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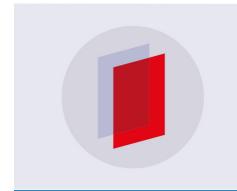
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Alternative power flow method for direct current resistive grids with constant power loads: A truncated Taylor-based method

O D Montoya[1]**, W Gil-González**[2]**, and J J Marulanda**[2]

[1] Programa de Ingeniería Eléctrica, Universidad Tecnológica de Bolívar, Cartagena, Colombia
[2] Programa de Ingeniería Eléctrica, Universidad Tecnológica de Pereira, Pereira, Colombia

E-mail: o.d.montoyagiraldo@ieee.org

Abstract. The power flow in electrical system permits analyzing and studying the steady-state behavior of any grid. Additionally, the power flow helps with the proper planning and management of the system. Therefore, it is increasingly necessary to propose power flows with fast convergence and high efficiency in their results. For this reason, this paper presents an alternative power flow approach for direct current networks with constant power loads based on a truncated Taylor-based approximation. This approach is based on a first-order linear approximation reformulated as a recursive, iterative method. It works with a slope variable concept based on derivatives, which allow few iterations and low processing times. Numerical simulations permit identifying the best power flow approaches reported in the specialized literature for radial and mesh dc grids, including the proposed approach. All the simulations were conducted in MATLAB 2015a.

1. Introduction

Power flow analysis is an essential tool for electrical systems [1, 2], since it allows determining the steady-state variables related with is operation given a particular load condition [3, 4]. Due to the nonlinear non-convexity of the power flow equations caused by the hyperbolic relation between voltage and power consumption [5–8], are required numerical methods for addressing its solution.

In specialized literature has been proposed multiple power flow approaches for power flow analysis to know: a Gauss-Seidel approach was proposed in [9], where its convergence was proved through the fixed point theorem in the Banach's space; in [5] was proposed and improved version of the Gauss-Seidel approach named successive approximation method that works with the admittance nodal matrix for being it speeder in terms of required iterations and processing times. Author of [2] presents the conventional Newton-Raphson approach applied on dc grids by demonstrating its convergence based on Kantarovich's theorem. In [5] was also offered a Taylor-based iterative method as an improvement of the linear version previously reported in [1]. In [10] is presented a triangular based approach for dc grids, its main problem is that it only works with the radial structure and only one slack node.

In the case of linear approximations, in [1] is presented a Taylor-based approach only solved for a first iteration; this approach was improved by the straight equation reported in [7]. Recently,

a modification of the logarithmic transformation of voltage magnitudes for ac grids developed in [11] was adapted for dc networks with the best numerical results when compared with previous linear approaches [12].

Based on the aforementioned state-of-the-art, in this paper, we propose an iterative procedure based on Taylor series expansion, which is truncated at the first linear term and corrected by modifying the linearizing point. The proposed approach is an extension of the convex optimal power flow approach reported in [12] for selecting the best candidate nodes for optimal location of distributed generators. The main contribution of our research lies in the possibility of having an alternative power flow approach faster than classical Gauss-Seidel and Newton-Raphson methods, even comparable with Taylor-based and Successive approximation methods recently reported in [5]

The remainder of this paper is ordered as follows: Section 2 presents the conventional power flow formulation for dc networks by highlighting its nonlinearities and non-convexities. Section 3 presents general concepts of Taylor series expansion for multivariable functions as well as the derivation of the proposed truncated Taylor-based method. Section 4 shows the main characteristics of the test systems employed in the numerical validation studied in section 5. Finally, section 6 presents the main conclusions derived from this work.

2. Mathematical model

The problem of power flow in dc networks is a common nonlinear problem of feasibility that models the relation between voltage profiles and loads in electrical systems. Its formulation is reached by using the classical nodal voltage method on conjunction with the first Tellegen's theorem, which determines the power balance in all nodes of the network as shown in Equation (1).

$$p_i^g - p_i^d = v_i \sum_{j=1}^{n} G_{ij} v_j, \ \forall i \in \mathcal{N}, \tag{1}$$

where p_i^g is the power generation at node i and p_i^d is its power demand, v_i and v_j are the voltage values at nodes i and j, respectively. G_{ij} is the component of the conductance matrix that relates nodes i and j. Note that n is the cardinality of the set $\mathcal{N}$ that contains all the nodes.

Note that a compact formulation of Equation (1) can be obtained if we rearrange all the slack nodes and the demand nodes, as shown in Equation (2) and Equation (3).

$$p_s = diag(v_s) \left[G_{ss} v_s + G_{sd} v_d \right], \tag{2}$$

$$-p_d = diag(v_d) \left[G_{ds} v_s + G_{dd} v_d \right], \tag{3}$$

where $p_s \in \mathbb{R}^{s \times 1}$ is the vector that contains all the power generated in the slack nodes; $p_d \in \mathbb{R}^{(n-g) \times 1}$ is the vector that contains all the constant power consumptions; $v_s \in \mathbb{R}^{g \times 1}$ and $v_d \in \mathbb{R}^{(n-g) \times 1}$ are the vector that contain all the voltages in slack and demand nodes, respectively. G_{ss}, G_{sd}, G_{ds} and G_{dd} are matrices with appropriate dimensions that define the conductance relations between slack nodes and demand ones. $diag(v_s)$ and $diag(v_d)$ are diagonal matrices composed by vectors v_s and v_d in their diagonals. Observe that g is the number of slack generators.

Note that Equation (2) is linear since voltage in the slack nodes v_s are perfectly known, which implies that the variables are v_d and p_s are related linearly, being later a free set of variables

that absorb the variations caused by v_d. Then, the power flow problem concentrates on the solution of Equation (3), next section is shown.

3. Truncated Taylor-based approximation
This section presents a Taylor-based series expansion truncated to the first-order approximation.

3.1. Linear approximation
Suppose that there exist a bi-dimensional function $g(v_i, v_j)$, such that is desired to be linearized around (v_i^0, v_j^0), then, its result is presented in Equation (4).

$$g(v_i, v_j) = g(v_i^0, v_j^0) + g_{v_i}(v_i^0, v_j^0)(v_i - v_i^0) + g_{v_j}(v_i^0, v_j^0)(v_j - v_j^0) + \mathcal{O}(v_i, v_j, v_i^0, v_j^0), \quad (4)$$

where $g_{v_i}()$ and $g_{v_j}()$ are the derivative function of the functions of $g()$ respect to v_i and v_j, respectively, and $\mathcal{O}()$ represents the high-order terms of the approximation. Here, we neglect those terms due to in power flow analysis they tend to zero speedily as demonstrating in [1].

Observing Equation (1), $g()$ is Equation (5).

$$g(v_i, v_j) = v_i v_j, \quad (5)$$

then, by applying Equation (4) on Equation (5), the following result yields, Equation (6).

$$g(v_i, v_j) \approx v_i^0 v_j + v_i v_j^0 - v_i^0 v_j^0, \quad (6)$$

Finally, if we extend Equation (6) to Equation (3), for a compact linear representation, then, we obtain Equation (7).

$$-p_d = diag(v_d)G_{ds}v_s + diag(v_d^0)G_{dd}v_d + diag(v_d)G_{dd}v_d^0 - diag(v_d^0)G_{dd}v_d^0, \quad (7)$$

Observe that if Equation (7) is solved for v_d, then, a linear approximation for power flow analysis be reached.

3.2. Iterative procedure
To achieve a recursive solution for power flow analysis based on a truncated Taylor-based method, let us used the properties of diagonal matrices and vectors as follows, being $diag(x)$ a matrix and y a vector with appropriate dimensions, as shown in Equation (8).

$$diag(x)y = diag(y)x, \quad (8)$$

If we apply Equation (8) on Equation (7), and solving for v_d, we reach rearrange the result for v_d, then, the following results is achieved, Equation (9).

$$v_d = \left[diag(G_{ds}v_s + G_{dd}v_d^0) + diag(v_d^0)G_{dd} \right]^{-1} \left[diag(v_d^0)G_{dd}v_d^0 - p_d \right]. \quad (9)$$

Note that Equation (9) is a linear approximation for power flow analysis being an alternative of the linear approach reported in [1]. To get an iterative procedure for solving the power flow

problem with exactness, just as Newton-Raphson or successive approximation methods [2,5], we change the linearizing point v_d^0 by using an iterative counter t as described in Equation (10).

$$v_d^{t+1} = \left[diag(G_{ds}v_s + G_{dd}v_d^t) + diag(v_d^t)G_{dd} \right]^{-1} \left[diag(v_d^t)G_{dd}v_d^t - p_d \right]. \tag{10}$$

Note that the iterative counter is increased from 0 to $t_{\max}$, until results be achieved with a minimum convergence error ϵ by doing $\max \left(\left| v_d^{t+1} \right| - \left| v_d^t \right| \right) \leq \epsilon$. Note that ϵ is typically selected in the specialized literature as 1×10^{-10} [5].

Finally, the main contribution of this work is the iterative formula (see Equation (10)) that helps solve power flow problems in dc grids with radial mesh grids with one, or multiple voltages controlled nodes as an alternative approach for classical power flow approaches. Here, this method is called truncated Taylor-based power flow (TTBPF) approach.

4. Test systems

Two test system are employed to validate the proposed power flow in this paper. The test are system are multi-terminal high-voltage direct current (HVDC) and 69-node systems. The schemes of the test systems and their parameters can be found in [12]. For multi-terminal HVDC system, a per-unit representation of this test system is employed by considering 400 kV and 1000 MVA as voltage and power bases, respectively. Additionally, we suppose that node 1 corresponds to the slack bus with a voltage of 1.02 p.u. While, for 69-node test feeder, we use 12.66 kV and 100 kW as voltage and power bases. In addition, the reactance component in all branches and reactive power consumption in all nodes are neglected.

5. Computational validation

Here, we present the numerical validation of the proposed TTBPF method for mesh and radial grids. The computational analysis was made in a personal computer with an AMD A10-8700P Radeon R6, 10 Compute Cores 4C + 6G (1.8 GHz) and 8 GB of RAM, running a 64-bit Windows 10 Home Single Language operating system in conjunction with the programming environment MATLAB 2015a.

For comparison purposes, only methods that work with radial or mesh networks by using an iterative procedure. These methods are: Gauss-Seidel power flow (GSPF) [9], Successive approximation power flow (SAPF) and Taylor-based power flow (TBPF) methods [5], and Newton-Raphson power flow (NRPF) approach [2]. The convergence error for all the methods was defined as $\epsilon = 1 \times 10^{-10}$. In addition, to obtain the averaged solution times, all the methods are evaluated 100000 consecutive times.

5.1. Multi-terminal high-voltage direct current

Table 1 reports the numerical performance of the comparison methods as well as the proposed method for power flow analysis. From these results, we can observe that the GSPF method is the slowest method in terms of convergence times, and also it takes the most significant number of iterations. When voltage profiles are observed, we can conclude that the first nine decimals are identical for all the power flow methodologies, which implies that in practical terms for the HVDC systems the voltage error between different methodologies will be in the order of tens of microvolts; which of course is negligible for any possible application in power systems. In the case of power loss estimation, we can affirm that the maximum difference between all the methods of some watts, which of course it is negligible, since power loss in the order of the tens of megawatts.

Table 1. Numerical performance of the power flow approaches on a mesh grid.

Method	Power loss (p.u)	Minimum voltage (p.u)	Average time (ms)	Iterations
GSPF [9]	0.254371646504838	0.968323258208860	0.664619691289139	47
SAPF [5]	0.254371647133838	0.968323258180689	0.319181127395381	8
TBPF [5]	0.254371647361616	0.968323258154582	0.314827622566145	4
NRPF [2]	0.254371647331213	0.968323258154582	0.828623248953276	4
TTBPF	0.254371647331215	0.968323258158700	0.339978176392202	4

Note that, even if the proposed approach is not faster, it can be comparable in terms of the number of iterations with any Taylor descendant approach, i.e., Taylor-based and Newton-Raphson power flow methods, since, they work with a variable slope for approximating voltage profiles, which reduces the required iterations to converge. In terms of processing times, from results of Table 1, we can affirm that for the multi-terminal HVDC systems, the proposed TTBPF approach is located at the third place, behind SAPF and TBPF, by moving the classical NRPF approach to the fourth place.

5.2. 69-node test feeder
Table 2 reports the numerical performance of the comparison methods as well as the proposed method for power flow analysis. Note that the number of iterations required by the GSPF approach is exaggerated large in contrast with the remainder methods. When the minimum voltage profiles are observed, then, we can conclude the first six decimals are equals, which implies that in practical terms that all the methods differ for this test systems in the order of milivolts, which is negligible for any practical application of radial distribution test systems.

Table 2. Numerical performance of the power flow approaches on a radial grid.

Method	Power loss (p.u)	Minimum voltage (p.u)	Average time (ms)	Iterations
GSPF [9]	1.538463553417373	0.927438796087894	3851.183057265740	46264
SAPF [5]	1.538475557921633	0.927438417525577	1.449723001478468	9
TBPF [5]	1.538475558217508	0.927438417489573	2.264500074664201	4
NRPF [2]	1.538475559117886	0.927438417488560	7.870733282341290	5
TTBPF	1.538475559083016	0.927438417489480	3.253316515837363	5

In the case of the processing times, the proposed approach remains located at the third position behind the SAPF, and TBPF approaches. In the case of power loss estimation, the maximum error between all the methods is about 2 W, which is minimum in comparison with power losses in distribution systems that are in hundreds of kilowatts. On the other hand, when the number of iterations is observed, the same tendency showed by the HVDC systems holds in this radial test system, i.e., the minimum number of iterations is reached by all the Taylor-based approaches.

6. Conclusions
This papers explored a new alternative methodology for power flow analysis in dc grids with radial or mesh structures, by using a truncated Taylor-based approach. This methodology was efficient in terms of voltage profile estimation, power loss calculation, number of iterations and processing time requirements when compared with classical numerical methods such as Gauss-Seidel, successive approximation, Taylor-based approach, and Newton-Raphson methods.

Numerical results confirms that all the numerical methods that use variable slope calculation (derivative information) converge in lower number of iterations; notwithstanding, they also confirm that Gauss-Seidel is th worst method for power flow analysis in terms of processing times, while the successive approximation is the most efficient method reported in the specialized literature, followed by the Taylor- and truncated Taylor-based power flow approaches, in descendant order.

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