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Efficient Day-Ahead Dispatch of Photovoltaic Sources in Monopolar DC Networks via an Iterative Convex Approximation

Oscar Danilo Montoya [1,2], Luis Fernando Grisales-Noreña [3,*] and Jesús C. Hernández [4,*]

1 Grupo de Compatibilidad e Interferencia Electromagnética (GCEM), Facultad de Ingeniería, Universidad Distrital Francisco José de Caldas, Bogotá 110231, Colombia
2 Laboratorio Inteligente de Energía, Facultad de Ingeniería, Universidad Tecnológica de Bolívar, Cartagena 131001, Colombia
3 Department of Electrical Engineering, Faculty of Engineering, Universidad de Talca, Curicó 3340000, Chile
4 Department of Electrical Engineering, University of Jaén, Campus Lagunillas s/n, Edificio A3, 23071 Jaén, Spain
* Correspondence: luis.grisales@utalca.cl (L.F.G.-N.); jcasa@ujaen.es (J.C.H.)

Abstract: The objective of this research is to propose an efficient energy management system for photovoltaic (PV) generation units connected to monopolar DC distribution networks via convex optimization while considering a day-ahead dispatch operation scenario. A convex approximation is used which is based on linearization via Taylor's series expansion to the hyperbolic relations between voltages and powers in the demand nodes. A recursive solution methodology is introduced via sequential convex programming to minimize the errors introduced by the linear approximation in the power balance constraints. Numerical results in the DC version of the IEEE 33-bus grid demonstrate the effectiveness of the proposed convex model when compared to different combinatorial optimization methods, with the main advantage that the optimal global solution is found thanks to the convexity of the solution space and the reduction of the error via an iterative solution approach. Different objective functions are analyzed to validate the effectiveness of the proposed iterative convex methodology (ICM), which corresponds to technical (energy losses reduction), economic (energy purchasing and maintenance costs), and environmental (equivalent emissions of CO_2 to the atmosphere in conventional sources) factors. The proposed ICM finds reductions of about 43.9754% in daily energy losses, 26.9957% in energy purchasing and operating costs, and 27.3771% in CO_2 emissions when compared to the benchmark case in the DC version of the IEEE 33-bus grid. All numerical validations were carried out in the MATLAB programming environment using the SEDUMI and SDPT3 tools for convex programming and our own scripts for metaheuristic methods.

Keywords: convex relaxations; efficient energy management system; photovoltaic generation; monopolar DC distribution networks; optimal power flow solution

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1. Introduction

Monopolar DC distribution grids show promise to interface substations with end users at medium- and low-voltage levels [1] as they are characterized by low energy losses and excellent voltage profiles [2]. The main advantage of DC networks when compared to classical AC grids is the absence of frequency and reactive power [3]. This makes them easily controllable as the primary variable corresponds to the voltage output at the terminals of the substation [4]. Monopolar DC networks can be analyzed from two perspectives. The first approach involves the dynamical analysis of the networks, which corresponds to studies associated with the control of power electronic converters [5] and studies regarding transient phenomena and protection coordination [6–8], among others. The second area of analysis is associated with the application of optimization methods to improve the electrical performance of DC networks by including renewable generation based on photovoltaic (PV) and wind power technologies [9–11] as well as energy storage devices [12,13]. Both research areas require efficient methodologies to address their main challenges, i.e., efficient control design methodologies or effective optimization techniques, in order to obtain the highest possible benefits while operating a network.

This research takes an interest in optimization applied to monopolar DC networks with high penetration of renewable generation based on PV technologies to improve the daily performance of distribution networks. The specialized literature has proposed multiple approaches in topics related to this field of study, which are detailed below.

The authors of [14] proposed the application of the salp swarm algorithm (SSA) to determine the optimal dispatch of PV generation units in DC distribution networks while considering multiple objective function formulations, i.e., daily energy losses, daily CO_2 emissions, and expected daily energy production costs. Numerical results in two distribution networks composed of 27 and 33 nodes demonstrated the effectiveness of the SSA approach when compared to different combinatorial optimizers, such as particle swarm optimization and the crow search algorithm. In [12], a semi-definite programming relaxation was proposed to operate renewable energy resources and batteries in monopolar DC networks. Numerical results demonstrate that the semidefinite approximation found the same optimal solution as the exact NLP model in the GAMS software. The main difficulty of the semi-definite programming method is the square increase in the number of variables with the number of nodes of the distribution grid under analysis, which can increase the time required by the semidefinite programming model to minutes or hours in some cases. The authors of [15] proposed the application of the second-order cone programming approach to determine the optimal dispatch of PV generation units in monopolar DC networks with the aim of minimizing the total greenhouse gas emissions in rural diesel-fed networks. Numerical results in different test feeders confirmed the efficiency of the conic model in dispatching these PV sources when compared to the exact solution provided by the GAMS software. The work by [16] proposed a general design of PV sources for residential applications in Bogotá while considering the Colombian laws regarding renewable generation and energy costs for residential users in four economic strata. Numerical validations demonstrated that end-user investments can be recovered in periods between 4 and 8 years. The authors of [17] proposed a convex approximation methodology to operate wind power and PV generation in medium-voltage DC distribution networks. Numerical results in two test feeders composed of 10 and 39 nodes showed the effectiveness of the proposed convex model when comparing their solution against different commercially available optimization tools. Other approaches used to study the problem regarding the optimal integration of renewable generators in electrical distribution grids include particle swarm optimization [18,19], ant colony optimization [20], the multiverse optimization approach [21], the krill herd algorithm [22], the whale optimization algorithm [23], evolutionary programming [24], and simulated annealing [25]. In addition, a complete review regarding the optimal design, modeling, and simulation of PV generation systems in distribution grids with grid-connected and standalone characteristics was presented by the authors of [26].

The main characteristic of the aforementioned combinatorial optimization methods for locating and sizing PV generation units in electrical distribution networks is that they all work with master-slave methodologies, where the master stage defines the nodes where the PV generation units must be installed. The slave stage (typically a power flow solver) is entrusted with defining the optimal sizes of these PV generation units. In addition, the most common objective function corresponds to minimizing the power/energy losses, which has a nonlinear nature and is a typical performance indicator used by regulatory entities in the sector to measure the efficiency of electrical networks.

Based on this review of the state of the art, this research makes the following contributions:

i. A convex approximation to the problem regarding the efficient dispatch of PV generation units in monopolar DC networks while considering three different objective functions. The convex approximation is reached by linearizing the hyperbolic relationship between voltage and powers in constant power terminals as well as a linear approximation to these relations in the case of renewable generation.

ii. An iterative solution methodology to reduce/eliminate the error induced in the final objective function value due to the use of linearization methods.

Note that the proposed optimization methodology assumes that the distribution company has characterized all the generation inputs of the PV sources and demand behaviors as they are included in the optimization model as external inputs with no uncertainties. In addition, the location of the PV generation units has also been predefined by the utility. This study takes interest in presenting an efficient methodology to operate these resources in order to minimize three objective functions, which are technical (daily energy losses), economic (daily energy purchasing and maintenance costs), and environmental (daily kilograms of CO_2 emissions). The main limitations of our approach are (i) the assumption that the demand and PV inputs are constant parameters with no uncertainties; (ii) the fact that the resolution of the day-ahead dispatch of the PV plants is defined as Δ_h, and it is assumed as 1 hour in a daily operation environment, which implies that the precision of the proposed convex depends on the exactness of the renewable and demand prediction for each hour; and (iii) the fact that the proposed solution method is only applicable to monopolar DC configurations, which implies that more research is required to include the stochastic nature of renewables and demand as well as considering bipolar DC configurations with different neutral-to-ground connections and reducing the time-step of Δ_h to minutes and allowing for more accurate results when compared to the daily variations of the PV plants and load measurements in the substation.

The remainder of this research is structured as follows. Section 2 describes the problem of mathematically designing an efficient day-ahead dispatch tool for renewable generators in monopolar DC distribution networks through a nonlinear programming formulation. Section 3 presents the proposed convexification approach and the iterative solution used to minimize the estimation error induced by the linearization method. Section 4 reveals the main characteristics of the test feeder, which corresponds to the DC version of the IEEE 33-bus grid adapted for the operative conditions of the city of Medellín, Antioquia, Colombia. Section 5 shows all the numerical validations for the proposed convex model and a comparison with different combinatorial optimization methods recently reported in the specialized literature. Finally, Section 6 presents the main conclusions of this study in addition to some possible future works.

2. General NLP Formulation

This section addresses the general NLP model that represents the optimal operation of PV plants in monopolar DC distribution networks. Firstly, the classical multi-period optimal power flow formulation is presented considering the total grid energy losses, which must be minimized as the objective function. Secondly, two alternative functions are presented, which can be considered in the day-ahead economic dispatch of PV plants in monopolar DC networks. These functions are economical and environmental. The economic function considers the energy purchasing costs at the terminals of the substation bus in addition to the operating cost of the PV sources, and the environmental function is related to the minimization of the CO_2 emissions into the atmosphere by the distribution network, i.e., as an equivalent of the emissions when the electrical energy comes from thermal sources.

2.1. Energy Loss Minimization

The problem regarding the day-ahead dispatch of PV generation units in monopolar DC networks can be modeled through a nonlinear programming model, where the nonlinearities appear in the power balance constraints and the objective function, while the remaining constraints belong to the family of linear functions. In this research, the objective function associated with the day-ahead dispatch of PV generation units corresponds to the minimization of the total grid energy losses as defined in Equation (1).

$$E_{\text{loss}} = \sum_{h\in\mathcal{H}} \sum_{k\in\mathcal{N}} \left(v_{k,h} \sum_{m\in\mathcal{N}} G_{km} v_{m,h} \right) \Delta_h, \tag{1}$$

where E_{loss} represents the objective function value, i.e., the total grid energy losses for a daily operation scenario; $v_{k,h}$ and $v_{m,h}$ are the decision variables regarding the voltages at nodes k and m for time h; G_{km} denotes the km element of the conductance matrix that relates nodes k and m (note that this value is different from zero only if there is a physical connection between both nodes); Δ_h is the fraction of time in which all the variables of the optimization problem take constant values; $\mathcal{H}$ is the set that contains all the periods considered in the day-ahead economic dispatch (in this research, $\Delta_h = 1$ h, and the cardinality of the set $\mathcal{H}$, i.e., its number of elements is 24, which implies that the proposed day-ahead dispatch is programmed while considering predictions regarding demand and PV generation inputs in fractions of 1 h); and $\mathcal{N}$ corresponds to the set that contains all the nodes of the network.

Remark 1. *The main characteristic of the objective function in (1) is that it corresponds to a quadratic function from the family of convex functions, so it can be represented as follows:*

$$E_{loss} = \sum_{h\in\mathcal{H}} V_h^{\top} \mathbf{G} V_h, \tag{2}$$

where V_h is a vector that contains all the voltage variables per period of analysis, and $\mathbf{G}$ is the conductance matrix, which is positive semi-definite if and only if the monopolar DC network has all its nodes connected at least in tree form, i.e., there are no isolated nodes [27].

The constraints associated with the day-ahead dispatch of PV generation units in monopolar distribution networks are listed from (3) to (8).

$$p_{k,h}^{cg} + p_{k,h}^{pv} - p_{k,h}^{d} = v_{k,h} \sum_{m\in\mathcal{N}} G_{km} v_{m,h}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{3}$$

$$v_{k,h} - v_{m,h} - r_{km} i_{km,h} = 0 \quad \{\forall km \in \mathcal{L},\ \forall h \in \mathcal{H}\} \tag{4}$$

$$p_{k,h}^{\min} \le p_{k,h} \le p_{k,h}^{\max}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{5}$$

$$p_{k,h}^{pv,\min} \le p_{k,h}^{pv} \le p_{k,h}^{pv,\max}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{6}$$

$$v_{k}^{\min} \le v_{k,h} \le v_{k}^{\max}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{7}$$

$$\left| i_{km,h} \right| \le i_{km}^{\max}, \quad \{\forall km \in \mathcal{L},\ \forall h \in \mathcal{H}\} \tag{8}$$

where $p_{k,h}^{cg}$ represents the power generation input in the conventional source (slack node) connected at node k in the period h; $p_{k,h}^{pv}$ is the power injection in the PV sources connected at node k in the period h; $p_{k,h}^{d}$ means the total constant power consumption for a load connected at node k in the period h; r_{km} corresponds to the resistive parameter associated with the distribution line that connects nodes k and m; $i_{km,h}$ is the current flowing through the route that connects nodes k and m at time h; $p_{k,h}^{\min}$ and $p_{k,h}^{\max}$ correspond to the lower and upper bounds allowed for the power generation of the conventional source at each time; $p_{k,h}^{pv,\min}$ and $p_{k,h}^{pv,\max}$ are the minimum and maximum generation bounds associated with PV generation; $v_{k}^{\min}$ and $v_{k}^{\max}$ are constant parameters related to the minimum and maximum voltage values allowed at all nodes of the network (i.e., the voltage regulation constraint); and $\mathcal{L}$ is the set that contains all the branches of the monopolar DC distribution grid.

Note that the set of constraints (3)–(8) can be interpreted as follows: Equation (3) expresses the power equilibrium at each node of the network at each time; equality constraint (4) represents Kirchoff's second law applied in each branch of the network, i.e., it allows determining the voltage drop at each distribution line; box-type constraints (4) and (5) are related to the generation capacities of the conventional and renewable generators connected to the monopolar DC network; box-type constraint (7) is associated with the voltage regulation bounds allowed by regulatory policies at any node of the distribution network; and inequality constraint (8) represents the maximum thermal bounds allowed for each conductor at each route that connects nodes k and m.

Remark 2. *The main characteristic of the set of constraints (3)–(8) is that the only non-convex restriction is the power equilibrium at each bus of the network, as shown in Equation (3), since it exhibits the product between voltages in all the buses interfaced by the conductance matrix [28].*

2.2. Economic and Environmental Objective Functions

Day-ahead dispatch studies typically involve economic objective functions associated with minimizing the total expected generation and operation costs. Here, an objective function associated with the maintenance costs of the PV plants, added with the energy purchasing costs at the substation terminals, is proposed as defined in (9).

$$\min E_{\text{costs}} = \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}} \left(C^{s}_{kWh} p^{s}_{k,h} + C^{pv}_{O\&M} p^{pv}_{k,h} \right) \Delta_h, \tag{9}$$

where E_{costs} denotes the expected operating costs for the analyzed period of operation; C^{s}_{kWh} corresponds to the energy purchasing costs in the conventional generator (thermal source) and/or the equivalent substation node, and $C^{pv}_{O\&M}$ is the expected maintenance and operation costs in the PV plant, which has a power output defined as $p^{pv}_{k,h}$.

An additional objective function for operating renewable energy resources in electrical distribution networks is minimizing the expected greenhouse gas emissions caused by energy production with thermal plants. In the case of distribution networks, the total CO_2 emissions are quantified as a factor that multiplies the energy production at the substation bus. This objective function is formulated as presented in (10).

$$\min E_{CO_2} = \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}} \gamma_s p^{s}_{k,h} \Delta_h, \tag{10}$$

where E_{CO_2} is the objective function value associated with minimizing the emissions of carbon dioxide into the atmosphere; γ_s denotes the emissions coefficient associated with the equivalent kilograms of CO_2 per kilowatt of electrical energy produced at the substation bus.

The objective functions (9) and (10) are linear functions, which implies that both of them are convex (also concave) and that the only one set of nonlinear non-convex constraints in the optimization model corresponds to the power balance equations in (3). The following section proposes a convexification approach that transforms constraint (3) into a linear equivalent using a linearization approach based on Taylor's series expansion.

3. Proposed Convexification Method

Here, a linear approximation is presented which is based on the convex OPF approach recently reported in [29] for obtaining a convex equivalent formulation of the power balance constraint. This approximation is carried out in two steps. The first step linearizes the hyperbolic relation between voltages and powers strictly for the demand nodes. The second step approximates the hyperbolic relation between voltages and powers strictly for renewable generation sources.

3.1. Linearization of the Demand Nodes

Let us consider that the demanded current in a constant power terminal can be expressed as follows:

$$i^{d}_{k,h} = \frac{p^{d}_{k,h}}{v_{k,h}}, \tag{11}$$

where it is possible to obtain a linear equivalent if Taylor's series expansion is applied [29]. This approximation is presented in Equation (12), which assumes that the linearizing point is $v^{0}_{k,h}$.

$$i^{d,0}_{k,h} \approx \left(\frac{2}{v^{0}_{k,h}} - \frac{1}{\left(v^{0}_{k,h}\right)^{2}} v_{k,h} \right) p^{d}_{k,h}. \tag{12}$$

Note that (12) is now a linear function of the voltage profiles. The initial values are defined by $v^{0}_{k,h}$, which implies that it is substituted into the power balance constraint (3). Thus, the component associated with the demand nodes becomes a convex function.

3.2. Linearization for PV Nodes

In the case of renewable generation, a linearization is considered in which both components of the PV net current injection (i.e., the numerator associated with PV power injection and the denominator corresponding to the voltage value), as presented below:

$$i^{pv}_{k,h} = \frac{p^{pv}_{k,h}}{v_{k,h}}, \tag{13}$$

where it is evident that, due to the voltage regulation constraint (7), the voltage variable has small variations regarding the initial value:

$$i^{pv}_{k,h} = \frac{p^{pv}_{k,h}}{v^{0}_{k,h} + \Delta_{v_{k,h}}}, \tag{14}$$

which means that $\Delta_{v_{k,h}} \approx 0$ during the iteration process, implying that (14) can be approximated as a linear equation, as defined in (15):

$$i^{pv,0}_{k,h} = \frac{p^{pv}_{k,h}}{v^{0}_{k,h}}. \tag{15}$$

3.3. Recursive Optimization Model

Once all the components of the power balance constraints have been linearized, Equation (3) can be rewritten as an affine equation, as presented in (16):

$$\frac{p_{k,h}}{v^{0}_{k,h}} + \frac{p^{pv}_{k,h}}{v^{0}_{k,h}} - \left(\frac{2}{v^{0}_{k,h}} - \frac{1}{\left(v^{0}_{k,h}\right)^{2}} v_{k,h} \right) p^{d}_{k,h} = \sum_{m \in \mathcal{N}} G_{km} v_{m,h}, \quad \{\forall k \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \tag{16}$$

Remark 3. *In the approximated power balance constraint (16), it is evident that using the linearizing point $v^{0}_{k,h}$ induces an estimation error in the final value of the power and voltages. However, to minimize this error, the complete optimization model (1)–(8) can be recursively solved by using an iterative counter t.*

The proposed recursive model is presented in (17), where the superscripts t and $t+1$ are introduced in order to define the linearizing point $v^t_{k,h}$ and the next set of voltage values, $v^{t+1}_{k,h}$, which are obtained after solving this model.

$$\begin{aligned}
&\text{Obj. func.:} \quad E_{\text{loss}} = \sum_{h\in\mathcal{H}}\sum_{k\in\mathcal{N}}\left(v^{t+1}_{k,h}\sum_{m\in\mathcal{N}}G_{km}v^{t+1}_{m,h}\right)\Delta_h,\\
&\text{Subject to:}\\
&\frac{p_{k,h}}{v^t_{k,h}} + \frac{p^{pv}_{k,h}}{v^t_{k,h}} - \frac{2p^d_{k,h}}{v^t_{k,h}} + \frac{p^d_{k,h}}{\left(v^t_{k,h}\right)^2}v_{k,h} = \sum_{m\in\mathcal{N}}G_{km}v^{t+1}_{m,h}, && \begin{Bmatrix}\forall k\in\mathcal{N}\\ \forall h\in\mathcal{H}\end{Bmatrix}\\
&v^{t+1}_{k,h} - v^{t+1}_{m,h} - r_{km}i_{km,h} = 0 && \begin{Bmatrix}\forall km\in\mathcal{L}\\ \forall h\in\mathcal{H}\end{Bmatrix}\\
&p^{\min}_{k,h} \le p_{k,h} \le p^{\max}_{k,h}, && \begin{Bmatrix}\forall k\in\mathcal{N}\\ \forall h\in\mathcal{H}\end{Bmatrix}\\
&p^{pv,\min}_{k,h} \le p^{pv}_{k,h} \le p^{pv,\max}_{k,h}, && \begin{Bmatrix}\forall k\in\mathcal{N}\\ \forall h\in\mathcal{H}\end{Bmatrix}\\
&v^{\min}_k \le v^{t+1}_{k,h} \le v^{\max}_k, && \begin{Bmatrix}\forall k\in\mathcal{N}\\ \forall h\in\mathcal{H}\end{Bmatrix}\\
&\left|i_{km,h}\right| \le i^{\max}_{km}, && \begin{Bmatrix}\forall km\in\mathcal{L}\\ \forall h\in\mathcal{H}\end{Bmatrix}
\end{aligned} \tag{17}$$

Regarding the minimization of the error induced by the linearization approach that allowed the NLP model (1)–(8) to become a convex approximation with the structure (17), Figure 1 presents the recursive solution methodology proposed in this research.

Remark 4. *Note that the iterative convex model (17) is recursively solved via a convex optimization tool until the desired convergence is achieved. This convergence criterion is defined as the difference in voltage magnitudes between two consecutive iterations, which fulfills the expected convergence error. The convergence criterion is presented below.*

$$\max_{k\in\mathcal{N},\, h\in\mathcal{H}}\left|v^{t+1}_{k,h} - v^t_{k,h}\right| \le \varepsilon, \tag{18}$$

where ε *means the maximum convergence error, defined as* 1×10^{-10} *[29].*

The main characteristic of the proposed iterative convex methodology shown in Figure 1 is the recursive solution of the approximated convex model (17), which is carried out until the desired convergence is reached. However, in order to implement this model in the convex disciplined tool (CVX) of the MATLAB programming environment, the pseudo-code in Algorithm 1 must be followed.

Note that the CVX implementation of the approximated convex model in (17) is based on the interpreted language programming structure, where the model is written as symbolic using parameters, vectors, and matrices, which allows researchers to focus on developing new efficient optimization models, not on the solution technique itself. For more details regarding the use of the CVX programming tool, see [30,31].

Algorithm 1: Implementation of the approximated convex model (17) in the CVX environment of MATLAB.

Data: Define grid parameters (Demands, PV inputs, and grid topology)
Obtain the per-unit representation of the system;
Define the upper and lower bounds of the optimization variables;
cvx_begin quiet;
    cvx_solver SDPT3;
    Define variables   Define the objective function;
    subject to;
    Write model equalities;
    Write model inequalities;
cvx_end;
Result: Report voltage variables

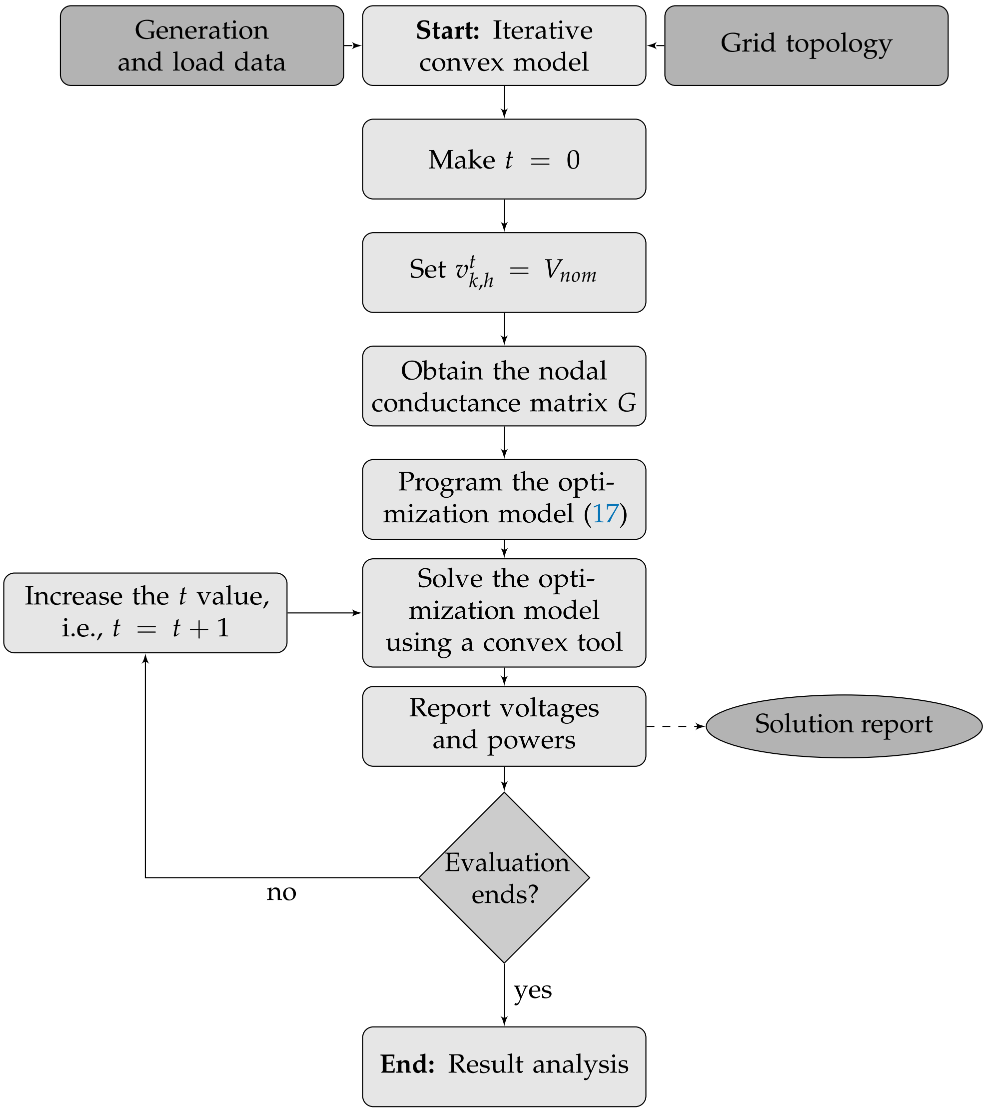

Figure 1. Iterative solution methodology for solving the optimization model (17).

4. Test Feeder Characteristics

In this section, a distribution network composed of 33 nodes characterized for an urban network in Colombia (Medellín) is considered as a test feeder. The electrical configuration of this distribution network is presented in Figure 2.

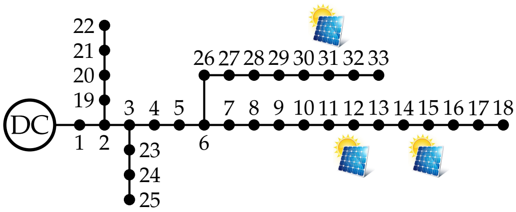

Figure 2. Urban distribution network composed of 33 nodes.

This test system includes three PV sources located at nodes 12, 15, and 31, with a nominal power of 2400 kW. In addition, it operates with a nominal voltage of 12.66 kV at the terminals of the substation. The parametric information regarding the loads and branches of the 33-bus grid is presented in Table 1.

Table 1. Parametric information of the urban distribution network.

Line l	Node i	Node j	R_{ij} (Ω)	P_j (kW)	$I_l^{\max}$ (A)
1	1	2	0.0922	100	320
2	2	3	0.4930	90	280
3	3	4	0.3660	120	195
4	4	5	0.3811	60	195
5	5	6	0.8190	60	195
6	6	7	0.1872	200	95
7	7	8	1.7114	200	85
8	8	9	1.0300	60	70
9	9	10	1.0400	60	55
10	10	11	0.1966	45	55
11	11	12	0.3744	60	55
12	12	13	1.4680	60	40
13	13	14	0.5416	120	40
14	14	15	0.5910	60	25
15	15	16	0.7463	60	20
16	16	17	1.2890	60	20
17	17	18	0.7320	90	20
18	2	19	0.1640	90	30
19	19	20	1.5042	90	25
20	20	21	0.4095	90	20
21	21	22	0.7089	90	20
22	3	23	0.4512	90	85
23	23	24	0.8980	420	70
24	24	25	0.8900	420	40
25	6	26	0.2030	60	85
26	26	27	0.2842	60	85
27	27	28	1.0590	60	70
28	28	29	0.8042	120	70
29	29	30	0.5075	200	55
30	30	31	0.9744	150	40
31	31	32	0.3105	210	25
32	32	33	0.3410	60	20

The PV generation and demand curves have been taken from [14], a complete study that deals with the characterization of the electrical behavior of Medellín (a city in Colombia). The demand and generation curves are presented in Figure 3.

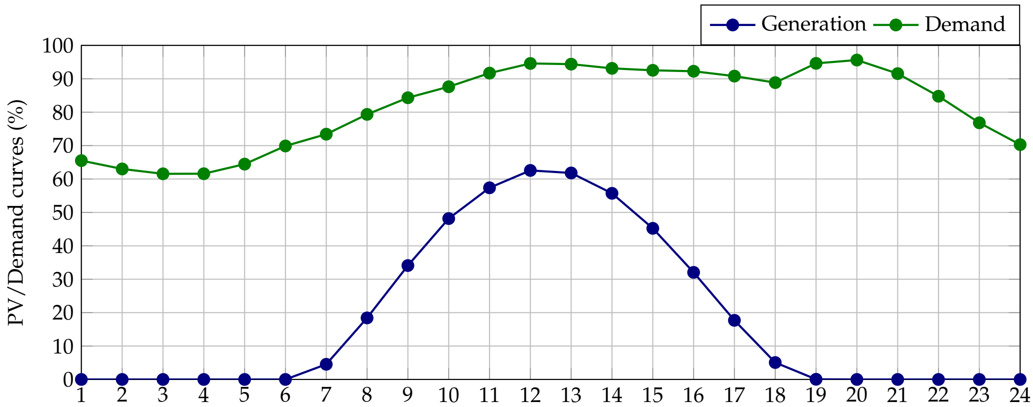

Figure 3. Behavior of the generation and demand curves for the municipality of Medellín.

5. Results and Discussion

This section presents and analyzes all the numerical results obtained for the 33-bus grid. During the computational implementation, the MATLAB programming environment (version 2021b) was used on a PC with an AMD Ryzen 7 3700 2.3 GHz processor and 16.0 GB RAM running the 64-bit version of Microsoft Windows 10 Single Language. The solution of the recursive convex approximation (15) was reached via the convex disciplined tool environment (also known as CVX) of MATLAB, using the SEDUMI and SDPT3 solvers.

5.1. Minimization of Daily Energy Losses

In this section, a complete comparison with different combinatorial optimizers is presented to demonstrate the effectiveness of the proposed iterative convex model (ICM). These combinatorial optimizers are the multi-verse optimization (MVO) approach, the particle swarm optimizer (PSO), the crow search algorithm (CSA), and the salp swarm algorithm (SSA). Note that all these methods have been recently reported in [14]. Table 2 presents the numerical comparisons between the proposed ICM and the combinatorial optimization methods. This table shows that the benchmark case (i.e., the scenario without PV generation) has expected daily energy losses of about 2186.2799 kWh/day.

Table 2. Numerical comparison between the ICM approach and the combinatorial optimization methods.

Method	P_{loss} (kWh/day)	Reduction (%)	Avg. Time (s)
Ben. Case	2186.2799	—	10.5630
SSA	1225.3323	43.9536	20.8476
MVO	1231.2531	43.6827	2.4479
PSO	1268.5973	41.9746	5.9597
CSA	1270.1562	41.9033	36.3663
ICM	1224.8548	43.9754	34.7228

The numerical results in Table 2 show that:

i. The proposed ICM finds a reduction of about 43.9754% concerning the benchmark case, (961.4251 kWh/day), which is the best objective function value obtained for the DC version of the IEEE 33-bus grid, i.e., the optimal solution. This solution is only closely followed by the SSA algorithm, as reported in [14], with a reduction of 43.9536%, i.e., 960.9476 kWh/day. This difference, even though it is small, shows that the proposed convex approximation can find the optimal solution to the studied problem due to the convexification applied to the solution space, which is not possible with random-based optimization algorithms, as is the case of the SSA approach, where statistical analyses must be conducted in order to define its average behavior in terms of solution quality and repeatability.

ii. The PSO and the CSA algorithms find solutions with expected reductions lower than 42%, demonstrating that both methods are stuck in locally optimal solutions. Note that the solution of the ICM allows for improvements of about 2.0214 and 2.0927% with respect to these methods.
iii. Regarding processing times, all the optimization methods take between 2 and 37 s. The SSA shows an average time of 20.8476 s, and the proposed ICM reports an average time of about 34.7228 s. Nevertheless, it is important to mention that, in order to find the solution with the SSA approach, 100 consecutive evaluations of the methodology were conducted, which implies that the real processing time was about 34.7460 min, thus demonstrating that the proposed ICM is the most efficient algorithm in this regard as only one evaluation is needed to obtain an optimal solution.

To illustrate the convergence characteristics of the proposed ICM, Figure 4 shows the curves for the benchmark case and the optimal solution found with the SEDUMI and SDPT3 tools in the CVX optimization package of the MATLAB programming environment.

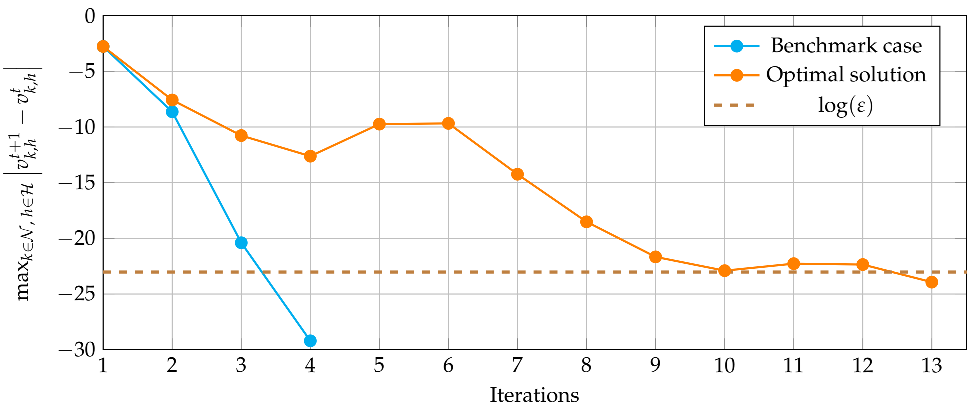

Figure 4. Evolution of the convergence error for the proposed ICM.

The evolution of the convergence error (Figure 4) shows that, in the benchmark case, the total number of iterations required to reach the expected convergence error (i.e., $\varepsilon = 1 \times 10^{-1}$) is four. In contrast, when the PV generation units are optimally dispatched, these iterations increase to 13. This is an expected behavior, given that in the benchmark case there is no presence of the dispersed generation variables, i.e., the solution space has fewer variables in comparison with the optimal operation of the PV sources, where 72 new variables must be assigned during the optimization process. This causes the voltage profiles to exhibit more oscillations during the optimization process. Regarding the convergence error, it is evidenced that the benchmark case converges quadratically, which is normal behavior for iterative methods based on derivatives, as is the case of the proposed approach for the multi-period power flow solution. However, in the case of the optimal dispatch of the PV sources, there is no evidence of a clear tendency, which is attributable to the fact that the hyperbolic relation between powers and voltages was relaxed without using derivatives, unlike the component associated with the constant power terminals.

5.2. Minimization of Economic and Environmental Functions

This section presents the results regarding the minimization of the economic (energy purchasing costs in the substation bus and the maintenance costs in the PV sources) and the objective environmental function (CO_2 emissions). Note that the values of the coefficients in the objective functions (9) and (10) are set as $C_{kWh}^{s} = 0.1302$ kWh/day, $\gamma_s = 0.1644$ kg/kWh, and $C_{O\&M}^{pv} = 0.0019$ USD/kWh.

Table 3 presents the numerical results regarding the minimization of E_{costs} and E_{CO_2}.

Table 3. Numerical results regarding the minimization of E_{costs} and E_{CO_2} with the comparative and proposed ICM.

Method	E_{costs} (USD/day)	E_{CO_2} (kg/day)
Benc. Case	9776.3892	12,345.1497
CSA	7407.9046	9328.7685
PSO	7392.0432	9282.4081
MVO	7298.7157	9187.9682
SSA	7297.9712	9166.6746
ICM	7137.1822	8965.4072

The numerical results in Table 3 show that (i) the proposed ICM allows finding the best numerical results for both objective functions, i.e., 7137.1822 USD/day and 8965.4072 kg/day, which are reductions of about 26.9957 and 27.3771% regarding the benchmark case, respectively; (ii) the best combinatorial optimizer is the SSA approach, but the proposed ICM outperforms these results by about 160.7890 USD/day and 201.2674 kg/day, respectively; and (iii) all the combinatorial optimization methods are stuck in locally optimal function values, which can be attributed to the number of variables associated with the studied problem and the random nature of their exploration. In contrast, the proposed convex-based optimization method allows for better numerical results, with the main advantage being the solution's repeatability.

Table 3 confirms that the proposed ICM is an efficient optimization methodology to deal with the optimal operation of PV plants in monopolar DC distribution networks while considering different objective functions, i.e., technical, economic, and environmental, with the main advantage that the global optimum is reached in each one of the possible PV dispatch scenarios.

6. Conclusions and Future Works

An iterative convex approximation was proposed in this research to solve the problem regarding the optimal dispatch of PV generation units in monopolar DC networks to minimize technical, economic, and environmental objective functions. Taylor's series expansion was used to obtain a linear relaxation of the hyperbolic relation between powers and voltages in constant power terminals to obtain a convex approximation. In the case of the dispersed generation units, the approximation was based on the slight variations of the voltage magnitudes compared to the expected variations in PV generation. To minimize the error induced by both relaxations, a recursive solution methodology involved the iterative solution of the proposed convex approximation, with an initial set of voltage values to be updated until the desired convergence error was reached.

Numerical results in the DC version of the IEEE 33-bus grid demonstrated that the ICM reached the optimal solution value regarding the final daily energy losses through an efficient dispatch of the PV generation units, with a value of 1224.4048 kWh/day, which was only followed by the SSA approach (1225.3323 kWh/day). As for the daily operating costs, the improvement reached by the proposed ICM was 160.7890 USD/day with respect to the SSA approach, while this difference was about 201.2674 kg/day for the CO_2 emissions. These results confirm the effectiveness and robustness of convex-based optimization methods to deal with multi-period optimal power flow problems in monopolar DC networks when compared to traditional and largely used combinatorial optimization methods.

Note that the main advantage of the proposed ICM is that, due to the convexification of the solution space, the final solution for the studied problem will always be the same (i.e., optimal solution value after applying the iterative convex solution method, which allows eliminating the error introduced by the linearization point), which is not possible with any of the combinatorial optimization techniques, given that their random-based nature requires multiple evaluations in order to determine their average behavior, with no guarantee that the optimal solution will be found.

In the future, it will be possible to carry out the following works: (i) extending the proposed ICM to the problem regarding the efficient operation of renewable energy resources and battery energy storage systems in monopolar and bipolar DC distribution networks; (ii) performing a comparative analysis with additional convex optimization methods such as semi-definite programming, second-order cone programming, and alternative recursive convex approximations; and (iii) modifying the proposed convex approximation to include binary variables with regard to the optimal location and sizing of renewable sources in monopolar DC networks.

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