilon K \qquad (11)$$

$$R_k \leq Tr_k; \forall k\epsilon K \qquad (12)$$

$$a_i \leqslant T_i \leq b_i; \forall i\epsilon N \qquad (13)$$

$$X_{ijk}\epsilon \{0, 1\} \qquad (14)$$

$$Y_{ijkm}\epsilon Z^+ \qquad (15)$$

$$T_i, R_k\epsilon R^+ \qquad (16)$$

The objective function 1 aims to minimize the distance covered in all the routes. Constrain 2 ensures that each customer is visited exactly once by a vehicle. Constrain 3 ensures that each customer is visited and left by the same vehicle. Constrain 4 ensures that each vehicle must leave exactly one deposit. Constrain 5 ensures the service in each customer. Loads delivered when returning to the origin is set to zero at constraint 6. Constrains 7 ensure that if a vehicle makes an arc ij a quantity must be discharged. The capacity constrain is imposed in 8. Constrains given in 9 and 10 ensure the travel times from the deposit to the first customer and between customers respectively. Vehicle route time is determined in equation 11 and limited by constraint 12. Time window constraint are embodied in 13. Finally, the types of variables are determined in 14, 15 and 16.

4. Computational results

We solved the model by mixed integer linear programming with initial test instances and real instances. The model was programmed using GAMS and was solved by obtaining optimal solutions through the CPLEX library. A computer with Intel ^{R} Core ^{TM} i7-5500 2.4 GHz CPU, 4.00 GB RAM, 64 Bit operating system and Windows 7 Professional was used.

For the initial test instances, we consider a set of nodes $N = 6$, where $N_e = \{1, 2\}$, vehicles $k = 2$, and products $m = 2$ to perform model validation. Similarly, we consider the capacity of the vehicles, the arrival times before, arrivals after, maximum route time, service time and demands, as shown in Table 1.

The solutions obtained with these instances have an optimal distance of 4.33 km, complying with all the deliveries in each node, in a otal time of 30 minutes for the vehicle $k1$ and 33 minutes for $k2$, with a GAP of 0%. Fig. 2.

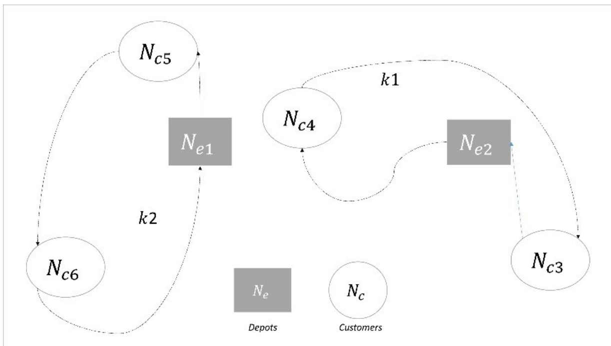

Fig. 2. Solution of the model for constructed instances

4.1. Solutions with real instances

We developed the model to be applied in a distribution company of treated water bottles and bales, which is located in the municipality of Sincelejo, Sucre, and has two deposits: the main one, where the entire administrative area is located, and the other, as an auxiliary warehouse. In total, the company has approximately 470 customers, made up of premises, companies, institutions and residential houses. Due to the complexity of the model, considered as NP- hard [30] we decided to carry out a Pareto analysis, organizing the customers in an accumulated sum, according to the total of products they demand, selecting only 46 customers.

In this sense, two constrains was applied for the Pareto analysis. First, using information of previous six month we calculate the total product sold and divided the total sales made to each of the 470 customers by the total orders to obtain the percentage representation. Subsequently, with

Table 1. Instances built for validation of the model.

Vehicles	Vehicle capacity	Nodes	Arrivals before	Arrivals after	Service time	Demand
K1	m1=30	N_e=1	0	0	0	m1=0,m2=0
	m2=10	N_e=2	0	0	0	m1=0,m2=0
		N_C=3	20	25	5	m1=3,m2=0
		N_C=4	16	20	5	m1=3,m2=1
K2	m1=15	N_C=5	14	22	5	m1=2,m2=0
	m2=5	N_C=6	20	25	5	m1=2,m2=0

the cumulative sum of percentage we identified that 187 customer correspond to 80% Fig.3.

To calculate the service time we take the data for each customer at the locations accompanying the current distribution route. We observed a problem derived from the scheduling of delivery schedules, for which a new organization of schedules with 1-hour slots was proposed, counted after starting the workday for reception of orders. There would be exactly 20 minutes to get the routes thrown by the model and an interval of 50 minutes for the vehicle to make the route. If an order cannot be delivered in the requested time, it can be sent in the next order, taking into account that the company has a maximum delivery period of 3 hours in compliance with its quality policy. This order can be prioritized to be delivered from the amount of minutes assigned to the Time window. Table 2

Given the complexity of the model and the number of customers, an optimal solution was obtained with a GAP of 0% at 13.15 hours, with the use of 3 of 4 vehicles available with which a total of 1751 was covered. Table 3.

The efficiency of the company consists in the organization of the daily routing, which is why, from the new organization of schedules, the model can generate optimal solutions with a small set of customers. This type of robust models present results in less computational time, being worked in a clustered way and not in a holistic way.

To verify the efficiency of the model in a clustered manner, we selected a real reference route formed by the customers $N_C = \{1,2,3,...,8\}$ that are served by the two depots $N_e = \{1,2\}$. The current route has a distance traveled of 10.8 km in a total time of 44.8 minutes. Fig.4.

In the previous route, only two vehicles were used. We assume the availability of the four vehicles that the company has, whose capabilities are evidenced in the previous Table 3, so that the model made the selection of the quantity and type of vehicle needed to optimize the route. For the eight customers, the demands of the products were $m1 = 3,3,2,1,6,1,1,6$ and $m2 = 2,0,0,2,0,0,0,1$. The model showed an optimal result with a traveled distance of 7.011 km a total time of 78 minutes (28 minutes for the vehicle $k1$ and 50 minutes for $k2$) and a GAP of 0%. Fig.5.

5. Conclusions and discussions

The organization of routes is one of the most common problems in transport processes, due to the several variables that must be considered, from the type of vehicles to the type of cargo and the conditions for unloading. With the development of this research, we built a vehicle routing model that groups the variants of multideposit, multiproduct, heterogeneous fleet and hard time windows, and evaluate it with real data from a company that distributes water bottles and bales.

The complexity of the model was observed, when trying to program the routing for all the prioritized customers by means of the Pareto analysis, because, when being considered Np-hard, with the increase of the nodes, the computational time for the execution of the model increases exponentially therefore, we conclude that it is better to work in a clustered manner by order of orders, through a new organization of schedules and deliveries that was proposed. To check the efficiency and optimality of the model, we select an order and accompany the route. Subsequently, we modeled the order obtaining an optimal route with a minimization of 35.08% of the distance traveled, evidencing that there is a notable improvement with the routing proposed by the designed model. For all this, we can see that the model developed in this research is considered an efficient tool, not only in terms of solving routing problems with small instances, but can be used as a basis for comparison to evaluate the performance and quality of solutions approximate methods such as heuristics and metaheuristics.

In this sense, when operations research (OR) is applied to solve real-life problems (i.e., as in the case of product distribution), it is common to design Np-hard mathematical models that become more complex as the size of input data increases. For these situations different solution approaches can be generated such as the application of

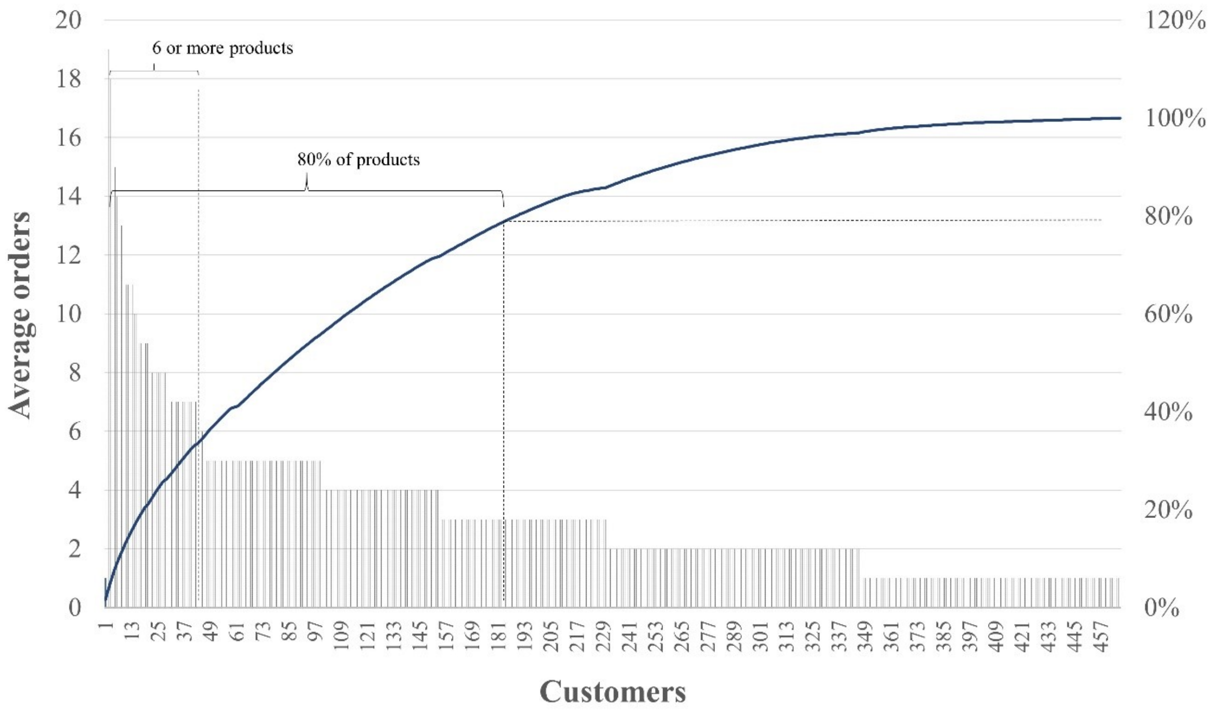

Fig. 3. Pareto diagram for 470 customers.

Table 2. Organization of order and service schedules.

Reception of orders		Time of organization of the route	Time of departure arrival of the vehicle		Maximum time delivery order (hours)
Start	End		Depot departure	Depot arrival	
8:00 a.m.	9:00 a.m.	20	09:20 a.m.	10:10 a.m.	3
9:00 a.m.	10:00 a.m.	20	10:20 a.m.	11:10 a.m.	3
10:00 a.m.	11:00 a.m.	20	11:20 a.m.	12:00 a.m.	3
11:00 a.m.	12:00 p.m.	20	2:20 p.m.	3:10 p.m.	3
2:00 p.m.	3:00 p.m.	20	3:20 p.m.	4:10 p.m.	3
3:00 p.m.	4:00 p.m.	20	4:20 p.m.	5:10 p.m.	3
4:00 p.m.	5:00 p.m.	20	5:20 p.m.	6:00 p.m.	3
5:00 p.m.	6:00 p.m.	20	8:20 a.m.	9:10 a.m.	3

Table 3. Instances built for validation of the model

Number of vehicles	Vehicle capacity	Time windows (min)		Route time (min)
		Before	After	
$K1$	m1=30 m2=10	5	180	195
$K2$	m1=30 m2=10			204
$K3$	m1=15 m2=5			209
$K4$	m1=15 m2=5			0

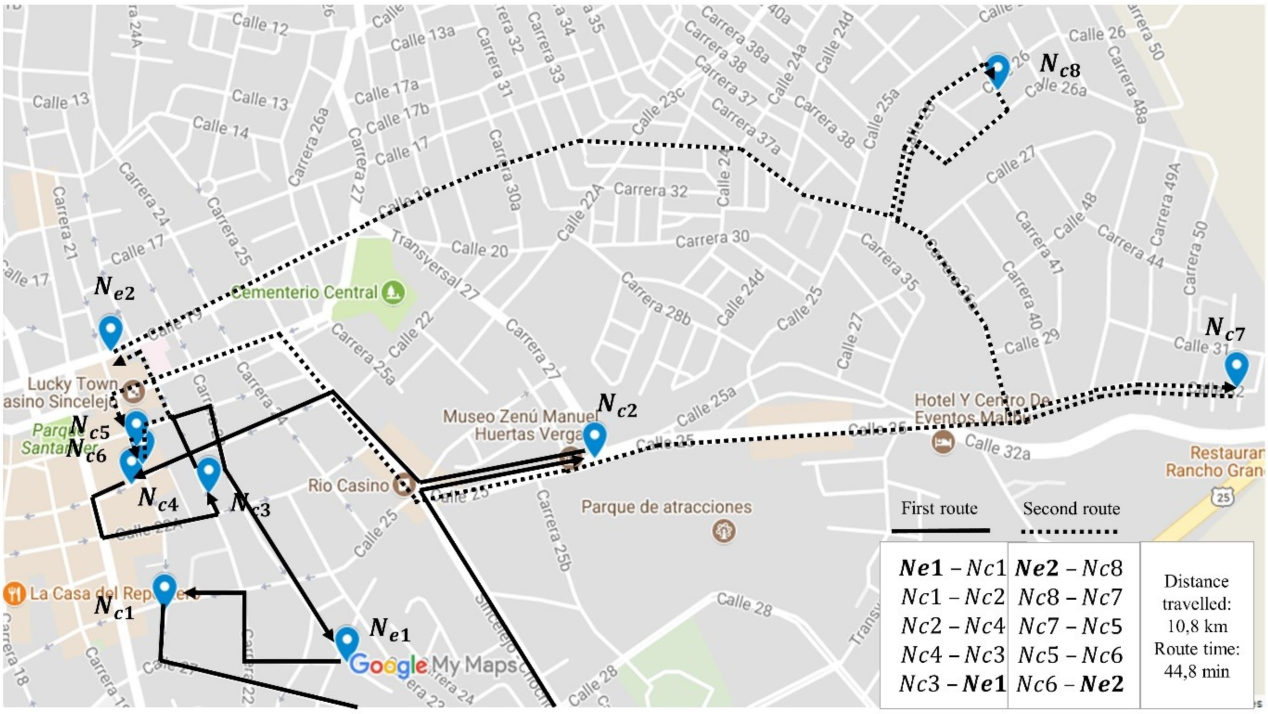

Fig. 4. Current route measured in terms of distance and travel time

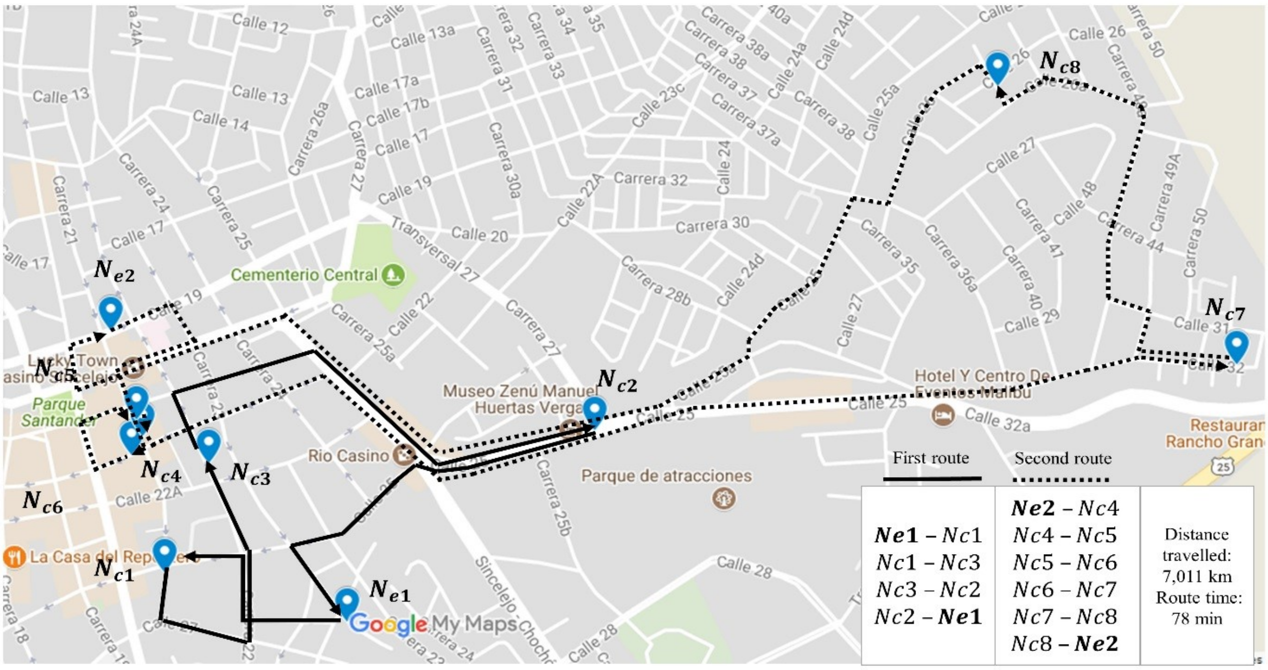

Fig. 5. Route obtained by the model.

approximate methods (heuristics and metaheuristics) that generate efficient solutions [31]. In the area of metaheuristics, the use of hybrid algorithms based on approximate and exact methods are a field of wide exploitation [32, 33] because they provide the advantages of each approach as the guarantee of obtaining quality solutions [34].

However, another solution approach is to design strategies that allow the problem to be relaxed in order to obtain optimal solutions using exact methods. In this paper, we developed a mathematical model Np-hard that generates solution in high computational time (13.15 hours) applied to the case study of the company considering all customers prioritized in Pareto analysis. In order to relax the problem and generate quick solutions for the calculation of short routes, we established an order time organization scheme that allowed us to make the company's delivery times more efficient and at the same time minimize the distances traveled.

6. Recommendations and future research

To obtain efficient solutions with large instances in a low computational time, the application of heuristic or metaheuristic algorithms is recommended, such as the hybrid algorithms that present good solutions for this type of problem. In the case of future research, we recommend the inclusion of costs in order to have better criteria for vehicle selection and route assignment. Likewise, we consider that the use of ecological variants is necessary due to the environmental problem derived to a large extent by the emission of greenhouse gases (GHG), in which transport presents an important contribution. On the other hand, we consider giving greater importance to the problems of routing multiproductos, because in the real distribution systems in the supply chains, it is very common to find this type of variables.

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