A Novel Full Boundary Element Formulation for Harmonic Analysis of Elastic Membranes Coupled to Acoustics Fluids

IOP Conf. Series: Materials Science and Engineering **844** (2020) 012061 doi:10.1088/1757-899X/844/1/012061

A Novel Full Boundary Element Formulation for Harmonic Analysis of Elastic Membranes Coupled to Acoustics Fluids

A J Narváez-Cruz[1], **J F Useche-Vivero**[1], **J A Martínez-Trespalacios**[2], **and J R Castro-Suarez**[3]

[1] Universidad Tecnológica de Bolívar, Cartagena de Indias, Colombia
[2] Institución Tecnológica Colegio Mayor de Bolívar, Cartagena de Indias, Colombia
[3] Universidad del Sinú, Cartagena de Indias, Colombia

E-mail: anarvaez@utb.edu.co

Abstract. *A novel full Boundary Element Formulation for the harmonic analysis of elastic membranes coupled to acoustics fluid is presented. The elastic membranes is modeled using the classical linear elastic pre-stretched membrane theory. The acoustic fluid is modeled using the acoustic-wave equation for homogeneous, isotropic, inviscid and irrotational fluids. Elastostatic fundamental solution is used in the boundary element formulation for the elastic membrane. The boundary element formulation for the acoustic fluid is based on the fundamental solution of three dimensional Poisson equation. Domain integrals related to inertial terms and those related with distributed pressure on membrane, were treated using the Dual Reciprocity Boundary Element Method. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles and dynamic pressure at fluid-structure interfaces. Results obtained shows the accuracy and efficiency of the proposed boundary element formulation.*

Nomenclature

$\ddot{p}$	second time derivative for pressure
$\ddot{w}$	second time derivative for displacement
Γ_f	fluid boundary surface
Γ_f^p	fluid boundary surface with stablished p
Γ_f^q	fluid boundary surface with stablished q
Γ_s	membrane boundary
Γ_s^f	membrane boundary with stablished f
Γ_s^w	membrane boundary with stablished w
Γ_{fs}	fluid-structure interface
$\hat{\dot{p}}_0(\mathbf{x})$	known initial first time derivative for pressure in Ω_f
$\hat{\dot{w}}_0(\mathbf{x})$	known initial velocity in Ω_s
$\hat{f}(\mathbf{x},t)$	known normal traction on Γ_s^f
$\hat{p}(\mathbf{x},t)$	known pressure on Γ_f^p

$\hat{p}_0(\mathbf{x})$ known initial pressure in Ω_f
$\hat{P}_j(\mathbf{x}', \mathbf{x})$ particular solutions for pressure as a poisson type equation
$\hat{q}(\mathbf{x}, t)$ known normal pressure gradient on Γ_f^p
$\hat{Q}_j(\mathbf{x}', \mathbf{x})$ particular solutions for gradient pressures as a poisson type equation
$\hat{T}_j(\mathbf{x}', \mathbf{x})$ particular solutions for tractions as a poisson type equation
$\hat{w}(\mathbf{x}, t)$ known displacement on Γ_s^w
$\hat{w}_0(\mathbf{x})$ known initial displacement in Ω_s
$\hat{W}_j(\mathbf{x}', \mathbf{x})$ particular solutions for displacements as a poisson type equation
κ bulk modulus
Ω_f fluid opened cavity domain
Ω_s flexible membrane domain
ρ_f fluid mass density
ρ_s membrane mass density
A_{ij} matrix of coefficients, obtained by taking the value of $p(t)$ at different DRM points
B_{ij} matrix of coefficients, obtained by taking the value of $\ddot{w}(t)$ at different DRM points
$c(\mathbf{x}')$ jump terms arising from the terms of $O(1/r)$ in the respective kernel
c_f fluid wave propagation velocity
c_w membrane wave propagation velocity
$f = f(\mathbf{x}, t)$ normal membrane traction
F_{ij} matrix of coefficients, obtained by taking the value of $\ddot{p}(t)$ at different DRM points
$f_j(r)$ radial basis function
h membrane thickness
N number of boundary elements
n_i^w components of the outward normal vector
$NDRF$ number of total DRM collocation points used in the fluid
$NDRM$ number of total DRM collocation points used in the membrane
$P(\mathbf{x}', \mathbf{x})$ fundamentals solutions for pressure for three dimensional acoustic problems
$p = p(\mathbf{x}, t)$ fluid pressure
p_w distributed pressure applied over the membrane
$p_{,\alpha\alpha}$ pressure laplacian
p_{max} distributed time harmonic pressure load amplitude
$Q(\mathbf{x}', \mathbf{x})$ fundamentals solutions for gradient pressures for three dimensional acoustic problems
$q = q(\mathbf{x}, t)$ normal fluid pressure gradient
q_n pressure gradient acting on the fluid-structure interface
$T(\mathbf{x}', \mathbf{x})$ fundamentals solutions for tractions
T_0 unit length tension
$W(\mathbf{x}', \mathbf{x})$ fundamentals solutions for displacements
$w = w(\mathbf{x}, t)$ membrane transversal displacement
$w_{,\alpha\alpha}$ displacement laplacian
α_j coefficients that relate A_{ij} with p_w
$\mathbf{A}$ fluid-membrane coupling sub-matrix of H

IOP Conf. Series: Materials Science and Engineering **844** (2020) 012061 doi:10.1088/1757-899X/844/1/012061

$\mathbf{C}_w$ and $\mathbf{C}_f$ connectivity matrices joining fluid and structural degree of freedom at fluid-structure interface

$\mathbf{H}$ and $\mathbf{G}$ global coupling influence matrices

$\mathbf{M}$ global coupling mass matrix

$\mathbf{n}_w$ outward normal vector

$\mathbf{S}$ fluid-membrane coupling sub-matrix of G

$\mathbf{x}$ field points

$\mathbf{x}'$ collocation points

$\ddot{\alpha}_j$ coefficients that relate F_{ij} with $\ddot{p}$

$\ddot{\beta}_j$ coefficients that relate B_{ij} with $\ddot{w}$

${}^f\mathbf{G}_{fs}$ and ${}^f\mathbf{G}_{ff}$ sub-matrices of ${}^f\mathbf{G}$ related with degrees of freedom defined on the interface Γ_{fs}

${}^s\mathbf{B}_{fs}$ sub-matrix of ${}^s\mathbf{B}$ related with pressure terms defined in Ω_s

t time

${}^f\mathbf{H}$ and ${}^f\mathbf{G}$ influence matrices for the fluid

${}^f\mathbf{M}$ fluid mass matrix

${}^s\mathbf{B}$ influence matrix related with distributed pressure $\mathbf{p}_w$ applied over the membrane

${}^s\mathbf{H}$ and ${}^s\mathbf{G}$ influence matrices for the membrane

${}^s\mathbf{M}$ membrane mass matrix

1. Introduction

Acoustic problems in real life applications involve considerations of structural and acoustic part together and thus calls for a coupled vibroacoustic treatment rather a pure acoustic approach. Thus, whenever an elastic structure is in contact with a fluid, the structural vibrations and the acoustic pressure field in the fluid are influenced by the mutual vibro-acoustic coupling interaction. The analysis of fluid-structure coupled systems is a challenging and complex task. In general, the use of experimentation or numerical methods represent the two uniques alternatives to obtain approximate solutions for these kind of problems. However, numerical methods based on domain discretization requires refined meshes for high frequency problems, since the length of the elements should be proportional to the size of the wavelength. This means a more time-consuming model. The boundary element method (BEM) is a modern numerical technique which has enjoyed increasing popularity over the last two decades, and is now an established alternative to traditional computational methods of engineering analysis [1, 2]. The main advantage of the BEM is its unique ability to provide a complete solution in terms of boundary values only, with substantial savings in modelling effort. Since consolidation of the boundary element method (BEM) as reliable numerical method for structural and fluid analysis, linear vibrations of structures coupled with an internal fluid has been carried out using hybrid BEM-FEM formulations. In these formulations BEM is used to model the fluid media and the FEM to model the structural response [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The main advantage of such formulations lies in a substantial reduction in the number of the degrees of freedom in the discretization of the fluid domain. However, in this formulations is necessary to discretize the entirely structure due to the use of the FEM. Few BEM-BEM coupled formulations for fluid structure-interaction has been published [14, 15, 16, 17]. However, despite the fact that the BEM has been used for dynamic analysis of membrane structures and for analyses of acoustic fluids, to the best of authors knowledge, these formulations do not have been used for the fluid-structure interaction problem analysis using a full boundary element formulation for such purpose. In this work, a new full boundary element formulation for the transient dynamic analysis of acoustic

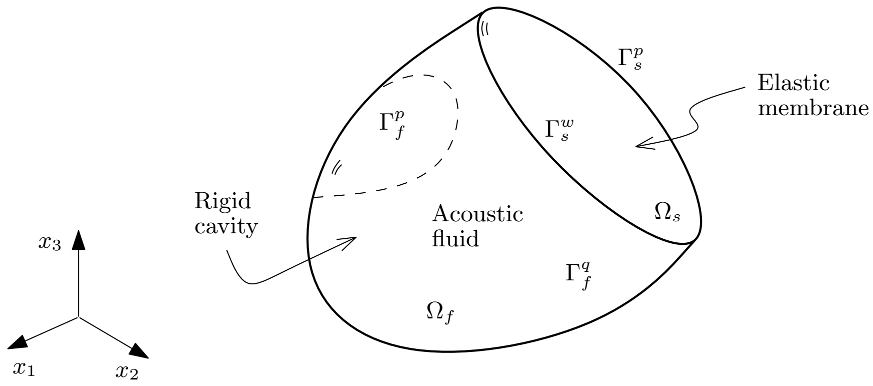

Figure 1. Fluid-structure interaction problem.

fluids coupled to elastic membranes is presented. Membranes are modeled using a boundary element formulation based on the linear elastic membrane theory under small deflection. The acoustic fluid is modeled using a boundary element formulation for the three dimensional acoustic wave equation. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles at fluid-structure interfaces. Domain integrals on both, fluid and structure equations, were treated using the Dual Reciprocity Boundary Element Method. The developed formulation was used to study the linear vibration response of elastic membranes coupled to acoustic fluids.

2. Structure subjected to a fluid pressure loading

Consider a partially opened cavity Ω_f with ridig walls and a flexible elastic membrane Ω_s with mass density ρ_s and thickness h (see Fig. 1). Cavity contains an homogeneous and isotropic acoustic fluid with mass density ρ_f. The membrane vibrations and the acoustic pressure field in the fluid are influenced by the mutual vibro-acoustic coupling interaction. In this work, the vibro-acoustic coupling interaction is modeled using an eulerian formulation where the acoustic response is described by the pressure, while the membrane response is described by the transversal displacement field.

2.1. Acoustic wave equation

The dynamic pressure of an ideal inviscid fluid under small perturbations in a spatial region Ω_f confined by the boundary surface $\Gamma_f = \Gamma_f^p \cup \Gamma_f^q$, is governed by the wave equation (see Fig. 2) [18]:

$$p_{,\alpha\alpha} = \frac{1}{c_f^2}\ddot{p} \tag{1}$$

In this equation p is the fluid pressure, $c_f^2 = \kappa/\rho_f$ stands for wave propagation velocity, κ is the bulk modulus. Above equation can be modified to include the effect of presence of an acoustic source. Double dot represents a second time derivative. Indicial notation is used throughout this work. Greek indices vary from 1 to 2 and Roman indices from 1 to 3. The boundary and initial conditions for equation (1) in the time interval $[0, t^*]$ are:

$$\begin{aligned} p(\mathbf{x}, t) &= \hat{p}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_f^p, \quad t \in [0, t^*] \\ q(\mathbf{x}, t) = n_i^f p_{,i} &= \hat{q}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_f^q, \quad t \in [0, t^*] \\ p(\mathbf{x}, 0) = \hat{p}_0(\mathbf{x}), &\quad \dot{p}(\mathbf{x}, 0) = \hat{\dot{p}}_0(\mathbf{x}), \quad \mathbf{x} \in \Omega_f \end{aligned} \tag{2}$$

where n_i^f are the components of the outward normal vector $\mathbf{n}_f$ at boundary. and t denotes time.

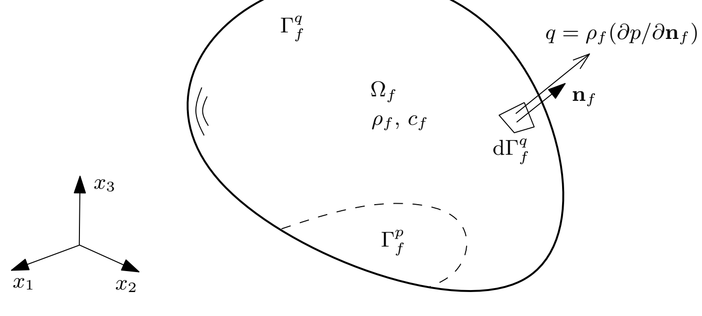

Figure 2. Acoustic fluid domain.

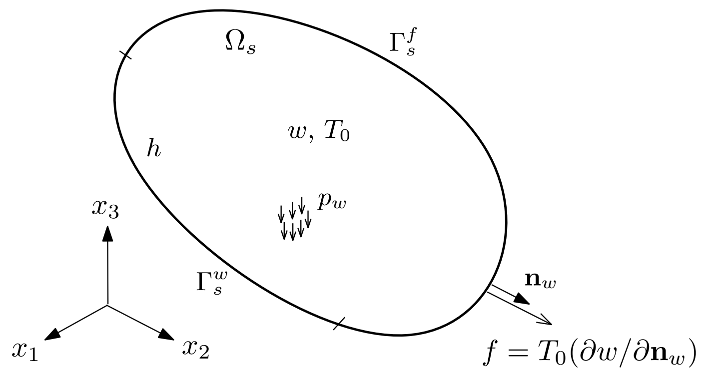

Figure 3. Elastic membrane domain.

2.2. Dynamic equation of an elastic membrane

Now consider a linear elastic membrane with thickness h ocuppying the spatial domain Ω_w confined by the boundary $\Gamma_s = \Gamma_s^w \cup \Gamma_s^f$ (see Fig. 3). An initial tension T_0 is uniformly applied to the membrane. In this work, the small deflection elastic membrane theory is considered. Thus, differential equation describing the trasversal displacement $w(\mathbf{x}, t)$ of this membrane in the time interval $[0, t^*]$ is given by:

$$w_{,\alpha\alpha} + \frac{p_w}{T_0} = \frac{1}{c_w^2}\ddot{w} \tag{3}$$

where $c_w^2 = T_0/\rho_s$ and $p_w(\mathbf{x}, t)$ is a distributed pressure applied over the membrane. The boundary and initial conditions for this equation are given by:

$$\begin{aligned} w(\mathbf{x}, t) &= \hat{w}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_s^w, \quad t \in [0, t^*] \\ f(\mathbf{x}, t) = n_i^w w_{,i} &= \hat{f}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma_s^f, \quad t \in [0, t^*] \\ w(\mathbf{x}, 0) = \hat{w}_0(\mathbf{x}), &\quad \dot{w}(\mathbf{x}, 0) = \hat{\dot{w}}_0(\mathbf{x}), \quad \mathbf{x} \in \Omega_s \end{aligned} \tag{4}$$

In these expressions, n_i^w are the components of the outward normal vector $\mathbf{n}_w$ at boundary.

3. Boundary element equations

3.1. Boundary element equations for acoustic wave equation

The derivation of the integral formulation for equation (1) is based on application of the Boundary Element Method to the acoustic wave equation as presented in [2]. Thus, by using the weighted residual method and making use of the Green's identity, the following equation is obtained:

$$c(\mathbf{x}')p(\mathbf{x}') + \int_{\Gamma_f} Q(\mathbf{x}',\mathbf{x})p(\mathbf{x})\mathrm{d}\Gamma_f = \int_{\Gamma_f} P(\mathbf{x}',\mathbf{x})q(\mathbf{x})\mathrm{d}\Gamma_f + \frac{1}{c_f^2}\int_{\Omega_f} P(\mathbf{x}',\mathbf{X})\ddot{p}(\mathbf{X})\mathrm{d}\Omega_f \tag{5}$$

In this equation, $\mathbf{x}'$ and $\mathbf{x}$ represent collocation and field points, respectively; $P(\mathbf{x}',\mathbf{x})$ and $Q(\mathbf{x}',\mathbf{x})$ are fundamentals solutions for pressure and gradient pressures for three dimensional acoustic problems, respectively, as presented in [2]. The value of $c(\mathbf{x}')$ is equal to $\frac{1}{2}$ when $\mathbf{x}'$ is located on a smooth boundary.

In order to threat the domain integral, the Dual Reciprocity Boundary Element Method (DRM) is used as presented in [19]. In this way, equation (5) can be re-written as:

$$\begin{aligned} c(\mathbf{x}')p(\mathbf{x}') + \int_{\Gamma_f} Q(\mathbf{x}',\mathbf{x})p(\mathbf{x})\mathrm{d}\Gamma_f &= \int_{\Gamma_f} P(\mathbf{x}',\mathbf{x})q(\mathbf{x})\mathrm{d}\Gamma_f + \\ +\frac{1}{c^2}\sum_{j=1}^{NDRF}\ddot{\alpha}_j(t)\Big[c_i\hat{P}_{ij} &+ \int_{\Gamma_f} Q(\mathbf{x}',\mathbf{x})\hat{P}_j\mathrm{d}\Gamma_f - \int_{\Gamma_f} P(\mathbf{x}',\mathbf{x})\hat{Q}_j\mathrm{d}\Gamma_f\Big] \end{aligned} \tag{6}$$

In this equation, $NDRF$ represents the number of total DRM collocations points used in the fluid; $\hat{P}_j(\mathbf{x}',\mathbf{x})$ and $\hat{Q}_j(\mathbf{x}',\mathbf{x})$ are the particular solutions to equivalent homogeneous equation (1). These particular solutions were obtained considering the function $f_j(r) = 1 + r_j$ for the approximation of $\ddot{p}(t)$, as presented in [19]. Coefficients $\ddot{\alpha}_j$ are related to $\ddot{p}$ through: $\ddot{p} = F_{ij}\ddot{\alpha}_j(t)$, where F_{ij} is a matrix of coefficients, obtained by taking the value of $\ddot{p}(t)$ at different DRM points.

In order to discretize boundary surfaces of the acoustic medium, N boundary quadrilateral elements were used and $p(\mathbf{x})$ and $q(\mathbf{x})$ were assumed to be constant over each element and equal to their values at the mid-element node. Thus, the discretised form of equation (6) is given by:

$$\begin{aligned} c_i(\mathbf{x}')p(\mathbf{x}',t) + \sum_{k=1}^{N}\Big[\int_{\Gamma k} Q(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma\Big]p_k(t) - \sum_{k=1}^{N}\Big[\int_{\Gamma k} P(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma\Big]q_k(t) = \\ \frac{1}{c_f^2}\sum_{j=1}^{NDRF}\ddot{\alpha}_j\Big[c_i(\mathbf{x}')\hat{P}_{ij}(\mathbf{x}',\mathbf{x}) + \sum_{k=1}^{N}\int_{\Gamma k} Q(\mathbf{x}',x)\hat{P}_j(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma \\ -\sum_{k=1}^{N}\int_{\Gamma k} P(\mathbf{x}',\mathbf{x})\hat{Q}_j(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma\Big] \end{aligned} \tag{7}$$

Applying this equation at each collocation point, the following linear system of equations is obtained:

$$^f\mathbf{M}\ddot{\mathbf{p}} + {}^f\mathbf{H}\mathbf{p} = {}^f\mathbf{G}\mathbf{q} \tag{8}$$

where $^f\mathbf{M}$ is the fluid mass matrix, $^f\mathbf{H}$ and $^f\mathbf{G}$ are boundary element influence matrices; $\mathbf{p}$ and $\mathbf{q}$ are vectors of nodal pressures and normal derivative of pressure, respectively.

3.2. Boundary element equations for an elastic membrane

The derivation of the integral formulation for equation (3) is based on the application of the BEM to the membrane equation as presented in [2]. Thus, by using the weighted residual method, and making use of the Green's identity, the integral formulation for equation (3) is given by:

$$\begin{aligned} c(\mathbf{x}')w(\mathbf{x}',t) + \int_{\Gamma s} T(\mathbf{x}',\mathbf{x})w(\mathbf{x},t)\mathrm{d}\Gamma_s - \int_{\Gamma s} W(\mathbf{x}',\mathbf{x})f(\mathbf{x},t)\mathrm{d}\Gamma_s = -\frac{1}{T_0}\int_{\Omega s} W(\mathbf{x}',\mathbf{x})p_w(\mathbf{x},t)\mathrm{d}\Omega_s \\ +\frac{1}{c_w^2}\int_{\Omega s} W(\mathbf{x}',\mathbf{x})\ddot{w}(\mathbf{x},t)\mathrm{d}\Omega_s \end{aligned} \tag{9}$$

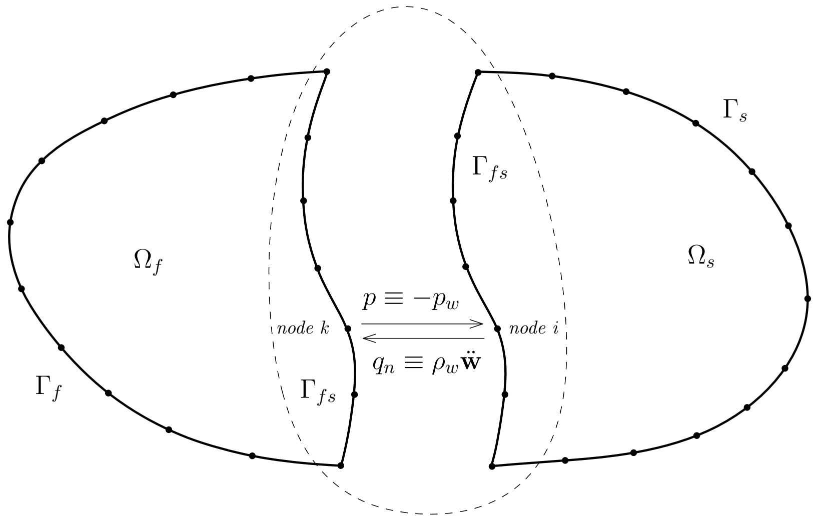

Figure 4. Discretized coupled problem using BEM.

where $\mathbf{x}$ and $\mathbf{x}'$ are field and collocation points respectively, $W(\mathbf{x}', \mathbf{x})$ and $T(\mathbf{x}', \mathbf{x})$ are fundamental solutions for displacement and traction, respectively as given in [1]. $c(\mathbf{x}')$ is the jump term arising from the terms of $O(1/r)$ in the kernel $T(\mathbf{x}', \mathbf{x})$.

In this work, the DRM was used to transform domain integrals related to inertial terms into boundary integrals. In this way, this equation (9) can be re-written as [19]:

$$c_i(\mathbf{x}')w(\mathbf{x}',t) + \int_{\Gamma s} T(\mathbf{x}',\mathbf{x})w(\mathbf{x},t)\mathrm{d}\Gamma_s - \int_{\Gamma s} W(\mathbf{x}',\mathbf{x})f(\mathbf{x},t)\mathrm{d}\Gamma_s = -\frac{1}{T_0}\sum_{j=1}^{NDRM} \alpha_j(t)\Big[c_i(\mathbf{x}')\hat{W}_j(\mathbf{x},t) + \int_{\Gamma s} T(\mathbf{x}',\mathbf{x})\hat{W}_j(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma_s - \int_{\Gamma s} W(\mathbf{x}',\mathbf{x})\hat{T}_j(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma_s\Big] + \frac{1}{c_w^2}\sum_{j=1}^{MDRM} \ddot{\beta}_j(t)\Big[c_i(\mathbf{x}')\hat{W}_j(\mathbf{x},t) + \int_{\Gamma s} T(\mathbf{x}',\mathbf{x})\hat{W}_j(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma_s - \int_{\Gamma s} W(\mathbf{x}',\mathbf{x})\hat{T}_j(\mathbf{x}',\mathbf{x})\mathrm{d}\Gamma_s\Big] \quad (10)$$

where $NDRM$ represent the total number of DRM collocation used in the membrane; $\hat{T}_j(\mathbf{x}', \mathbf{x})$ and $\hat{W}_j(\mathbf{x}', \mathbf{x})$ are the particular solutions to equivalent homogeneous equation (3). These particular solutions were obtained considering the function $f_j(r) = 1 + r_j$ for the approximation of $\ddot{w}(t)$ and $p_w(t)$ terms, as presented in [19]. Coefficients α_j are related to $p_w(t)$ through: $p_w = A_{ij}\alpha_j(t)$, where A_{ij} is a matrix of coefficients, obtained by taking the value of $p(t)$ at different DRM points. Similarly, coefficients $\ddot{\beta}_j$ are related to $\ddot{w}$ through: $\ddot{w} = B_{ij}\ddot{\beta}_j(t)$, where B_{ij} is a matrix of coefficients, obtained by taking the value of $\ddot{w}(t)$ at different DRM points.

Applying this equation at each collocation point, the following linear system of equations is obtained:

$$^s\mathbf{M}\ddot{\mathbf{w}} + {}^s\mathbf{H}\mathbf{w} = {}^s\mathbf{G}\mathbf{f} - {}^s\mathbf{B}\mathbf{p}_w \quad (11)$$

Here $^s\mathbf{M}$ is the membrane mass matrix, $^s\mathbf{H}$ and $^s\mathbf{G}$ are the influence matrices, $^s\mathbf{B}$ is the influence matrix related with distributed pressure $\mathbf{p}_w$ applied over the membrane.

IOP Conf. Series: Materials Science and Engineering **844** (2020) 012061 doi:10.1088/1757-899X/844/1/012061

4. Fluid-structure coupling equations

Fluid-structure coupling equations are given by compatibility considerations about normal pressure and dynamic pressure force acting at fluid-structure interface. Mathematically, these conditions can be written as follows [20]:

$$\mathbf{n}_f \cdot \nabla p = \mathbf{q}_n \quad \equiv \quad -\rho_w \mathbf{C}_w \ddot{\mathbf{w}} \tag{12}$$

$$\mathbf{p}_w \quad \equiv \quad -\mathbf{C}_f \mathbf{p} \tag{13}$$

That is, pressure gradient acting on the fluid-structure interface Γ_{fs} are related to normal acceleration of the plate and the acoustic pressure is equilibrated with pressure on the membrane (see Fig. 4). In these equations, $\mathbf{C}_w$ and $\mathbf{C}_f$ represent connectivity matrices joining fluid and structural degree of freedom at fluid-structure interface.

Replacing equations (13) and (13) into equations (8) and (11) we obtain the coupled fluid-structure equation problem:

$$\begin{bmatrix} {}^s\mathbf{M} & \mathbf{0} \\ \mathbf{S} & {}^f\mathbf{M} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{w}} \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} {}^s\mathbf{H} & -\mathbf{A} \\ \mathbf{0} & {}^f\mathbf{H} \end{bmatrix} \begin{Bmatrix} \mathbf{w} \\ \mathbf{p} \end{Bmatrix} = \begin{bmatrix} {}^s\mathbf{G} & \mathbf{0} \\ \mathbf{0} & {}^f\mathbf{G}_{ff} \end{bmatrix} \begin{Bmatrix} \mathbf{f} \\ \mathbf{q} \end{Bmatrix} \tag{14}$$

In this equation, the off-diagonal sub-matrices $\mathbf{A} = {}^s\mathbf{B}_{fs}\mathbf{C}_f$ and $\mathbf{S} = \rho_w {}^f\mathbf{G}_{fs}\mathbf{C}_w$ are fluid-membrane coupling matrices. ${}^f\mathbf{G}_{fs}$ and ${}^f\mathbf{G}_{ff}$ are sub-matrices of ${}^f\mathbf{G}$ related with degrees of freedom defined on the interface Γ_{fs}. ${}^s\mathbf{B}_{fs}$ is a sub-matrix of ${}^s\mathbf{B}$ related with pressure terms defined in Ω_s.

Equations (14) can be rewritten in a general way as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{H}\mathbf{u} = \mathbf{G}\mathbf{r} \tag{15}$$

where, $\ddot{\mathbf{u}} = \{\ddot{\mathbf{w}}, \ddot{\mathbf{p}}\}^T$, $\mathbf{u} = \{\mathbf{w}, \mathbf{p}\}^T$ and $\mathbf{r} = \{\mathbf{f}, \mathbf{q}\}^T$.

5. Numerical examples

5.1. Box-shaped structure containing an acoustic fluid coupled to a membrane

Consider a partially opened box-shaped structure with ridig walls and a flexible elastic membrane with $c_w = 22.36$ m/s as presented in Fig. 5. The structure contains an acoustic fluid with mass density $\rho_f = 100$ kg / m^3 and a bulk's modulus $\kappa = 2.5 \times 10^5$ Pa. A distributed time harmonic pressure load $p_{max} = 1.0$ Pa is applied over the face of the structure located at $x_3 = 2$ m. Pressure gradient $q = n_i^f p_{,i} =$ is considered zero in the other faces.

Four meshes were used to show the convergence. Table 1 shows the respective meshes.

Table 1. Mesh characteristics for example 1

Meshes	Elements	Collocation Points
Mesh 1	54 (12)	125 (9)
Mesh 2	150 (20)	125 (25)
Mesh 3	294 (28)	343 (49)
Mesh 4	486 (36)	343 (81)

The collocation points are coincidents with fluid collocation points located in the fluid-membrane interface. A membrane tensile load $T_0 = 500$ N/m was considered in the analysis. The frequency-history response of the central deflection for frequency interval 0.039788 Hz. to 20.0 Hz. is shown in figure 6.

The collocation points are coincidents with fluid collocation points located in the fluid-membrane interface. A membrane tensile load $T_0 = 500$ N/m was considered in the analysis.

IOP Conf. Series: Materials Science and Engineering **844** (2020) 012061 doi:10.1088/1757-899X/844/1/012061

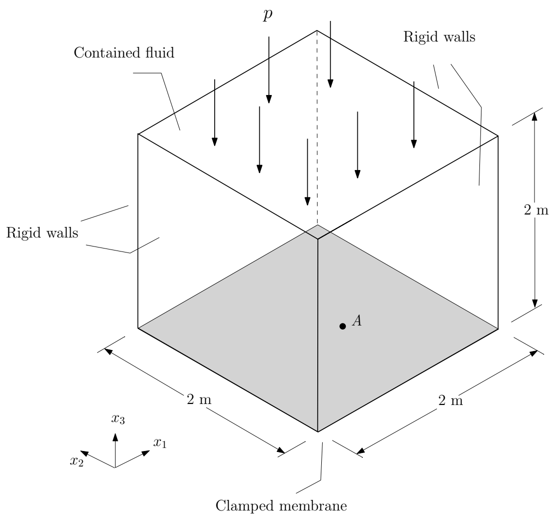

Figure 5. Discretized coupled problem using BEM.

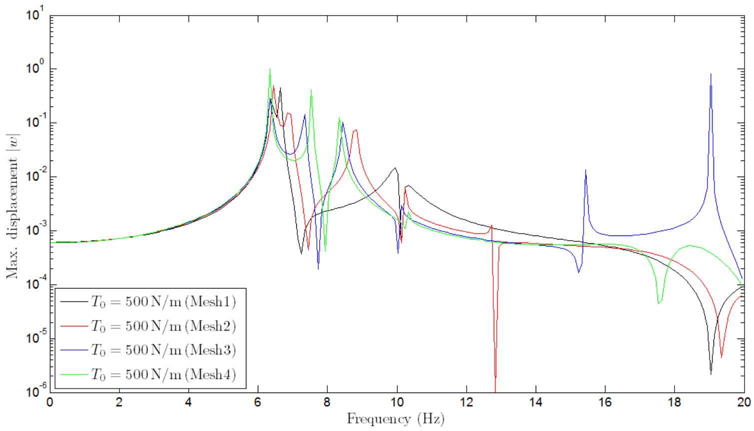

Figure 6. Maximum displacement frequency history at point A(0.25,-0.25,-1) using DR-BEM

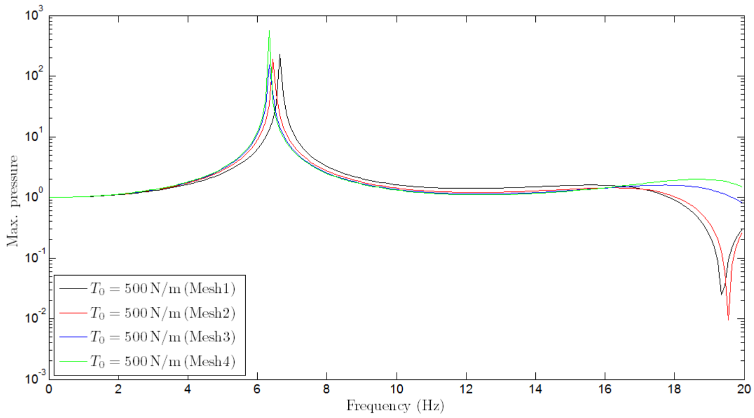

Figure 7. Pressure frequency history at point A(0.25,-0.25,-1) using DR-BEM

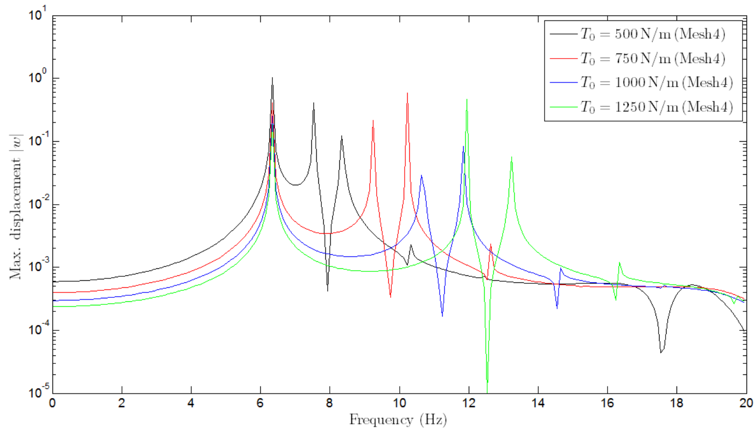

Figure 8. Maximum displacement frequency history at point A(0.25,-0.25,-1) using DR-BEM

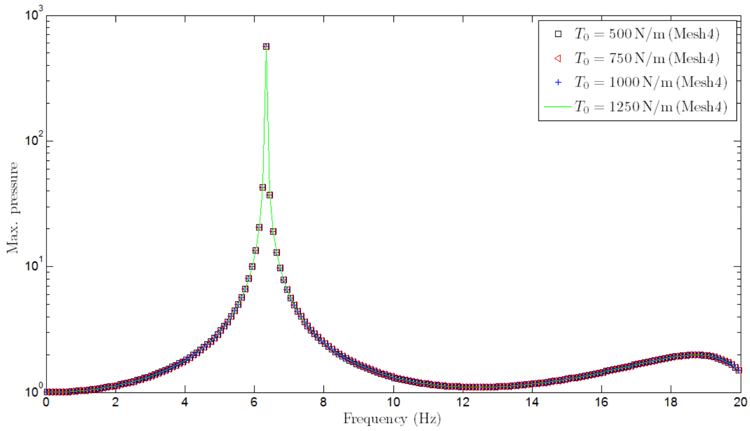

Figure 9. Pressure frequency history at point A(0.25,-0.25,-1) using DR-BEM

5.2. Rigid channel containing an acoustic fluid coupled to a membrane

In this problem, a 10 m. long channel containing an acoustic fluid under a distributed time harmonic pressure load and coupled with a homogeneous elastic membrane with $c_w = 22.36$ m/s and density $\mu_w = 1.0$ kg/m^2 is analyzed (see Fig. 10). The fluid has a density ρ_f 100 kg/m^3 and a bulk's modulus of $\kappa = 2.5 \times 10^5$ Pa. The membrane is inclined 20.56° with respect to the vertical axis. A distributed time harmonic pressure load $p_{max} = 1.0$ Pa is applied on the membrane. Pressure gradient $q = n_i^f p_{,i} =$ is considered zero in the other faces.

Four meshes were used to show the convergence. Table 2 shows this meshes.

Table 2. Mesh characteristics for example 2

Meshes	Elements	Collocation Points
Mesh 1	162 (12)	90 (9)
Mesh 2	330 (20)	90 (25)
Mesh 3	658 (28)	171 (49)
Mesh 4	1098 (36)	171 (81)

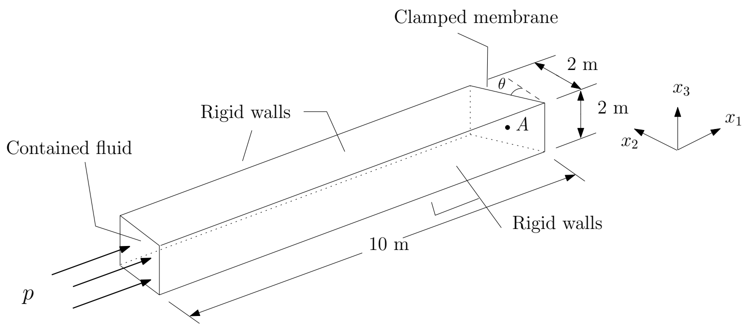

Figure 10. Discretized coupled problem using BEM.

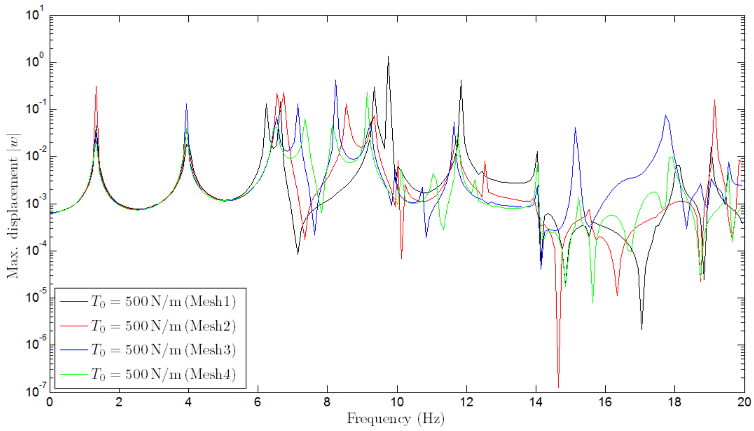

Figure 11. Maximum displacement frequency history at point A using DR-BEM

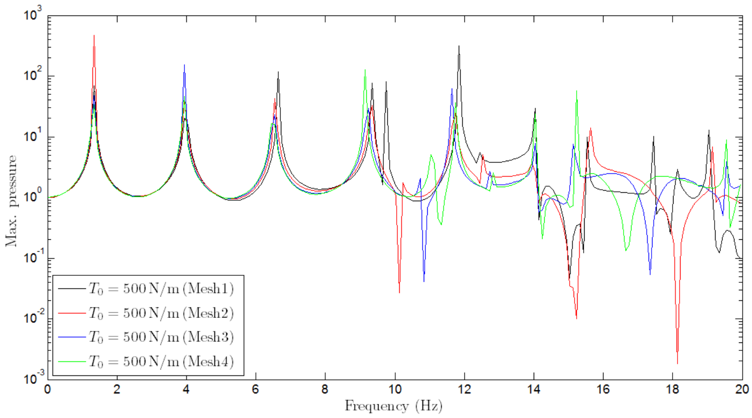

Figure 12. Pressure frequency history at point A using DR-BEM

IOP Conf. Series: Materials Science and Engineering **844** (2020) 012061 doi:10.1088/1757-899X/844/1/012061

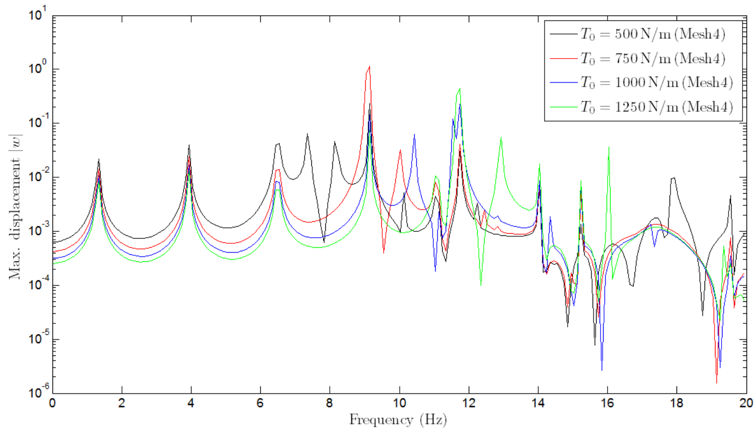

Figure 13. Maximum displacement frequency history at point A using DR-BEM

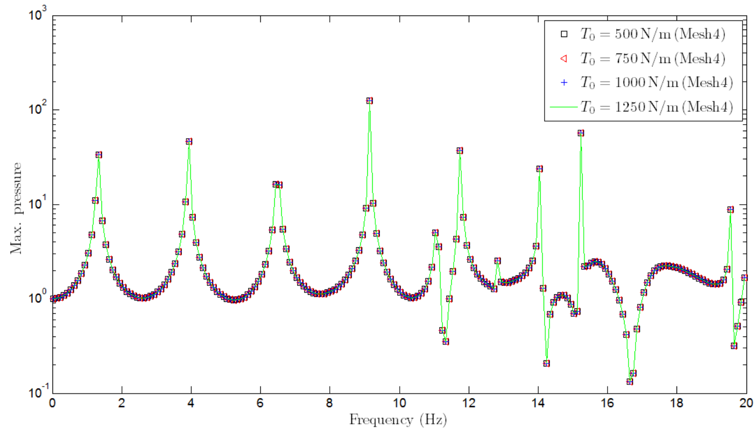

Figure 14. Pressure frequency history at point A using DR-BEM

IOP Conf. Series: Materials Science and Engineering **844** (2020) 012061 doi:10.1088/1757-899X/844/1/012061

6. Conclusions

A new full boundary element formulation for the harmonic dynamic analysis of acoustic fluids coupled to elastic membranes is presented. Membranes were modeled using a boundary element formulation based on the linear elastic membrane theory under small deflection. The acoustic fluid was modeled using a boundary element formulation for the three dimensional acoustic wave equation. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles at fluid-structure interfaces. Domain integrals on both, fluid and structure equations, were treated using the Dual Reciprocity Boundary Element Method. Results show good agreement with those obtained from finite difference models, turning proposed formulation a reliable and an alternative numerial engineering tool for the dynamic analysis of acoustic fluids coupled to flexible elastic membranes.

References

[1] Wrobel L C and Aliabadi M H 2002 *The Boundary Element Method Volume 2: Applications in Solid and Structures* (New York, NY: Wiley)
[2] Wrobel L C 2002 *The Boundary Element Method Volume 1: Applications in Thermo-Fluids and Acoustics* (New York, NY: Wiley)
[3] Mackerle J 1999 *Finite Elements in Analysis and Design Journal* **31** 231–240
[4] Shekari M R, NKhaji and Ahmadi M T 2009 *Journal of Fluids and Structures* **25** 567–585
[5] Naumenkob V, Strelnikovac E and Yeselevac E 2010 *Engineering Analysis with Boundary Elements Journal* **34** 856–862
[6] Gaul L and Wenzel W 2002 *Engineering Analysis with Boundary Elements* **26** 629–636
[7] Soares D 2009 *International Journal for Numerical Methods in Engineering* **78** 1076–1093
[8] Everstine G C and Henderson F M 1990 *The Journal of the Acoustical Society of America* **87** 1938–1947
[9] Fritze D, Marburg S and Hardtke H J 2005 *Computers and Structures Journal* **83** 143–154
[10] Citarella R, Federico L and Cicatielloa A 2007 *Engineering Analysis with Boundary Elements* **31** 248–258
[11] He Z, Liu G, Zhong Z, Zhang G and Cheng A 2011 *Engineering Analysis with Boundary Elements* **35** 140–147
[12] Chen P T, Ju S H and Cha K C 2000 *Journal of Sound and Vibration* **233** 407–422
[13] Marburg S and Nolte B 2010 *Computational Acoustics of Noise Propagation in Fluids - Finite and Boundary Element Methods* (New York, NY: Springer)
[14] Tanaka M and Masuda Y 1988 *Computer Methods in Applied Mechanics and Engineering Journal* **71** 225–234
[15] Jr D S and Mansur W 2006 *Journal of Computational Physics* **219** 498–512
[16] Burgschweiger R, Ochmann M and Nolte B 2008 *The Journal of the Acoustical Society of America* **123** 3757–3760
[17] Jr D S, von Estorff O and Mansur W J 2005 *International Journal for Numerical Methods in Engineering* **64** 1416–1431
[18] Kinsler L E, Freg A R, Coppens A B and Sanders J V 2005 *Fundamental of Acoustics* (London, UK: Jhon Wiley and Sons Inc.)
[19] Partridge P, Brebbia and Wrobel 1992 *The Dual Reciprocity Boundary Element Method* (Boston: Southampton)
[20] Morand P and Ohayon R 1995 *Fluid Structure Interaction* (New York, NY: John Wiley and Sons)