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To cite this article: L F Grisales-Noreña *et al* 2020 *J. Phys.: Conf. Ser.* **1448** 012012

Application of the backward/forward sweep method for solving the power flow problem in DC networks with radial structure

L F Grisales-Noreña[1], O D Garzon-Rivera[1], C A Ramírez-Vanegas[2], O D Montoya[3], and C A Ramos-Paja[4]

[1] Departamento de Electromecánica y Mecratrónica, Instituto Tecnológico Metropolitano, Medellín, Colombia
[2] Facultad de Ciencias Básicas, Universidad Tecnológica de Pereira, Pereira, Colombia
[3] Programa de Ingeniería Eléctrica e Ingeniería Electrónica, Universidad Tecnológica de Bolívar, Cartagena de Indias, Colombia
[4] Facultad de Minas, Universidad Nacional de Colombia, Medellín, Colombia

E-mail: `luisgrisales@itm.edu.co`

Abstract. This paper presents the application of the backward/forward sweep iterative method for solving the power flow problem in direct current networks with radial structure, considering resistive and constant power loads. The validation of the effectiveness and robustness of the proposed method is made by using six comparative methods proposed in literature for power flow analysis in radial direct current networks: Gauss-Jacobi, Gauss-Seidel, Newton-Raphson, linear approximation based on Taylor series, successive approximations, and the triangular matrix formulation. Those methods are evaluated using four test systems formed by 10, 21, 33 and 69 nodes. Simulation results, obtained in MATLAB, show that the proposed approach is efficient for radial direct current grids in terms of solution quality and processing time, increasing the efficiency for larger number of nodes increases.

1. Introduction

In the last years the microgrids (MGs) have been used to integrate distributed generators based on renewable energy sources and energy storage systems, which are operated by energy management strategies in order to improve the technical aspects of the grid and to reduce the environmental pollution [1, 2]. Direct current (DC) MGs are the most used solution due to its advantages in terms of efficiency, lower mathematical complexity and implementation simplicity [3]. An essential tool for analysing DC networks is the formulation and solution of the power flow (PF) Equations, which enable to define the operative characteristics of the grid such as voltages profiles and power losses [4].

To solve the PF problem in DC grids, different solutions have been proposed: for example, the work presented in [5] used the Gauss-Jacobian (GJ) and Gauss-Seidel (GS) methods, where both methods provide similar solution quality, but the Gauss-Seidel method was the fastest one. Similarly, in [4] was used the Newton-Raphson method (NR) for solving the PF problem in DC networks, it using the Kantarovich theorem to demonstrate convergence. In addition, in [3] was presented a linear approximation via Taylor series expansion as solution method, it requiring shorter processing times in comparison with the GJ, GS and NR methods. In [6] was proposed two iterative approaches for solving the PF problem in DC grids: an iterative approach of the linear approximation (TBM) and an iterative process based on a successive approximation (SA); both methods provide accurate solutions and acceptable processing times in comparison with the other solution methods. In addition, in [7] was proposed a triangular matrix (TM) formulation for power flow analysis in DC resistive grid with constant power loads.

In AC grids, multiple authors have proposed solution methods focusing in electrical systems with radial structure, achieving to reduce the complexity and processing times required. These type of solution methods are known as sweep iterative power flow methods [8,9], which are based on a simple mathematical formulation and iterative processes. In these methods, the most used is the Back Forward sweep iterative (BF) [10], due to allow obtaining excellent results with processing times reduced [11]. This method employs an ordering stage, the Kirchhoff laws and a simple iterative process for solving the power flow problem; presenting as main advantage that not required matrix inversions. Is important highlight that to the best knowledge of the authors, the BF has not been applied for solving the power flow problem in DC microgrids.

This paper proposes the application of the back/forward sweep iterative method (BF) to solve the power flow problem in DC MGs with radial structure, it considering resistive and constant power loads. This method is easy to implement by calculating currents in the backward stage and recovering voltages in the forward stage using an iterative procedure. To demonstrate the effectiveness and robustness of the backward/forward method, four test systems of 10, 21, 33 and 69 nodes are used; it providing comparative simulations with other six approaches proposed in specialized literature as: GJ, GS, NR, TBM, SA and TM. The selection of these methods is based on the excellent results reported by the authors in terms of solution and processing time required. The simulations results obtained in this work, show the effectiveness and robustness of the solution method proposed.

2. Back/forward sweep method

The backward/forward power flow method is a well-known strategy for solving power flow problems in AC grids with radial structure. Its mathematical foundation is supported by graph theory applied to grids with tree structure [10]. To apply the BF method in DC networks, it is necessary to make the following assumption: the DC electrical network must have radial structure, *i.e.*, the number of lines l and nodes n fulfills that $l = n - 1$; in addition, the grid has only one voltage controlled node that behaves as an ideal power source. A general formulation of the BF method is based on the relation between nodal and branch currents (voltages). Here, is presents an brief description of the BF method for DC grids.

Assuming that $\mathcal{H} \in \mathcal{R}^{l\times l}$ is a reduced node-to-branch incidence matrix (the row related with the slack node is eliminated), which produces an square invertible matrix that relates nodal injected currents I with branch currents J. In addition, considering that $\mathcal{Z} \in \mathbb{R}^{l\times l}$ is a square matrix (also invertible) that contains in its diagonal all the resistive effects of each line, then the Equation (1) and the Equation (2) are reached, where U represents the vector of branch voltages.

$$J = \mathcal{H}^{-1} I, \tag{1}$$

$$U = \mathcal{Z} J, \tag{2}$$

To perform the PF analysis in DC grids, it is proposed an iterative procedure based on the previous relations [5]: assuming that the voltages in all the nodes are defined as $V^k = [v_1^k, ..., v_{f-1}^k, v_{f+1}^k, ..., v_n^k]^T$, where k is the iterative counter. Then, condition $k = 0$ corresponds to the initial solution V^0 (typically plane voltages in per-unit representation). The slack node f was removed from this vector since its voltage v_f is known. Finally, starting from the initial solution (i.e. $V^0 = \mathbf{1} \rightarrow$ vector filled by ones with dimensions $l \times 1$), the Algorithm 1 enables to implement the BF approach for PF solutions in DC grids.

Algorithm 1 presents a compact formulation for power flow analysis that was initially proposed in [12] for AC grids; nevertheless, in this paper it is implement for DC grids. The algorithm starts by reading the parameters that define the test system and making the nodal ordering. Then, are assigned the convergence error (ϵ) and the maximum number of iterations k_{max}. Subsequently, the iterative process is carry out, calculating the nodal currents in busses of the electrical system associated to the resistive and constants power load. Then, are calculated the branch currents and voltages, and the nodal voltages by using the Equations (1) and (2), respectively. Finally, if the stopping criterion is reached ($\max\left(\left|\mathrm{V}^{k+1} - \mathrm{V}^k\right|\right) \leq \epsilon$), the power losses are calculated and the iterative process stops; otherwise a new iteration of the algorithm is executed.The calculation of power losses was made with the aim to compare the quality of the solution obtained with comparison methods used.

Algorithm 1. Backward/forward power flow method.

Data: Define the test system and make the nodal ordering
Data: Define the convergence error (ϵ) and the maximum number of iterations $k_{\max}$
for $k = 0 : k_{max}$ **do**
  Calculate the nodal currents as: $I^k = \left[\frac{p_1}{v_1^k}, ..., \frac{p_{f-1}}{v_{f-1}^k}, \frac{p_{f+1}}{v_{f+1}^k}, ..., , \frac{p_n}{v_n^k}\right]^T$;
  Determine the branch currents by applying (1) as: $J^k = \mathcal{H}^{-1} I^k$ →**backward sweep**;
  Calculate all the branch voltages by using (2) as: $U^k = \mathcal{Z} J^k$;
  Determine the nodal voltages as: $V^{k+1} = \mathbf{1} v_f + \mathcal{H}^{-T} U^k$ →**forward sweep**;
  if $\max\left(\left| V^{k+1} - V^k\right|\right) \leq \epsilon$ **then**
    Solution reached;
    Calculate power losses as: $p_{loss} = \left(J^{k+1}\right)^T \mathcal{Z} J^{k+1}$;
    break;
  end
end
Result: Return V^{k+1} and p_{loss}.

3. Numerical validation

To verify the effectiveness and robustness of the proposed method, four test system were used: 10, 21, 33 and 69 nodes. The test systems with 10 and 21 nodes were taken from [3, 5, 13]. Test systems with 33 and 69 nodes are modifies versions of test systems used in AC networks [14, 15]; the complete information for both DC equivalent systems are reported in [16]. In addition, to validate the performance of the BF in terms of solution quality and processing time, six comparison methods reported in literature are used: Gauss-Jacobi (GJ) [17], Gauss-Seidel [3], Newton-Raphson (NR) [4], triangular method (TM) [7], Taylor-based method (TBM) and successive approximations (SA) [6].

The simulations were carried out on a Dell Precision T7600 Workstation with 32 GB of RAM memory and with an Intel(R) Xeon(R) CPU ES-2670 at 2.50 GHz, using the software MATLAB. To ensure a fair comparison between all the solution methods, it is considered a convergence error equal to 1×10^{-10} and 10^4 consecutive executions for each one. Finally, the NR method was selected as base case for analyzing the power losses error and processing times; this method was selected since in [4] it was demonstrated the method converge to the power flow solution in DC grids [4]. In this paper it is selected the power losses error as indicator of the quality of the PF solution, which depends on the square form of the voltage profiles. Table 1 presents the

Journal of Physics: Conference Series **1448** (2020) 012012 doi:10.1088/1742-6596/1448/1/012012

numerical results for the proposed and the comparative methods; from left to right: the method, the total power losses and the averaged processing time.

Table 1. Results obtained for the different test systems.

Method	p_{loss} (p.u)	Time (s)	Method	p_{loss} (p.u)	Time (s)
	10 nodes test system			21 nodes test system	
NR	0.064475001	4.98×10^{-4}	NR	0.026527688	8.31×10^{-4}
GJ	0.064474982	9.63×10^{-3}	GJ	0.026527688	1.70×10^{-2}
GS	0.064474986	5.02×0^{-3}	GS	0.026527688	8.18×10^{-3}
TBM	0.064475001	4.00×10^{-4}	TBM	0.026527688	4.91×10^{-4}
SA	0.064475001	1.47×10^{-4}	SA	0.026527688	1.81×10^{-4}
TM	0.064475001	7.27×10^{-5}	TM	0.026527688	7.85×10^{-5}
BF	0.064475001	8.06×10^{-5}	BF	0.026527688	9.61×10^{-5}
	33 nodes test system			69 nodes test system	
NR	1.352509249	1.48×10^{-3}	NR	1.538475559	3.23×10^{-3}
GJ	1.352508282	1.47×10^{-1}	GJ	1.538454359	4.83
GS	1.352508749	8.13×10^{-2}	GS	1.538463557	3.99
TBM	1.352509249	9.90×10^{-4}	TBM	1.538475559	3.94×10^{-3}
SA	1.352509247	2.29×10^{-4}	SA	1.538475557	6.72×10^{-4}
TM	1.352509249	7.35×10^{-4}	TM	1.538475559	1.75×10^{-3}
BF	1.352509249	1.45×10^{-4}	BF	1.538475559	2.50×10^{-4}

Calculating the power losses error for the different methods, with respect to the NR method, the maximum and minimum errors were obtained by GJ $\left(2.12 \times 10^{-5}\right)$ and BF $\left(1.77 \times 10^{-12}\right)$, respectively. In addition, the average power losses errors obtained with each method are: 5.59×10^{-6} (GJ), 3.28×10^{-6} (GS), 2.47×10^{-10} (TBM), 1.52×10^{-8} (AS), 1.39×10^{-9} (TM) and 1.32×10^{-9} (BF)

Figure 1 presents on the horizontal axis the test systems used for the validation of the proposed method, and on the vertical axis the processing time required by each method with respect to the NR method, i.e. the time required by the NR method corresponds to 100 %. For the 10 and 21 nodes test systems, the BF requires only 1.58% and 2.58% more time than the best solution (TM), but it is faster than the other methods (NR, GJ, GS, TBM, and SA). In the case of 33 and 69 nodes test systems, the BF provides the best solution in terms of processing time, with an average reduction of 91.22% (NR), 95.60% (GJ), 95.59 % (GS), 90.72 % (TBM), 51.66 % (SA) and 83.14 % (TM). In general terms, the BF provides the best results with an average reduction in processing time of 79.38 % when it is compared with the other methods: 86.67% (NR), 94.26% (GJ), 94.18% (GS), 86.21% (TBM), 48.30 %(SA) and 64.66 % (TM).

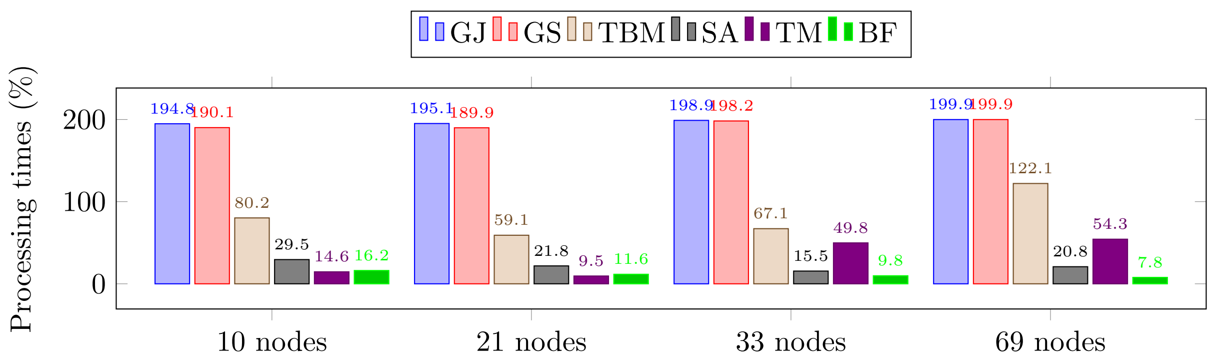

Figure 1. Processing time performance for all solution methods with respect to the base case (NR).

4. Conclusions

This paper proposed the application of the traditional backward/forward sweep iterative method to the power flow problem in DC networks with radial structure. By using different comparison methods and four test systems, the effectiveness and robustness of the proposed solution was verified. The simulation results demonstrated that, due to the power losses error of the methods can be neglected, any method considered in this work can be used for solving the power flow problem in DC grids. However, the processing times required by the methods show that, as the size of the DC network increases, the BF method provides the best performance. Therefore, the BF method corresponds to the best option for solving power flow problem in DC microgrids with radial structure, specially for large systems. As future work, the BF method could be integrated with a planing and energy management system for DC networks with radial structure, which will enable to reduce the processing times, hence improving the exploration space and the quality of the solutions.

Acknowledgment

This work was supported by the Instituto Tecnológico Metropolitano, Universidad Tecnológica de Bolívar, Universidad Nacional de Colombia, and Colciencias (Fondo nacional de financiamiento para ciencia, la tecnología y la innovación Francisco José de Caldas) under the projects C2018P020 and "Estrategia de transformación del sector energético Colombiano en el horizonte de 2030 - Energética 2030" - "Generación distribuida de energía eléctrica en Colombia a partir de energía solar y eólica" (Code: 58838, Hermes: 38945).

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