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A mixed-integer second-order cone model for optimal siting and sizing of dynamic reactive power compensators in distribution grids

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A R T I C L E I N F O

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A B S T R A C T

The problem of the optimal placement and sizing of dynamic reactive power compensators in AC distribution networks is addressed in this paper from convex optimization. The exact mixed-integer nonlinear programming (MINLP) model is transformed into a mixed-integer second-order cone programming (MISOCP) model. The main advantage of the MISOCP formulation is the possibility of finding a global optimum with branch & cut combined with interior-point method due to the convex structure of the continuous part of the problem, i.e., the multi-period branch optimal power flow. The dynamic reactive power compensators are sized and dimensioned considering daily load curves and variable reactive power injections. Numerical validations are tested in the 33- and 69-bus test feeders using the CVX tool available for MATLAB with the MOSEK solver. These simulations demonstrate the effectiveness and robustness of the MISOCP approach when compared with the solution of the exact MINLP obtained in the GAMS software.

1. Introduction

The constant growth of demands on distribution systems leads them to implement new devices and strategies to operate appropriately, satisfying all system requirements [1,2]. This growth in demand has been greater in industrial loads in recent years than residential loads [3]. In addition, network operators have challenges in their demands, e.g., their industrial loads, which are constantly growing and mainly equipped with many rotating machines, which consume a large amount of reactive power. Generally degrading the factor distribution systems power and transmission lines congestion reduces the power transfer capacity causing low voltage problems [4–6].

Typically, the installation of the capacitor banks on the distribution network has been used to improve the power factor and its consequences. However, this is not an optimal solution since both the demand and the reactive power consumption are not constant or discrete during a typical day of operation [7]. Although capacitor banks improve the power factors of the distribution system, they are not currently the most suitable solution since many power electronics devices are available, such as (i) flexible AC transmission systems (FACTS), (ii) static distribution compensators (STATCOM), (iii) dynamic voltage restorers (DVR), and (iv) unified power quality conditioner (UPQC), among others [8]. These devices have appeared as a solution to this problem due to the growth of power electronics, creating opportunities to implement and achieve good equipment performance. Such power electronics devices can function as dynamic reactive power compensators to improve distribution system performance in many ways. Improving load power factor and voltage profiles, increasing power transfer capacity, controlling power oscillations, and improving system stability [9,10].

Siting and sizing of the dynamic reactive power compensators must be performed appropriately; if this is not the case, they can negatively affect the electrical distribution networks. Effects as lack of protection schemes coordination, increase in the level of current faults, transmission line overload, and deterioration of the voltage profile [11,12]. Therefore, an optimization model for optimal compensators siting and

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sizing is necessary to avoid these potential problems.

In the specialized literature, the siting and dimensioning of dynamic reactive power compensators have been approached with different formulations and mathematical methodologies. However, this problem is difficult to solve since its formulation combines discrete and continuous variables with nonlinear and non-convex constraints [13]. Hence, the global optimum cannot be guaranteed. This problem has typically been solved using a master-slave methodology. The master stage chooses the siting and sizing of the dynamic compensators when the slave stage solves an optimal power flow problem.

For the master stage, metaheuristic optimization techniques have been commonly implemented, such as the Chu-Beasley genetic algorithms [14,15], algorithms based on differential evolution [16,17], particle swarm optimization [11], gray wolf optimization algorithm [18], among others. Although these optimization techniques can find great solutions, none of them can guarantee the optimal solution, and some are not mathematically well-justified [19]. Furthermore, these techniques include a certain degree of randomness, which lead to slightly different solutions from one run to another. Additionally, these techniques also contain many tuning parameters that make them highly dependent on the programmer, adding to the fact that a statistical analysis to determine its reproducible capabilities [20]. Unlike the metaheuristic techniques typically used, this article proposes an exact formulation based on convex optimization where the location problem and optimal sizing of dynamic reactive power compensators are transformed into a second-order mixed-integer conic programming problem (MISOCP) [13]. The problem is solved by a modified version of the Branch & Cut (B&C) method, which guarantees an optimal global optimization if each optimization sub-problem is convex [21]. For this purpose, the optimal power flow becomes a convex problem through a second-order conic relationship [22].

This paper is organized as follows: Section 2 shows the formulation of the location problem and optimal sizing of dynamic reactive power compensators in distribution systems. Section 3 describes the proposed convex reformulation transforming the original nonlinear model into a second-order conic equivalent. Section 4 represents the methodology for solving the location problem and optimal sizing of dynamic reactive power compensators in distribution systems. Section 5 displays the main characteristics of the test systems. Section 6 presents the computational validation for the different proposed scenarios. Furthermore finally, section 7 portrays the main conclusions derived from this study, as well as some possible future research.

2. Exact formulation

Siting and sizing optimal of dynamic reactive power compensators are formulated through a mixed-integer nonlinear programming model [23]. The integer part, i.e., binary, is associated with selecting the nodes where the compensators will be located. In contrast, the continuous section is associated with the solution of the resulting multi-period optimal power flow. The objective function is the reduction of daily energy losses in the distribution network associated with the dissipation of electrical energy into heat due to the resistance of the conductors. The objective function takes the following form:

$$\min\ C_{\text{losses}} = C_{\text{mean}}^{\text{energy}} \sum_{t\in\mathcal{T}} \sum_{ij\in\mathcal{L}} R_{ij} I_{ij,t}^2 \Delta t, \tag{1}$$

where C_{losses} corresponds to the value of the objective function associated with the average daily cost of energy losses. $C_{\text{mean}}^{\text{energy}}$ represents the average cost of energy in kilowatt-hours, R_{ij} corresponds to the resistance of the network section connecting nodes i and j, where $I_{ij,t}$ is the magnitude of the current electricity that flows through it in each time t. Note that ΔT is the length of the period under analysis (this is typically associated with ranges of 15 min, 30 min, or 60 min); moreover, $\mathcal{T}$ is the set that contains all the periods, and $\mathcal{L}$ is the set that incorporates all the network segments.

The constraints of the optimal reactive power compensation problem correspond to the apparent power balance equations, the capacity limits of the devices, and the voltage regulation, among others. The set of constraints of the problem under study is presented below.

$$p_{ij,t} - R_{ij} I_{ij,t}^2 - \sum_{k:(j,k)\in\mathcal{L}} p_{jk,t} = P_{j,t}; \{j\in\mathcal{N},\ t\in\mathcal{T}\}, \tag{2}$$

$$q_{ij} - X_{ij} I_{ij,t}^2 - \sum_{k:(j,k)\in\mathcal{L}} q_{jk,t} = Q_{j,t} - q_{j,t}^{cr}\ \{j\in\mathcal{N},\ t\in\mathcal{T}\}, \tag{3}$$

$$V_{j,t}^2 = V_{i,t}^2 - 2\left(R_{ij} p_{ij,t} + X_{ij} q_{ij,t}\right) + \left(R_{ij}^2 + X_{ij}^2\right) I_{ij,t}^2 \quad \{(i,j)\in\mathcal{L},\ t\in\mathcal{T}\}, \tag{4}$$

$$I_{ij,t}^2 = \frac{p_{ij,t}^2 + q_{ij,t}^2}{V_{i,t}^2},\ \{(i,j)\in\mathcal{L},\ t\in\mathcal{T}\}, \tag{5}$$

$$-x_j q^{cr,\max} \le q_{j,t}^{cr} \le x_j q^{cr,\max}\ ,\ \{t\in\mathcal{T},\ j\in\mathcal{N}\}, \tag{6}$$

$$\sum_{j\in\mathcal{N}} x_j \le N_{\max}^{cr} \tag{7}$$

$$V^{\min} \le V_{j,t} \le V^{\max},\ \{t\in\mathcal{T},\ j\in\mathcal{N}\}, \tag{8}$$

$$x_j \in \{0,1\},\ j\in\mathcal{N}, \tag{9}$$

where $p_{ij,t}$ and $q_{ij,t}$ represent the flows of the active and reactive power from node i to node j at time t, $P_{j,t}$ and $Q_{j,t}$ correspond to the active and reactive power consumption at node j, at time t. Note that these loads have been modeled as constant power loads. $q_{j,t}^{cr}$ corresponds to the injection of reactive power by the dynamic compensator connected at node j, at time t. $V_{i,t}$ and $V_{j,t}$ correspond to the magnitudes of the voltages at nodes i and j, at time t, respectively. $q_j^{cr,\max}$ represents the maximum reactive power injection limit associated with the dynamic compensator connected at node j; and $V^{\min}$ and $V^{\max}$ correspond to the minimum and maximum limits associated with voltage regulation in all nodes of the distribution network. The variable x_j determines the installation ($x_j = 1$) or not ($x_j = 0$) of a dynamic reactive power compensator at node j. In addition, $N_{\max}^{cr}$ corresponds to the maximum number of reactive compensators available for installation in the distribution system. Notice that $\mathcal{N}$ represents the set that contains all nodes in the system.

The mathematical model interpretation for the location and optimal sizing of dynamic reactive power compensators in power distribution systems is defined from (1) to (9) as follows: Eq. (1) represents the objective function of the problem, which corresponds to the daily minimization of beneficial energy losses due to the dissipation of electrical energy in heat in all conductors of the network. Eqs. (2) and (3) represent the active and reactive power balance at system nodes for each period under study. Eq. (4) presents the voltage drop in each one of the branches as a function of their power flow, the current flowing through them, and their impedance parameters. Eq. (5) represent the definition of average apparent power calculated at the distribution line dispatch. The box constraint defined by (6) determines the possibility of injecting/absorbing reactive power by the dynamic compensator if the binary variable associated with its location is activated.

Likewise, the inequality defined by (7) determines the maximum number of reactive compensators that can be installed in the electrical network, which is a limit typically defined by the network operator. The box constraint described in (8) defines the maximum and minimum allowed voltage limits for the distribution network, which are typically assigned between 5% and 10% for the case of Colombia. Finally, Eq. (9) defines the binary nature of the decision variable associated with the installation and sizing of reactive compensators.

Fig. 1 represents the characterization of the mathematical model (1)–

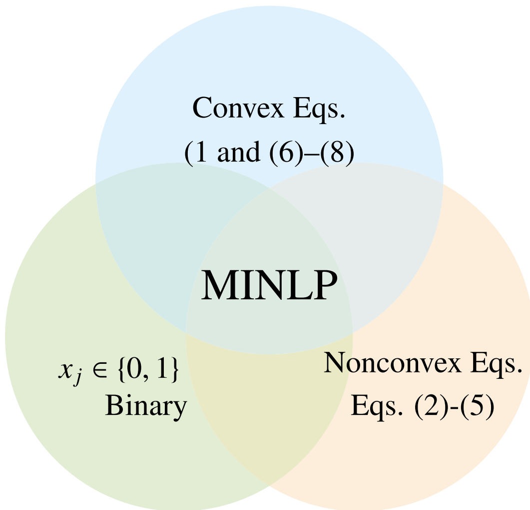

Fig. 1. Characterization of the MINLP model for the location and sizing of dynamic reactive power compensators.

(9) that defines the optimal location and size of dynamic reactive power compensators in distribution systems.

According to the characterization presented in Fig. 1, the main challenge to obtain a convex formulation with binary variables corresponds to the subset of Eqs. (2)–(5), which is mainly associated with the power balance equations and voltage drops in the electrical network. Therefore, the following section proposes a reformulation of the MINLP model through second-order conic programming (SOCP), which results in a MISOCP model, which guarantees the optimal global solution of the problem.

3. Convex reformulation

In order to solve the optimal placement and sizing of dynamic reactive compensators problem in radial distribution systems, this section presents a second-order conic reformulation that allows transforming the exact MINLP model into a convex model of the MISOCP type. To perform this transformation, consider the definition of the following auxiliary variables: $l_{ij,t} = I_{ij,t}$, and $z_{i,t} = V_{j,t}$, whereby smoothly the objective function (1) and the constraints (2) a (4) become linear expressions, and therefore convex, as will be shown below. However, constraint (5) is the most complicated since it represents the product between voltage and current, which is approximated by a second-order cone. To obtain this approximation, Eq. (5) using the auxiliary variables takes the following form (the set notation is omitted for clarity purposes).

$$p_{ij,t}^2 + q_{ij,t}^2 = l_{ij,t} z_{i,t}. \tag{10}$$

Now, to approximate the right-hand side of Eq. (10), we employ its hyperbolic equivalent [22], thereby arriving at:

$$\begin{aligned} p_{ij,t}^2 + q_{ij,t}^2 &= \frac{1}{4}\left(l_{ij,t} + z_{i,t}\right)^2 - \frac{1}{4}\left(l_{ij,t} - z_{i,t}\right)^2, \\ \left(2p_{ij,t}\right)^2 + \left(2q_{ij,t}\right)^2 &= \left(l_{ij,t} + z_{i,t}\right)^2 - \left(l_{ij,t} - z_{i,t}\right)^2, \\ \left(2p_{ij,t}\right)^2 + \left(2q_{ij,t}\right)^2 + \left(l_{ij,t} - z_{i,t}\right)^2 &= \left(l_{ij,t} + z_{i,t}\right)^2, \\ \sqrt{\left(2p_{ij,t}\right)^2 + \left(2q_{ij,t}\right)^2 + \left(l_{ij,t} - z_{i,t}\right)^2} &= l_{ij,t} + z_{i,t}. \end{aligned} \tag{11}$$

Note that Eq. (11) can now be written using rule 2 as follows:

$$\left\| \begin{matrix} 2p_{ij,t} \\ 2q_{ij,t} \\ l_{ij,t} - z_{i,t} \end{matrix} \right\| = l_{ij,t} + z_{i,t}. \tag{12}$$

The conic constraint defined in (12) remains nonconvex due to the equality imposition; however, as demonstrated in Ref. [22], this can be relaxed by a least-equal symbol, which immediately makes this a convex approximation, as follows:

$$\left\| \begin{matrix} 2p_{ij,t} \\ 2q_{ij,t} \\ l_{ij,t} - z_{i,t} \end{matrix} \right\| \leq l_{ij,t} + z_{i,t}. \tag{13}$$

It is relevant to highlight that second-order conic programming, i.e., SOCP, corresponds to a subarea of mathematical optimization that works with affine linear constraints and conic expressions [21]. In the specialized literature, there are multiple reports related to trouble-shooting comics that take milliseconds to achieve, independent of the number of variables involved in these problems [24]. As mentioned above, a conic constraint is a convex expression that, in general, takes the following form:

$$\|y\| \leq w. \tag{14}$$

The vector y is defined in an $n-$ dimensional space, i.e., $y \in \mathbb{R}^n$ and w in a scalar, i.e., $w \in \mathbb{R}$. Moreover, $\|y\|$ is known as the $2-$ norm of the vector y. Fig. 2 represents a second-order cone defined on a three-dimensional space $\mathbb{R}^3$, which is a convex set. For more details about convex optimization theory, review reference [25].

Given the convex structure of the conic representation for the apparent power equation sent from node i to node j, obtained in (13), the mathematical model (1)–(9) is transformed from an MINLP model to a MISOCP model, as shown below:Objective function:

$$\min C_{\text{losses}} = C_{\text{mean}}^{\text{energy}} \sum_{t \in \mathcal{T}} \sum_{ij \in \mathcal{L}} R_{ij} l_{ij,t} \Delta t.$$

Subject to:

$$p_{ij,t} - R_{ij} l_{ij,t} - \sum_{k:(j,k) \in \mathcal{L}} p_{jk,t} = P_{j,t}; \{j \in \mathcal{N},\ t \in \mathcal{T}\},$$

$$q_{ij} - X_{ij} l_{ij,t} - \sum_{k:(j,k) \in \mathcal{L}} q_{jk,t} = Q_{j,t} - q_{j,t}^{cr}\ \{j \in \mathcal{N},\ t \in \mathcal{T}\},$$

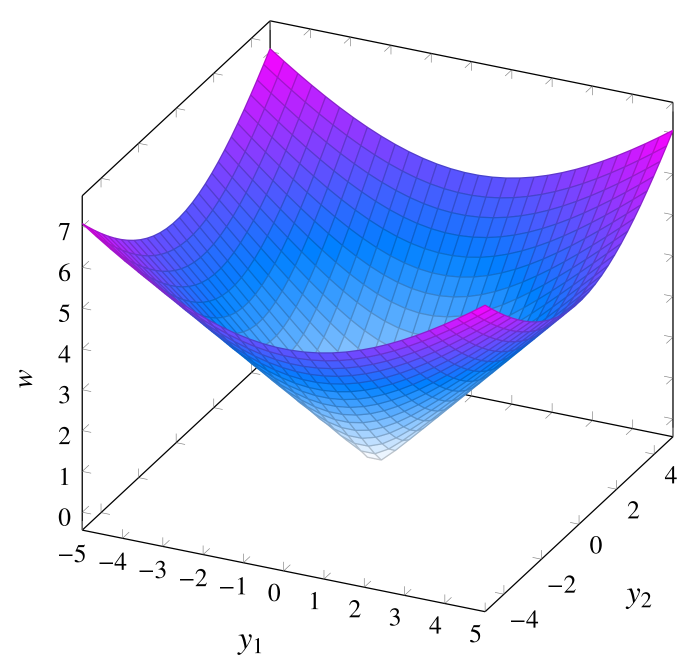

Fig. 2. Graphical representation of a second-order cone, where $\Omega = \{\|y\| \leq w\}$, $y \in \mathbb{R}^2$ and $w \in \mathbb{R}$.

$$z_{j,t} = z_{i,t} - 2\left(R_{ij}p_{ij,t} + X_{ij}q_{ij,t}\right) + \left(R_{ij}^2 + X_{ij}^2\right)l_{ij,t} \quad \{(i,j) \in \mathcal{L},\ t \in \mathcal{T}\}, \tag{15}$$

$$\left\| \begin{matrix} 2p_{ij,t} \\ 2q_{ij,t} \\ l_{ij,t} - z_{i,t} \end{matrix} \right\| \leq l_{ij,t} + z_{i,t},\ \{(i,j) \in \mathcal{L},\ t \in \mathcal{T}\},$$

$$-x_j q^{cr,\max} \leq q_{j,t}^{cr} \leq x_j q^{cr,\max},\ \{t \in \mathcal{T},\ j \in \mathcal{N}\},$$

$$\sum_{j \in \mathcal{N}} x_j \leq N_{\max}^{cr},$$

$$V^{\min} \leq V_{j,t} \leq V^{\max},\ \{t \in \mathcal{T},\ j \in \mathcal{N}\},$$

$$x_j \in \{0,1\},\ j \in \mathcal{N}.$$

The main advantage of the MISOCP model defined in (15) is that it is possible to guarantee the finding of the global optimum of the problem by applying the B&C method combined with the interior-point method. In the following section, (15) represents a brief explanation of the solution methodology for the MISOCP model proposed.

4. Solution methodology

Second-order conic programming problems involving mixed integers, i.e., MISOCP-type models, have within their constraints mathematical structures of the following form:

$$\|A_k x + b_k\| \leq \alpha_k^\top x + \beta_k^\top z_k + \gamma_k. \tag{16}$$

The decision variables include continuous x and binary z variables. A_k being real domain matrices, b_k, α_k, β_k actual domain vectors, and γ_k original constants for the $k-$-th constraint.

Like most programming problems involving integer variables, the MISOCP model can be solved using a modified version of the B&C, as illustrated in Fig. 3. In each one of the iterations, the B&C method solves a SOCP model by employing an interior-point method specially designed for conic problems. This solution methodology benefits from the properties of SOCP problems related to their convexity and fast convergence of interior-point method [21].

It is relevant to highlight that the main advantage of a MISOCP model is that it can be solved by combining the B&C method and the interior-point method. In each possible combination of binary variables, a second-order conic model is obtained that has a unique solution, and it is the global optimum for each binary input [21]. So, it guarantees that this methodology can assure the finding of the global optimum of the problem, which is not possible in mathematical models of the MINLP type [27], being this the main contribution of this research.

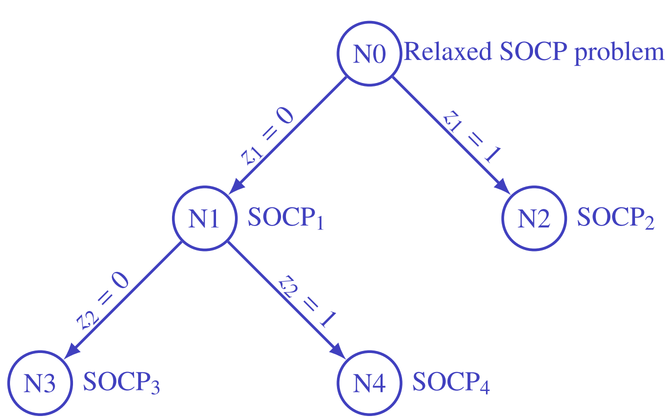

Fig. 3. Exploration of the solution space with the B&C method to solve a MISOCP-type model with two binary variables [26].

In this work, the MATLAB optimization tool known as CVX and the solvers MOSEK and GUROBI, which combine the B&C and interior-point method [28] are used to solve the MISOCP model proposed in (15).

5. Test systems

To validate the methodology of location and optimal sizing of dynamic reactive power compensators, two classical distribution systems of radial configuration operated at the substation node with a voltage equivalent to 12.66 kV are considered from the specialized literature. The first test system contains 33 nodes, and the second test system of 69 nodes. Fig. 4 represents the configurations of these test systems.

Peak consumption and line impedance data for these test systems can be found in Ref. [29]. In addition, for the operation during a typical day, it is considered the demand curve presented in Fig. 5. The data for this curve can be consulted in Ref. [30].

For the test systems under study, the maximum size of each of the dynamic reactive power compensators is 2000 kVAr, and three can be installed in each distribution network.

6. Computational validation

The implementation of the proposed MISOCP model is performed on a desktop computer with an Intel(R) Core(TM) $i5-3550$ 3.5-GHz, 8 GB RAM in a 64-bit version under Microsoft Windows 7 Professional environment, using MATLAB 2020*b* software with the convex optimization tool CVX and the solvers MOSEK and GUROBI. Furthermore, to demonstrate that the MISOCP model guarantees the finding of the global optimum, the exact MINLP model is implemented in the GAMS software with the solvers BONMIN, COUENNE, and DICOPT, respectively.

6.1. Results of the 33-node test system

Table 1 shows the results obtained with the different solvers for MINLP type problems in GAMS and the results obtained with the proposed MISOCP model.

The results presented in Table 1 show that:

✓ The MISOCP model allows finding the best solution to the problem by assigning dynamic reactive power compensators to nodes 13, 24, and 30 with an injection level equivalent to 1.6773 MVAr, thus achieving a reduction in the daily cost of losses of US$/day 80.7784, i.e., a 26.15% reduction concerning the base case.

✓ The closest solution to the global optimum found by the MISOCP model is obtained with the BONMIM solver, which identifies two of the three nodes of the optimum solution, i.e., nodes 24 and 30. This solution achieves a 25.54% reduction with a nominal reactive power injection equivalent to 1.6375 MVAr. However, like the COUENNE and DICOPT solvers, this solution co++rresponds to an optimal local solution, which demonstrates the complexity of the exact MINLP model.

On the other hand, Fig. 6 shows the dynamic behavior in reactive power injection by the compensators installed at nodes 13, 24, and 30, which correspond to the optimal global solution found by solving the proposed MISOCP model.

From this performance, it is relevant to highlight that the dynamic generation of reactive power in the compensators follows the general performance of reactive power presented in Fig. 5. This performance is because the active power losses are partly related to the reactive current flow in the distribution lines; this implies that as these change throughout the day, they must be compensated. Hence, as the dynamic reactive power compensator varies hourly, its reactive power generation reduces these losses to a minimum.

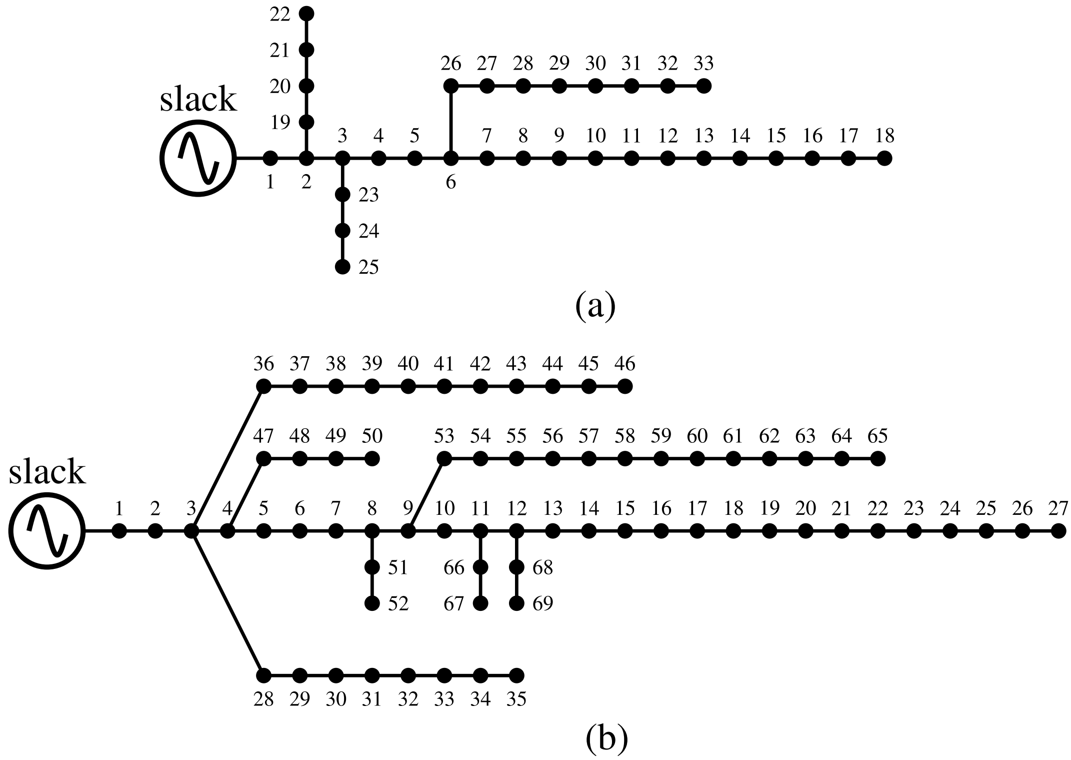

Fig. 4. The electrical connection of the test systems: (a) 33-node test system, (b) 69-node test system.

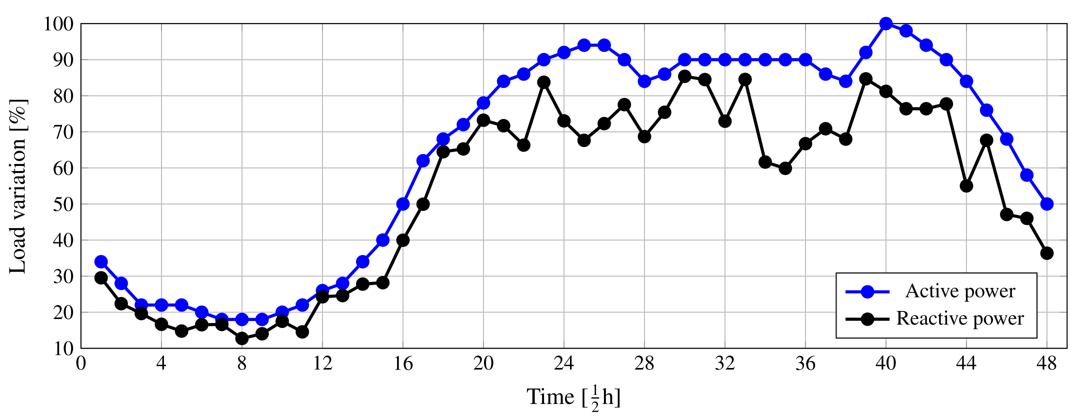

Fig. 5. Typical demand behavior for a day of operation in Colombia (taken from Ref. [30]).

Table 1
Location and size of reactive compensators in the 33-node system.

Method	Localization (node)	Size (MVAr)	Price (US $/day)
Base case	–	–	308.8791
COUENNE	{5, 6, 11}	{0.2887, 1.0346, 0.3166}	249.0945
DICOPT	{8, 18, 33}	{0.4858, 0.1487, 0.6409}	237.3643
BONMIN	{18, 24, 30}	{0.2352, 0.4805, 0.9218}	229.9880
MISOCP	{13, 24, 30}	{0.3304, 0.4630, 0.8839}	228.1007

6.2. Results of the 69-node test system

Table 2 presents the results obtained with the different solvers for MINLP type problems in GAMS and the results obtained with the proposed MISOCP model.

From the results reported in Table 2, it is possible to notice that:

✓ All GAMS solvers are trapped in optimal local solutions due to the complexity of the MINLP model, in which the 69-node system has 50,116 possible installation options for dynamic reactive power compensators. As in the 33-node test system, the BONMIN solver is the best performer with a solution that differs from the optimal global solution achieved by the MISOCP model by 1.8722 per day.

✓ The overall optimal result achieved by the MISOCP model shows that the best nodes to locate dynamic reactive power compensators correspond to nodes 11, 21, and 61, with a reactive power injection equivalent to 1.5973 MVAr, which allows a reduction in the cost of energy losses equals 27.10% during each day of operation.

On the other hand, Fig. 7 shows the daily dynamic performance of the reactive power compensators obtained in the MISOCP model solution. It is important to note that, as in the 33-node system, the performance of the compensators follows the reactive power demand curve presented in Fig. 5. This performance is explained by the fact that active power losses are partly associated with reactive power demand. This

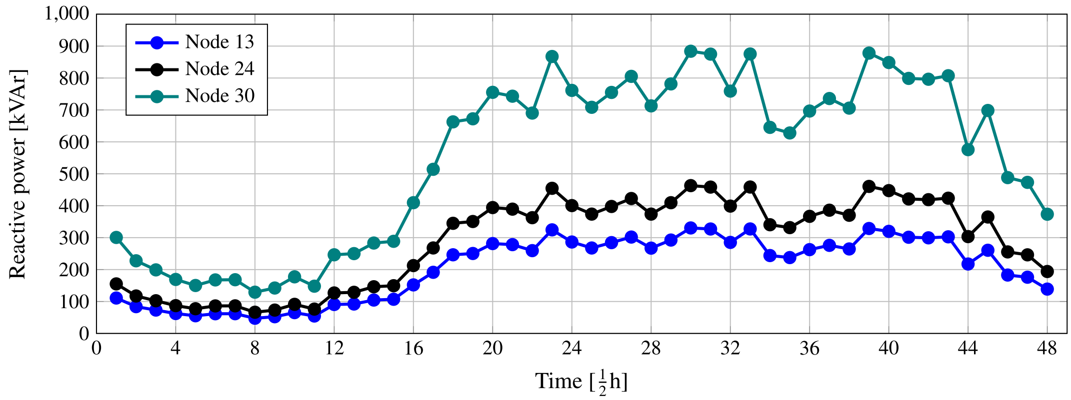

Fig. 6. Performance of dynamic reactive power compensators during the performing day in the 33-node test system.

Table 2
Location and size of reactive compensators in the 69-node system.

Method	Localization (node)	Size (MVAr)	Price (US $/day)
Caso base	–	–	327.9880
COUENNE	{5, 6, 11}	{0.2268, 0.8822, 0.8805}	302.8234
DICOPT	{27, 47, 64}	{0.2439, 0.8974, 0.9486}	249.9691
BONMIN	{27, 49, 61}	{0.2291, 0.4882, 1.0987}	240.9820
MISOCP	{11, 21, 61}	{0.3502, 0.1968, 1.0503}	239.1098

performance involves that for each demand hour, there is an optimal reactive injection that minimizes power losses. So that, for each point of the reactive power curve presented in Fig. 7 for each of the compensators, these points correspond to the optimal global solution for the optimal reactive power flow in each period t.

6.3. Supplementary analysis

In order to verify that installing dynamic reactive power compensators is a more efficient strategy for reducing the daily cost of losses in distribution systems, the MISOCP model defined in (15) considers the possibility of installing capacitive step banks in identical locations of the compensators reported in Tables 1 and 2 for 33 and 69-node systems, respectively. This analysis scenario yields the following results.

✓ In the 33-node test system, fixed-pitch capacitors with sizes of 0.2120 MVAr, 0.2872 MVAr, and 0.5653 MVAr are selected at nodes 13, 24, and 30, respectively, thus achieving an equivalent reduction of 21.72% concerning the base case. This result implies that the solution with dynamic reactive power compensators achieves an additional improvement of 4.43% during each day of operation.

✓ In the 69-node test system, fixed-pitch capacitors with sizes of 0.2137 MVAr, 0.1249 MVAr, and 0.6729 MVAr are selected at nodes 11, 21, and 61, respectively, thus achieving an equivalent reduction of 17.76% concerning the base case. This result implies that the solution with dynamic reactive power compensators achieves an additional improvement of 4.83% during each day of operation. .

The results of additional improvement achieved with dynamic compensators compared to fixed pitch capacitors imply during a continuous year of operation a saving of US$ 4994.9155 and US$ 5511.2810 for the 33 and 69 node test systems, respectively. It is relevant to highlight that this saving, associated with reactive compensator use, justifies the initial investment that these may require, contrary to the fixed pitch capacitor banks, which undoubtedly can be cheaper in the market.

Finally, concerning the computational times, it is relevant to mention that the MISOCP model solved with the GUROBI solver in the MATLAB CVX tool takes on average 190 s s to find the optimal solution in the 33-node system and about 2200 s for the 69-node system. These times are efficient since installing a dynamic compensator problem in a distribution network can take several days or even weeks; thus, understanding its optimal global solution in less than 1 h demonstrates the efficiency and quality of the proposed optimization methodology.

7. Conclusions and future work

This study proposes a convex optimization methodology for the location and optimal sizing of reactive compensators in distribution systems reconfiguring the MINLP model into a MISOCP-type conic

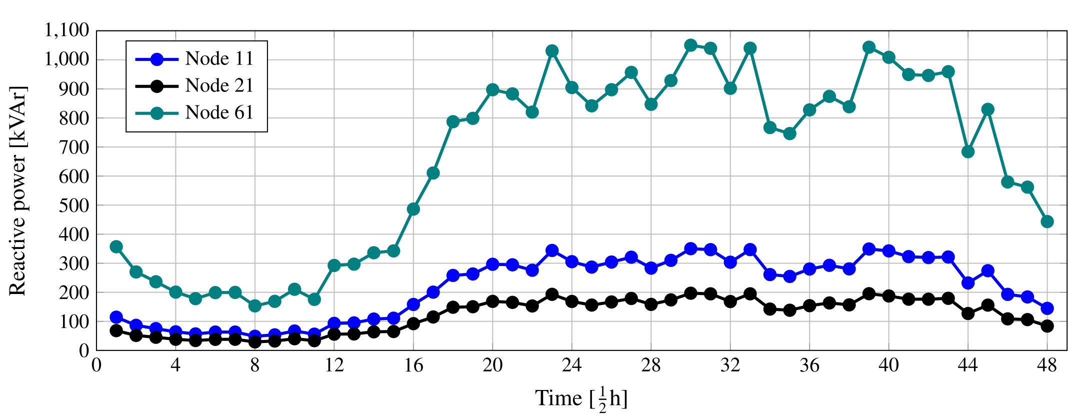

Fig. 7. Performance of dynamic reactive power compensators during the performing day in the 69-node test system.

model. The main advantage of the proposed model is that the finding of the optimal global solution is assured due to the convex structure of the objective function and the solution space of the problem. The objective function was obtained by relaxing the hyperbolic relationship between voltage, current, and power through a convex cone of second order. In order to solve the mathematical design, a B&C method with the interior-point method was used available for the GUROBI solver in the MATLAB CVX tool. We could verify that the MISOCP model finds the best possible solution for the 33- and 69-node test systems, even improving the results found by the GAMS software and the BONMIN, COUENNE, and DICOPT solvers trapped in optimal premises.

Considering the variable characteristic of the active and reactive demanded power for the studied system test, it was possible to verify that the dynamic reactive power compensators perform as variable pitch capacitive banks. These compensators vary the reactive power injection in the system throughout the day. This variation is achieved by controlling the converter that integrates them into the network reducing losses by an additional 4.43% for the 33-node system and 4.83% for the 69-node system compared to the fixed reactive power injection scenario.

CRediT author statement

W. Gil-Gonzalez: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data Curation, Writing-Original Draft, Writing-Review & Editing, Visualization. **O.D. Montoya**: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data Curation, Writing-Original Draft, Writing-Review & Editing, Visualization, Supervision, Project administration, Funding acquisition. **L. F. Grisales-Noreña**: Writing-Review & Editing. **C. L. Trujillo**: Writing-Review & Editing. **D. A. Giral-Ramírez**: Writing-Review & Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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