Journal of Energy Storage 21 (2019) 1–8

Contents lists available at ScienceDirect

Journal of Energy Storage

journal homepage: www.elsevier.com/locate/est

Economic dispatch of energy storage systems in dc microgrids employing a semidefinite programming model

Walter Gil-González[a,*], Oscar Danilo Montoya[b], Erika Holguín[a], Alejandro Garces[a], Luis Fernando Grisales-Noreña[c]

[a] Universidad Tecnológica de Pereira. AA: 97, 660003 Pereira, Colombia
[b] Programa de Ingeniería Eléctrica e Ingeniería Electrónica, Universidad Tecnológica de Bolívar. Km 1 vía Turbaco, Cartagena, Colombia
[c] Departamento de Mecatrónica y Electromecánica, Instituto Tecnológico Metropolitano, Medellín, Colombia

ARTICLE INFO

Keywords:
Economic dispatch
Dc microgrids
Energy storage systems
Semidefinite programming
Convex reformulation

ABSTRACT

A mathematical optimization approach for the optimal operation focused on the economic dispatch for dc microgrid with high penetration of distributed generators and energy storage systems (ESS) via semidefinite programming (SDP) is proposed in this paper. The SDP allows transforming the nonlinear and non-convex characteristics of the economic dispatch problem into a convex approximation which is easy for implementation in specialized software, i.e., CVX. The proposed mathematical approach contemplates the efficient operation of a dc microgrid over a period of time with variable energy purchase prices, which makes it a practical methodology to apply in real-time operating conditions. A nonlinear autoregressive exogenous (NARX) model is employed for training an artificial neural network (ANN) for forecasting solar radiation and wind speed for renewable generation integration and dispatch considering periods of prediction of 0.5 h. Four scenarios are proposed to analyze the inclusion of ESS in a dc microgrid for economic dispatch studies. Additionally, the results are compared with GAMS commercial optimization package, which allows validating the accuracy and quality of the proposed optimizing methodology.

1. Introduction

Electric power generation has employed mainly hydro-power and thermo-power plants, and nowadays distributed generation resources, such as wind and solar power [1], appears as the third most important generation technology. However, despite having numerous hydro-power plants (e.g., Colombia) to supply the energy demand, hydro-electric generation is highly dependent on weather conditions, which makes it necessary to use thermo-power plants and thus, contribute the increase greenhouse emissions continuously. This has motivated, in recent decades, to research for effective methods to reduce the different environmental impacts and expand the use of distributed generators and energy storage systems (ESS) for supporting power demand increments in a sustainable form [2–4].

Electrical power distribution systems with high penetration of distributed generators are called microgrids and can be classified into two groups: ac microgrids and dc microgrids, as shown in Fig. 1 for the dc case [5].

Notice that the integration of renewable energy resources or ESS in microgrids, requires power electronic converters/inverters [6]. In the case of microgrids operating under dc paradigm, main ac grids and interfacing to the dc network via voltage source converters are depicted in Fig. 1 [7]. In case of ESS and photovoltaic generators, it is usual to employ dc-dc converters based on buck-boost technologies [8,9]; nevertheless, in case of wind generation an ac-dc inverter based on voltage source technology is needed [10]. The inclusion of power electronic converters on the grid modeling is particularly important from the dynamical analysis point of view (control design) to manage the energy interchange between different grid elements [7]. However, in specialized literature for optimal dispatch problems, ideal elements are assumed [11,12], which implies that, the distributed energy resources can be considered as direct inputs/outputs (generators and demands) connected to the dc system [13].

From the regulation point of view, governments worldwide have proposed and implemented regulatory and economic policies to encourage network operators using renewable energy resources, whose objective is to reduce the emission of greenhouse gases with the purpose of minimizing the harmful consequences of climate change

* Corresponding author.
E-mail addresses: wjgil@utp.edu.co (W. Gil-González), omontoya@utb.edu.co (O.D. Montoya), emkarou@gmail.com (E. Holguín), alejandro.garces@utp.edu.co (A. Garces), luisgrisales@itm.edu.co (L.F. Grisales-Noreña).

https://doi.org/10.1016/j.est.2018.10.025
Received 15 May 2018; Received in revised form 12 October 2018; Accepted 29 October 2018
Available online 12 November 2018
2352-152X/

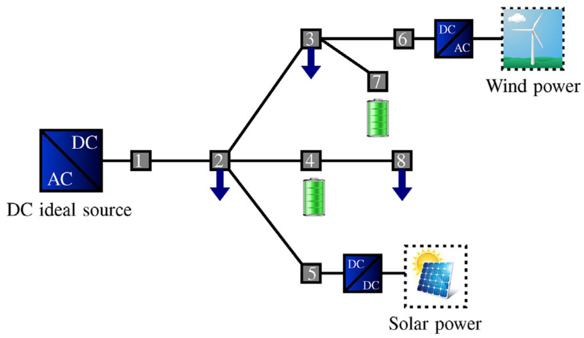

Fig. 1. Electrical configuration of the low-voltage dc power grid.

[14–16].

On the other hand, the use of these distributed generators can generate technical and operational problems if they are integrated into the grid in an uncoordinated form [3]. Some of the most important problems that can be introduced to the electrical power distribution systems are deterioration of the voltage profile and congestion in the transmission lines [3]. Also, a reduction in the operational efficiency of the wind and/or solar generators and ESS can be originated. This is due to that the peaks of generation do not match to the peaks in the demand curve, since it is essential to use devices that store energy in high generation hours and low demand, to be supplied to the system in low generation hours and high demand, and thus improve the efficiency in the operation of the electrical power systems [17,18,11].

The introduction of distributed generators (solar power) can become the users of the distribution, active entities since they can deliver energy to the network. This participation can fundamentally transform, the behavior and classic model of the electrical system [19–21].

It is important to empathize those classic methods for optimal operation in power systems, adapting the economic dispatch in the dc microgrids [22]. Also, their mathematical models are still non-linear non-convex problems, just like the ac power systems [20]. The non-convex problems have many local minimums, making them difficult to implement with optimization techniques. One solution to this type of inconvenience is to use convex optimization formulations which allows finding the global optimum [23]. Convex optimization formulations contain linear programming, quadratic optimization with linear constraints, and semidefinite optimization (SDP) [24].

Several investigations have proposed formulations for optimal operation of power grids. In [25], an optimal power flow for microgrids is proposed, based on genetic algorithm considering dc distribution network. In [26], an on-line optimal power flow of a networked dc microgrid is employed based on Lagrange multiplier to quantify the range of networked microgrids that can be accommodated by new control schemes. In [27], a primal-dual interior point method is developed for the generation and transmission loss problem minimization in dc power system considering power reservation constraints in a subset of generating units. However, none of these works consider ESS. In [16] and [28], ESS for minimizing energy losses in power grids have been developed with formulations of genetic algorithms and optimal programming, respectively. Furthermore, when exploring the problem of optimal operation in dc microgrid with ESS, it has not been presented a work that proposes a solution by applying SDP. The works presented in [29] and [30] have proposed formulations for optimal operation using SDP, but none of these consider GDs and ESS. The authors of [11] and [12] have proposed stochastic approaches to solve the problem of optimal operation of DGs based on solar and wind power in conjunction with ESS for reduction of the operating costs. These approaches are interesting in terms of the possibility to include the uncertainties associated to the primary energy resources as wind speed and solar radiation for wind and solar generation; notwithstanding, the mathematical model that represents these problems remains nonlinear, which the optimal global solution cannot be guaranteed. Additionally, stochastic approaches have been proposed in [31,32] and [33] for analyzing the impact of the uncertainties in renewable generation over the power grid, nevertheless, they use approximated linear models based on conventional dc power flow for ac grids and their applications are concentrated in large-scale ac power systems.

Unlike other investigations, in this paper an SDP model to solve the problem of economic dispatch in a dc microgrid with ESS is proposed, using a convex approximation where the results obtained are comparable with the exact mathematical model. Average generation values for DGs are considered in this paper to compare the performance of the proposed convex model with the conventional nonlinear non-convex formulation, since they are the most likely generating conditions for tropical countries like Colombia [34,35]. Notice that uncertainties of renewable generation based on wind or solar photovoltaic technologies are addressed in this paper via artificial neural networks (ANN) by applying the nonlinear autoregressive exogenous (NARX) model [11,36], in conjunction to Bayesian regularization as training algorithm. This methodology allows predicting solar radiation and wind speed considering as inputs of the ANN the following parameters: time, temperature, humidity and pressure; whit higher precision and low computational requirements; in addition, the main objective of this research is to reformulate the economic-dispatch problem in dc microgrids as linear problem guaranteeing the global optimum of the problem; which is identified in the paper as an opportunity of research that this paper tries to fulfill. Four scenarios are proposed to analyze the inclusion of energy storage in a dc microgrid for an economic dispatch in real-time to decrease the operating costs of the network. Additionally, the results obtained are compared with GAMS commercial optimization package to evaluate the performance of the proposed methodology.

The remain of this paper is organized as follows: Section 2 presents economic dispatch model for a dc microgrid with ESS. Section 3 shows SDP model and its application for economic dispatch model. Later, Section 5 presents the test system and the simulation scenarios. In Section 6 the general results are shown. Finally, Section 7 draws the conclusions followed by the list of references.

2. Economic dispatch model

An economic dispatch model of non-linear non-convex programming for a dc microgrid is presented in (1). The objective of this model is to minimize the purchase costs of energy in the stock market of conventional generators (generators based on the use of fossil fuels) in a dc microgrid, considering variable costs in the study period. Notice that this model corresponds to a dc grid adaptation of the mathematical model proposed by [16,37].

$$\min z = \sum_{t\in\Omega_T}\sum_{i\in\Omega_N} C(i,t)p_{\mathrm{CG}}(i,t)\Delta t, \tag{1}$$

where $C(i, t)$ and $p_{CG}(i, t)$ represent the purchase costs of energy and power generation of a conventional generator (CG) connected to node i for each period t, respectively. Ω_N and Ω_T denote sets that contain all nodes and operating horizons of the dc microgrid, respectively. Δt is delta of the time period. Besides, the characteristics of the electrical networks, i.e., operational and technical constraints are considered:

Load-generation balance:

$$\begin{aligned} p_{\mathrm{CG}}(i,t) + p_{\mathrm{DG}}(i,t) + p_{\mathrm{ESS}}(i,t) + p_L(i,t) \\ = \textstyle\sum_{j\in\Omega_N} g(i,j)v(i,t)v(j,t), \\ \forall\, i\in\Omega_N \cup \forall\, t\in\Omega_T, \end{aligned} \tag{2}$$

where (i, t) are the indices associated with with the node i in the period t; p_{DG}, p_{ESS}, and p_L are the distributed generators (DGs), energy storage systems (ESS) and demands powers, respectively. The parameter $g(i, j)$ is the conductance value extracted from the admittance matrix, and v

corresponds to the variable associated with the voltage profile.

Battery state of charge (SoC):

$$\mathrm{SoC}(i,\,t) = \mathrm{SoC}(i,\,t-1) - \phi(i)p_{\mathrm{ESS}}(i,\,t)\quad \forall\ i\in\Omega_N \cup \forall\ t\in\Omega_T \tag{3}$$

$$\mathrm{SoC}(i,\,t=1) = \mathrm{SoC}^{o}(i) \tag{4}$$

$$\mathrm{SoC}(i,\,t=24) = \mathrm{SoC}^{f}(i), \tag{5}$$

where *SoC* is the state of charge for each ESS; ϕ is the charging efficiency of ESS, SoC^o and SoC^f are initial and final state of charge of the ESS.

Eq. (3) represents the load/discharge power ratio of each ESS and its state of charge, Eqs. (4) and (5) control the condition initial and final state of charge of each ESS.

Minimum and maximum capacity:

$$\begin{aligned} p_{\mathrm{CG}}^{\min} &\le p_{\mathrm{CG}}(i,\,t) \le p_{\mathrm{CG}}^{\max}, &&\forall\ i\in\Omega_N \cup \forall\ t\in\Omega_T,\\ p_{\mathrm{DG}}^{\min} &\le p_{\mathrm{DG}}(i,\,t) \le p_{\mathrm{DG}}^{\max}, &&\forall\ i\in\Omega_N \cup \forall\ t\in\Omega_T,\\ p_{\mathrm{ESS}}^{\min} &\le p_{\mathrm{ESS}}(i,\,t) \le p_{\mathrm{ESS}}^{\max}, &&\forall\ i\in\Omega_N \cup \forall\ t\in\Omega_T, \end{aligned} \tag{6}$$

where superscripts *min* and *max* represent minimum and maximum capacity of the generator *i*, respectively.

Voltage limits:

$$v^{\min} \le v(i,\,t) \le v^{\max},\quad \forall\ i\in\Omega_N \cup \forall\ t\in\Omega_T, \tag{7}$$

where $v^{\min}$ and $v^{\max}$ represent the upper and lower operating limits of the voltage in each node *i*.

Power flow limits:

$$-f_{(i,j)}^{\max} \le f_{(i,j)(t)} \le f_{(i,j)}^{\max},\quad \forall\ \mathrm{ij}\in\Omega_N, \tag{8}$$

where $f_{(i,j)(t)}$ represent the power flow through line *ij* and $f_{(i,j)}(t)^{max}$ is its maximum power flow. Ingeneral, $f_{(i,j)(t)} \neq f_{(j,i)(t)}$, with

$$f_{(i,j)} = v_i(v_i - v_j)g(i,\,j) \tag{9}$$

SoC limits:

$$0 \le \mathrm{Soc}(i,\,t) \le 1,\quad \forall\ i\in\Omega_N \cup \forall\ t\in\Omega_T, \tag{10}$$

The above constraint represents capacity limits of stored energy for each ESS, which is between 0 to 100%. Additionally, it is important to mention that the battery life can be increased by varying the lower storage limits according to the IEEE 1561-2007 standard [38].

Observe that this problem is basically a non-linear non-convex model, due to (2), which is a non-affine equality constraint. Therefore, conventional optimization techniques cannot guarantee global optimum. In addition, a fast and reliable algorithm to be implemented in real-time operation for dc microgrids is required.

3. Semidefinite programming

Semidefinite programming (SDP) is part of the mathematical optimization, which is a field that has taken a great interest in recent years due to its theoretical and practical implications (control systems, communication and power systems) [39,13]. The SDP is a reformulation for nonlinear problems with aspects similar to linear programming in the sense that both have a solid mathematical basis and are effective using the interior point methods [40,41].

In some problems it is possible to affirm that SDP can reach to global optimum. However, this cannot be generalized for all problems. Even so, the solutions found are of good quality and can be implemented as initial points of non-linear optimization algorithms, improving the convergence times of these algorithms. Finally, in regards with non-linear problems, the SDP presents a formulation with greater ease of implementation, regardless of the number of variables associated with the model [42].

An SDP model corresponds to an optimization problem which can be described as:

$$\begin{aligned} \min\quad & z = \mathrm{Trace}(\mathrm{CX})\\ s.\,t.\quad & \\ & A_iX = B_i,\quad i = 1,\,2,\,\ldots,n\\ & X \succeq 0 \end{aligned} \tag{11}$$

where $X,\ C \in R^{n\times n}$ represent decision variables and cost function i.e., losses, profits, according to the objective of the problem), respectively. $A_i \in R^{n\times n}$ and $B_i \in R^{n\times n}$ are matrices that represent operating constraints of the system and $\succeq$ denotes positive semi-definiteness. Note that this model is similar to the formulation for linear programming problems (or some quadratic programming problems). Actually, linear programming problems are special cases of the SDP [23,24]. The difference of linear programming (and quadratic programming) and SPD comes from the decision variable which is a matrix for SDP instead of a vector.

3.1. Economic dispatch model as an SDP

A matrix formulation given in (12), is necessary to transform economic dispatch model for dc microgrids presented by (1)-(10) to the canonical formulation of the SDP defined in (11).

$$X(t) = V(t)V(t)^T,\quad \forall\ t\in\Omega_T \tag{12}$$

where $V(t) \in R^n$ is a column vector that contains the voltages of each node *i* in the study time period *t*. This definition allows to transform economic dispatch model described by (1)-(10) into an SDP model as follows,

$$\min\ z = \sum_{t\in\Omega_T} C(t)^T p_{\mathrm{GC}}(t)\Delta t \tag{13}$$

$$P_{\mathrm{CG}}(t) + P_{\mathrm{DG}}(t) + P_{\mathrm{ESS}}(t) + P_L(t) = \mathrm{diag}(\mathrm{GX}(t)) \tag{14}$$

$$\mathrm{SoC}(t) = \mathrm{SoC}(t-1) - \phi P_{\mathrm{ESS}}(t) \tag{15}$$

$$\begin{aligned} \mathrm{SoC}(t=1) &= \mathrm{SoC}^{o}\\ \mathrm{SoC}(t=24) &= \mathrm{SoC}^{f} \end{aligned} \tag{16}$$

$$\begin{aligned} P_{\mathrm{CG}}^{\min} &\le P_{\mathrm{CG}}(t) \le P_{\mathrm{CG}}^{\max}\\ P_{\mathrm{DG}}^{\min} &\le P_{\mathrm{DG}}(t) \le P_{\mathrm{DG}}^{\max} \end{aligned} \tag{17}$$

$$\begin{aligned} P_{\mathrm{ESS}}^{\min} &\le P_{\mathrm{ESS}}(t) \le P_{\mathrm{ESS}}^{\max}\\ J_N v_{\min}^2 &\le X(t) \le J_N v_{\max}^2 \end{aligned} \tag{18}$$

$$-F^{\max} \le F(t) \le F^{\max} \tag{19}$$

$$0 \le \mathrm{Soc}(t) \le 1 \tag{20}$$

$$X_{1,1}(t) = 1 \tag{21}$$

$$X \succeq 0 \tag{22}$$

$$\mathrm{Rank}(X(t)) = 1 \tag{23}$$

where $P_{CG}(t)$, $P_{DG}(t)$, $P_{ESS}(t)$, and $F(t)$ are column vectors with entries $[P_{\mathrm{CG}}(t)] = p_{\mathrm{CG}}(i,\,t)$, $[P_{\mathrm{DG}}(t)] = p_{\mathrm{DG}}(i,\,t)$, $[P_{\mathrm{ESS}}(t)] = p_{\mathrm{ESS}}(i,\,t)$, and $[F(t)] = f(\mathrm{ij},\,t)$, respectively. $J_N \in R^{n\times n}$ is an all-ones matrix, and $Diag(\cdot)$ and $Rank(\cdot)$ denote the main diagonal and rank of the corresponding matrix. *G* represents the conductance matrix of the system.

At this point, both models for economic dispatch are equivalent, i.e., both are non-convex. Notice that the only non-convex constraint is given by (23). When this constraint is relaxed, an SDP problem is achieved. This problem can be efficiently solved by the interior point method [30]. Although SDP model increases the number of variables, this does not imply that a larger number of variables can increase the computational cost [30].

It can be observed that the result of the SDP is a rank N matrix, which represents a problem. Therefore, it is necessary to employ a general decomposition to recover the vector *V*(*t*) from matrix *X*(*t*). To achieve this, it uses the decomposition method by means of eigenvalues and eigenvectors [13,42], as follows:

$$X(t) = \sum_{k=1}^{n} \lambda_k(t) W_k(t) W_k(t)^T, \; \forall\, t \in \Omega_T \tag{24}$$

where $\lambda_k(t)$ and $W_k(t)$ represent eigenvalues and its corresponding eigenvectors in each time period, respectively. If the representation for problem as an SDP is good enough, $N-1$ eigenvalues close to zero are expected. Therefore, rank of $X(t)$ is approximation to one and can be achieved as:

$$X(t) \approx \lambda_m(t) W_m(t) W_m(t), \; \forall\, t \in \Omega_T \tag{25}$$

where $\lambda_m(t)$ represents the maximum eigenvalue (i.e other eigenvalues are close to zero) in each time period. According to this approximation, it is possible to recover the vector $V(t)$ as

$$V(t) \approx \sqrt{\lambda_m(t)}\, W_m(t)^T, \; \forall\, t \in \Omega_T \tag{26}$$

The main advantage of this methodology is its efficiency and precision [13,42].

4. Receding horizon control

Receding horizon control (RHC) is a general-purpose control scheme that involves repeatedly resolving a restricted optimization problem, using predictions of some variables (i.e., wind speed or solar radiation) and restrictions on a moving time horizon. This idea of control is also well-known as a predictive control model. Fig. 2 depicts the main concept of RHC.

The RHC is used to minimize the forecasting errors with the following methodology:

- The predictions of wind speed and solar radiation by periods p based on a nonlinear autoregressive exogenous model (NARX) are determined. p represents prediction horizon.
- The economic dispatch is calculated from period t to period $t+p$ as presented in Section 3.1, using the predictions of wind speed and solar radiation.
- In period $t+1$ the predictions of wind speed and solar radiation are recalculated by periods p with measured of them in period t.

4.1. Nonlinear autoregressive exogenous

The nonlinear autoregressive network with exogenous inputs is a recurrent dynamic network, with feedback connections enclosing several layers of the network, that helps to predict stochastic variables by using historic [11]. These data are used for training an ANN following the next nonlinear learning rule:

$$y(t) = f(y(t-1), \ldots y(t-n_y), x(t-1), \ldots x(t-n_x)) \tag{27}$$

where x corresponds to the set of exogenous inputs, while y is the desired output, which depends on the last n_y values of the variable under

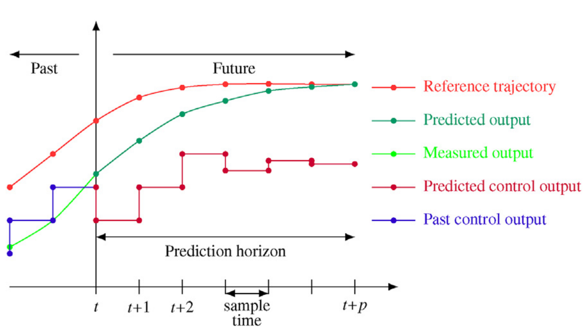

Fig. 2. RHC concept.

Table 1
Input parameters and desired outputs for wind and solar generation applications.

Photovoltaic		Wind	
Inputs	**Output**	**Inputs**	**Output**
Temperature Time	Solar Radiation	Temperature Humidity Pressure Time	Wind speed

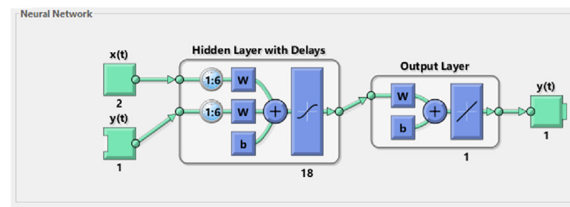

Fig. 3. NARX for solar radiation prediction.

prediction. Table 1 presents the set of inputs for predicting solar radiation and wind speed for photovoltaic and wind generation applications.

The training process for the NARX for predicting solar radiation and wind speed are implemented in MATLAB software via *ntstool* considering 2 inputs, 6 delays and 18 hidden neurons for the photovoltaic case, and 4 inputs, 4 delays and 12 neurons for the wind generation case. Fig. 3 shows the schematic implementation of the NARX for the photovoltaic case.

For both NARX, we use as training data 70% of the total data and 15% for adjusting and validation. Finally, Fig. 4 depicts historic solar radiation and wind speed used during ANN training process.

Notice that the solar radiation and wind speed presented in Fig. 4 cover a wide range of possibilities of generation during a normal day, which implies that the ANN has the possibility to have enough information to achieve a successful training process, in order to predict with low error (less than 3%) the power output in a typical day, as will be presented in results' section. Finally, for simplicity and readability, input components presented in Table 1 can be consulted to the authors.

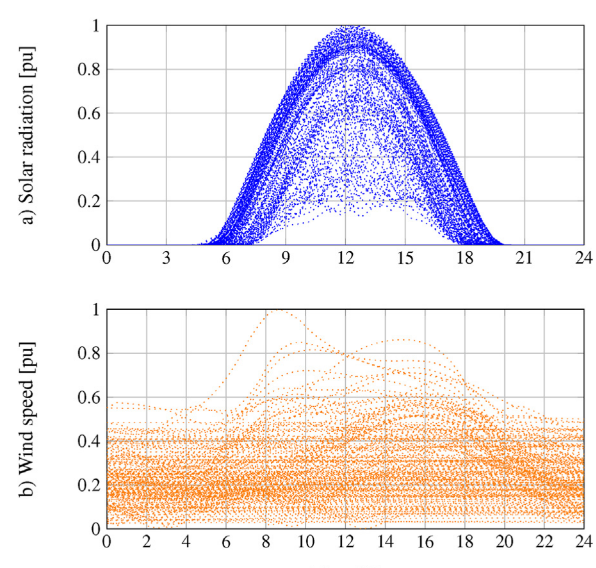

Fig. 4. Historic data for the NARX training process: a) solar radiation, and b) wind speed.

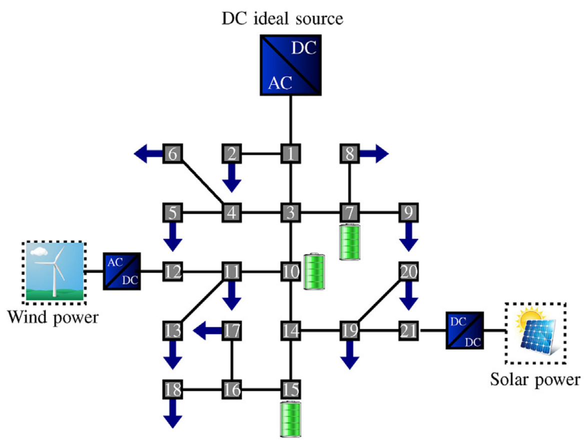

Fig. 5. Test system of the 21-node.

5. Test system and scenarios

5.1. The system under analysis

The proposed mathematical model is tested in a dc microgrid as presented in Fig 5 , which is an adaptation of the 21-node test system presented in [5]. The test system contains a radial topology of 21 nodes, 11 loads, a conventional generator, a wind power source and a solar power source connected to nodes 1, 12 and 21, respectively. Additionally, it has three energy storage systems connected to nodes 7, 10, and 15, respectively. Table 2 lists the resistance and load values.

Table 3 shows the costs to buy energy from the conventional generator for each time period *t*. These costs are considered in the context of prices in real time, which indicate that they are variable during the course of the day. In addition, the demand variation in each period *t* is also shown. For simplicity, the variation of each of the demands is considered equal.

Parameters of ESS are shown in Table 4, which have a storage capacity of 0.25 pu (node 7), 0.2 pu (node 10), and 0.2 pu (node 15) with charge times of 5 hours and discharge periods of 4 hours under nominal rates.

The first column of Table 4 indicates the node where the ESSs are connected. $P_{\text{charging}}^{\text{max}}$ and $P_{\text{discharging}}^{\text{max}}$ are the maximum powers of charging and discharging of each ESS along a period of time, respectively.

5.2. Simulation scenarios

Four simulation scenarios to examine the impact of the ESS in economic dispatch of a dc microgrid are proposed.

- **First scenario (S1):** This is the baseline scenario in which the energy storage in the economic dispatch is not considered.

Table 2
Parameters for the 21-nodes test system.

From	To	R_{line} [pu]	Load [pu]	From	To	R_{line} [pu]	Load [pu]
1	2	0.0053	0.25	11	12	0.0079	-
1	3	0.0054	-	11	13	0.0078	0.31
3	4	0.0054	-	10	14	0.0083	-
4	5	0.0063	0.04	14	15	0.0065	-
4	6	0.0051	0.14	15	16	0.0064	-
3	7	0.0037	-	16	17	0.0074	0.17
7	8	0.0079	0.13	16	18	0.0081	0.13
7	9	0.0072	0.32	14	19	0.0078	0.04
3	10	0.0053	-	19	20	0.0084	0.1
10	11	0.0038	0.17	19	21	0.0082	-

All parameters are in per unit. S_{Base} = 1 MW, V_{Base} = 13.2 kV.

Table 3
Test system data.

t [h]	C [$/kWh]	Demand variation [%]	t [h]	C [$/kWh]	Demand variation [%]
0	0.77	0.17	12	0.9	0.47
0.5	0.74	0.14	12.5	0.9	0.47
1	0.71	0.11	13	0.895	0.45
1.5	0.7	0.11	13.5	0.89	0.42
2	0.69	0.11	14	0.895	0.43
2.5	0.695	0.10	14.5	0.9	0.45
3	0.7	0.09	15	0.9	0.45
3.5	0.71	0.09	15.5	0.9	0.45
4	0.72	0.09	16	0.925	0.45
4.5	0.76	0.10	16.5	0.95	0.45
5	0.8	0.11	17	0.945	0.45
5.5	0.835	0.13	17.5	0.94	0.45
6	0.87	0.14	18	0.925	0.43
6.5	0.89	0.17	18.5	0.91	0.42
7	0.91	0.20	19	0.905	0.46
7.5	0.895	0.25	19.5	0.9	0.50
8	0.88	0.31	20	0.875	0.49
8.5	0.895	0.34	20.5	0.85	0.47
9	0.91	0.36	21	0.825	0.45
9.5	0.91	0.39	21.5	0.8	0.42
10	0.91	0.42	22	0.755	0.38
10.5	0.91	0.43	22.5	0.71	0.34
11	0.91	0.45	23	0.685	0.29
11.5	0.905	0.46	23.5	0.66	0.25

Table 4
ESS parameters.

ESS	$P_{\text{charging}}^{\text{max}}$ [pu]	$P_{\text{discharging}}^{\text{max}}$ [pu]	ϕ [%]
Node 7	0.008	0.013	40
Node 10	0.010	0.010	60
Node 15	0.010	0.010	60

- **Second scenario (S2):** It proposes to analyze the operation of energy storage considering a state of charge of 0% (ESS totally discharged) at the beginning and at the end of the dayily operation.
- **Third scenario (S3):** The operation of energy storage is analyzed considering a state of charge of 50% of its nominal capacity at the beginning and at the end of the dayily operation. However, during the period of operation, energy storage can take values between 0 and 100% of its capacity.
- **Fourth scenario (S4):** The recommendation of Standard IEEE 1561-2007 is considered in this scenario, where the standard defines that in order to increase the useful life for any energy storage connected to an electrical network, it should not have a stored energy lower than 50% of its nominal capacity [38].

The generation available of the wind and solar power source for four simulation scenarios is illustrates in Fig. 6.

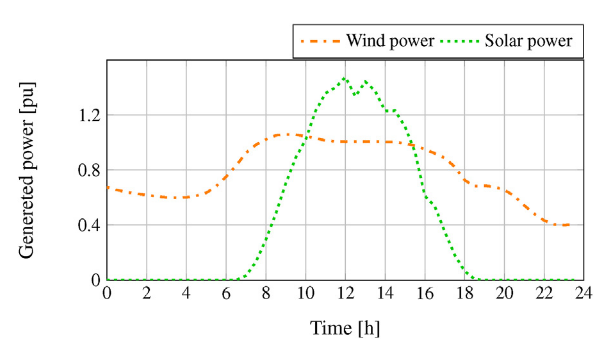

Fig. 6. Generation available for wind and solar power.

Table 5
Comparison between SDP and GAMS models for economic dispatch.

Model	S1 [$]	S2 [$]	S3 [$]	S4 [$]
SDP	7364.02	6755.18	6972,40	7058,6
GAMS	7364.02	6755.18	6972,40	7058,6
Error	8.62×10^{-6}	9.17×10^{-5}	6.48×10^{-5}	3.51×10^{-6}

6. Results

The SDP model proposed for a dc microgrid with ESS is solved using CVX programming software [43] on MATLAB environment [44]. Additionally, the SDP model is compared with GAMS commercial optimization package to evaluate the accuracy and efficiency of the proposed methodology [16].

All simulations were carried-out in a desk-computer INTEL(R) Core (TM) *i*7 − 7700, 3.60 GHz, 8 GB RAM with 64 bits Windows 10 Pro by using MATLAB 2017*b*.

6.1. Comparison between mathematical models

This part shows the results obtained with the SDP model and the model implemented in GAMS for the four proposed scenarios. Table 5 presents the results of objective function for each one of the proposed scenarios of the SDP model and the non-convex model in GAMS. In addition, the absolute error of SDP model for each scenario is also shown.

Notice in Table 5 that the SDP model presents a very good approach to solve the economic dispatch problem of a dc microgrid with energy storage since the absolute errors are less than $6.48 \cdot 10^{-5}$. The two first maximum average eigenvalues during the study period for each proposed scenario are shown in Table 6. Note that results of the eigenvalues confirm that the linear approximation used for the SDP model, matches the non-linear response of the economic dispatch problem of a dc microgrid with ESS, as defined in (26).

It is important to mention that in this part does not consider the forecast errors since the objective is analyzed the performance the SDP model regarding GAMS model.

6.2. Scenarios analysis

In this part, we analyze the inclusion of ESS in a dc microgrid for an economic dispatch in real-time to decrease the operating costs of the network. However, the forecast of the wind speed and solar radiation are not considered since the objective in this part is to show the effect that has the ESS in the operation of dc microgrid.

As can be seen in Table 5, the worst scenario of operation is S1. This is because the total power consumption need to be supported by a combination of the distributed generators and the conventional generator. Since ESS are not connected to the grid, the CG needs to provide the remaining power to guarantee the power balance independent of the energy price (including support for power oscillations), which increments significantly the total operating grids costs per day.

In scenario S2, it is possible to observe that the inclusion of ESS in the system operation is more efficient than presented in scenario S1 since it is feasible to store energy when there is greater distributed generation than demand in the system. The inclusion of ESS provides a reduction of 690.0 US$ on objective function, i.e., a saving of 8.27% in daily operating costs.

Table 6
Eigenvalues averaged.

λ_m	S1	S2	S3	S4
λ_m^1	21.05	21.06	21.06	21.06
λ_m^2	1.1×10^{-3}	1.3×10^{-3}	1.4×10^{-3}	1.4×10^{-3}

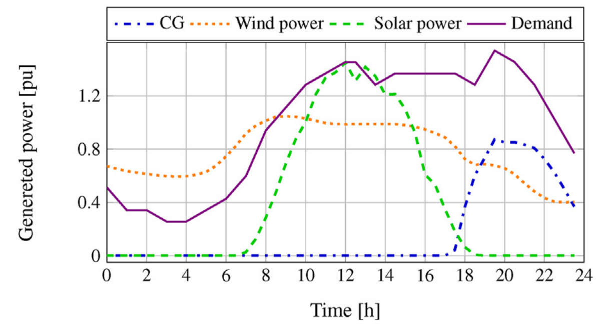

Fig. 7. Generated power by conventional generator, solar and wind power.

In scenario S3, a slight increase in operating costs can be observed in regards with the scenario S2 (a saving of 4.89% in daily operating costs with respect to scenario S1). This is because at the end of the period, the ESS need to have a stored energy of 50%, which coincides with the hours where there is less generation availability (photovoltaic generation is zero).

Scenario S4 shows an increase of the 9.37% in operating costs in relation with the scenario S2. This situation is expected, since the ESS are limited to maintaining a minimum load of 50% at any period of time, for this reason, the energy that can be injected into the network is significantly reduced in contrast with scenarios S2 and S3. However, this operation restriction in the ESS have the advantage of increasing the useful life of the batteries (Standard IEEE 1561-2007).

On the other hand, Figs. 7 and 8 illustrate results for the best scenario of proposed operation, i.e., scenario S2.

Fig. 7 depicts generated power of the conventional generator, wind and solar power for each period of time. State of charge and transferred power for each ESS are shown in Fig. 8.

Notice in Fig. 7 that conventional generator is only required at hours where demand is greater than the availability of distributed generators (time periods from 18:00 to 5:00, and at 16:00). This situation also occurs when ESS do not have enough stored energy or they are in process of charging (see Fig. 8a)).

Note in Fig. 8(a) that ESS are charged in the periods where the energy costs have on average the lowest prices (period from 00:00 to

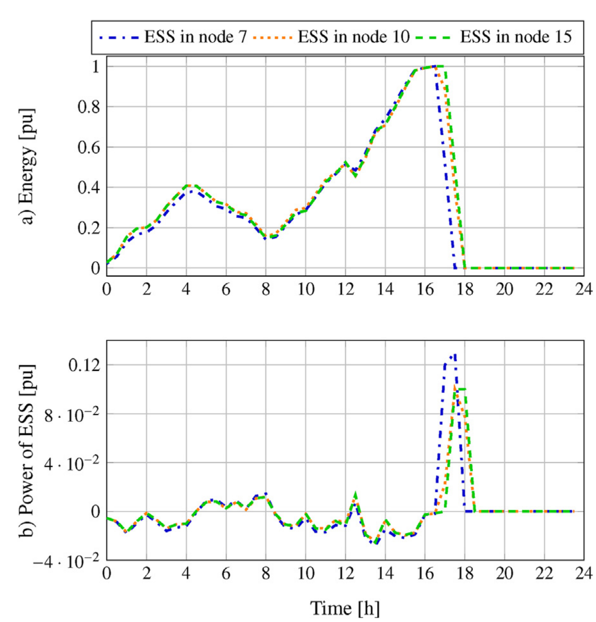

Fig. 8. Stored energy and transferred power by ESS.

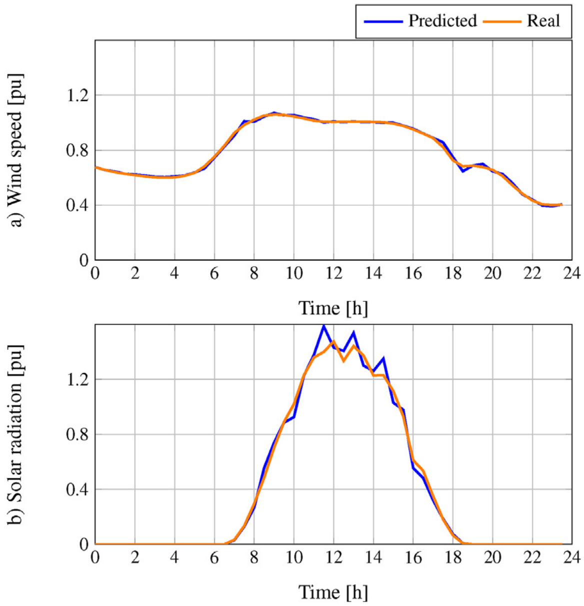

Fig. 9. Real and prediction of the wind speed and solar radiation.

Table 7
Comparison in the objective function for real and RHC

	S2 [$]	S3 [$]	S4 [$]
Real	6755.19	6972.41	7058.60
RHC	6755.19	6972.41	7058.60

04:00, see Table 3) or when the distributed generators are at their maximum values (period between 08:00 to 17:00, see Fig. 7).

Consequently, the discharge periods occur when the energy costs are much more expensive and/or when it is not enough with the distributed generators to attend the demanded power (periods between 04:00 and 08:00 and after 17:00). The other proposed scenarios are not shown graphically, because their analysis is similar to that presented for scenario S2.

Note in Fig. 8(b) that each one of ESS fulfills the charging and discharging power limits established in Table 4.

6.3. Stochastic optimization

In this part, we include the forecast of the wind speed and solar radiation where its effect is diminished with the RHC and NARX described in Section 4.

The NARX has been trained with 7776 data which were registered every 30 minutes for a total of 162 days (see Fig. 4). The prediction of the wind speed employs characteristics such as time, temperature-relative humidity, and pressure, while the temperature and time are used for the prediction of solar radiation (see Table 1). All data were taken of solar radiation data in [45]. Fig. 9 depicts the speed wind and solar radiation for an any day. Additionally, their predictions are also shown which are determined by the RHC.

Observe in Fig. 9 that the prediction for the wind speed and solar radiation by the RHC is suitable with a mean squared error of $1.83\ 10^{-5}$ and $5.44\ 10^{-4}$ for the wind speed and solar radiation, respectively. This demonstrates that the RHC is a suitable methodology to minimize the forecast errors. Table 7 presents the results of objective function when there is no error in the forecast (Real) as well as the RHC is implemented.

Observe in Table 7 that the operating costs of the network are not affected when the economic dispatch is programmed by the RHC. This entails the operation of ESS continues being appropriate for the grid. It is important to notice that the RCH can be extended for the time periods shorter than the used in this paper.

7. Conclusions

A mathematical approach for the optimal operation focused on the economic dispatch of a dc microgrid with ESS, through a mathematical model of SDP was proposed. This formulation allowed transforming the non-linearity and non-convexity of the economic dispatch into a convex approach that is easy to implement in specialized software, such as the CVX. The proposed mathematical approach contemplated the efficient operation of the dc microgrid over a period of time with variable energy purchase price, which makes it a practical methodology to apply in real-time, because the results obtained clearly showed that the solutions found by the SDP were very close to those presented by the exact model implemented in GAMS. For this reason, the SDP model is an efficient methodology that can be applied to problems of planning and operation of dc microgrid with a good level of precision, reducing the computational effort in solving the problem.

It was proved that installing ESS in dc microgrids reduce operating costs. However, it is necessary to have an efficient operation strategy that allows maximizing the use of the ESS, since the inadequate programming of the ESS affects their useful life. Additionally, several operational strategies can be obtained according to the objectives set by the network operator (e.g., loss minimization, investment costs, operating costs, etc.).

It was proposed a methodology based on a RHC to reduce the forecasting errors of the wind speed and solar radiation. The predictions for the RHC were determined using a NARX. Methodology demonstrated to be suitable to minimize the forecast errors and the operation of ESS continues being appropriate for economic dispatch.

As future works, it would be possible to extend the proposed model to a dc-ac microgrid and also include active power losses on the power electronic converters.

Financial support

This work was partially supported by the National Scholarship Program Doctorates of the Administrative Department of Science, Technology and Innovation of Colombia (COLCIENCIAS), by calling contest 727-2015.

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