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A successive approximations method for power flow analysis in bipolar DC networks with asymmetric constant power terminals

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ABSTRACT

This paper deals with the power flow problem in bipolar direct current distribution networks with unbalanced constant power loads. The effect of the neutral wire is considered in two prominent cases: (i) when the system is solidly grounded at each load point and (ii) when the neutral terminal is only grounded at the substation bus. The problem is solved using the successive approximation power flow method. Numerical results in two test feeders composed of 4 and 25 nodes demonstrate that the successive approximation power flow approach is adequate to solve the problem. It is also demonstrated that it is equivalent to the backward/forward power flow in matrix form. The main advantage of both power flow approaches is that they can work with radial and meshed distribution networks. Additionally, they do not require inverting matrices at each iteration, making them efficient in terms of computational processing times requirements. All the simulations are carried out in the MATLAB programming environment.

1. Introduction

In recent years, the study of direct current (DC) distribution networks has gained relevance in electrical power systems involving medium to low-voltage levels. These networks have advantages over alternating current (AC) distribution networks, such as a higher power transfer capability, no need for synchronizing generators, and a better voltage profile [1–3]. Most of the control and stability requirements in AC distribution networks come from the frequency and reactive power, which are nonexistent in DC networks. Additionally, the advances in power electronics technology have allowed for increased integration of DC distributed energy resources (DERs) in electrical networks, such as battery energy storage devices, solar photovoltaic systems, and fuel cells [4]. These resources are inherently DC and can be effortlessly integrated into DC networks [5,6].

Two types of DC networks can be designed: monopolar and bipolar [3,7]. Monopolar networks are only constructed with one voltage level and two wires (positive and neutral). In contrast, bipolar networks have two-level voltage with three wires (positive, negative, and neutral), as shown in Fig. 1 [8]. A bipolar network allows transferring twice as much power as a monopolar network at the expense of one additional neutral cable. Under balanced conditions, *e.g.*, when the load in the positive pole is equal to the load in the negative pole, it is possible to eliminate the neutral cable while saving an additional 33% of conductive material and reducing power losses [9,10].

As in all electrical analyses, it is essential to know the operating state of the bipolar DC network in order to make decisions adequately. For this reason, a power flow analysis is required, which calculates the voltage and power flow in current conditions. This is not an easy task since bipolar DC networks have a complex nonlinear system of algebraic equations [11]. Furthermore, a bipolar DC network may have asymmetric power flows due to unbalanced transmission/distribution lines and loads connected between a pole, and the neutral cable [12, 13].

In general, the specialized literature on bipolar DC networks has studied the problem of optimal power flow (OPF) in high-voltage DC (HVDC) networks using a hybrid approach between AC and DC networks [14–16]. However, these studies have considered bipolar HVDC transmission to be symmetrically operated. Hence, they oversimplify the problem, and single-line approximations for OPF can be employed. These approaches cannot be applied in bipolar DC networks due to the unbalanced power flows that can occur in them.

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Nomenclature

$\mathcal{D}$	Set that contains all the demand nodes.
$\mathbf{A}_{d,pon}$	Component of the branch-to-node incidence matrix that relates all the branches to the demand nodes.
$\mathbf{A}_{pon}$	Branch-to-node incidence matrix.
$\mathbf{A}_{s,pon}$	Component of the branch-to-node incidence matrix that relates all the branches to the slack node.
$\mathbf{E}_{pon}$	Voltage drops in the branches ordered by pole (V).
$\mathbf{G}_{dd,pn}$	Component of the conductance matrix that relates the positive and negative poles of the demand nodes (S).
$\mathbf{G}_{dd,pon}$	Component of the conductance matrix that relates the positive, neutral, and negative poles of the demand nodes (S).
$\mathbf{G}_{ds,pn}$	Component of the conductance matrix that relates the positive and negative poles of the demand and voltage-controlled nodes (S).
$\mathbf{G}_{ds,pon}$	Component of the conductance matrix that relates the positive, neutral, and negative poles of the demand and voltage-controlled nodes (S).
$\mathbf{G}_{pon}^{\text{prim}}$	Primitive conductance matrix that relates all the nodes for positive, neutral, and negative poles (S).
$\mathbf{G}_{sd,pn}$	Component of the conductance matrix that relates the positive and negative poles of the voltage-controlled and demand nodes (S).
$\mathbf{G}_{ss,pn}$	Component of the conductance matrix that relates the positive and negative poles of the voltage-controlled nodes (S).
$\mathbf{I}_{d,pn}$	Vector with the net current consumption in the positive and negative poles of the constant power loads (A).
$\mathbf{I}_{d,pon}$	Vector with the net current consumption in the positive, neutral, and negative poles of the constant power loads (A).
$\mathbf{I}_{dk,pn}^{t}$	Vector with the currents demanded in the positive and negative poles at node k in iteration t (A).
$\mathbf{I}_{s,pn}$	Vector with the net current injection in the positive and negative poles of the voltage-controlled nodes (A).
$\mathbf{I}_{s,pon}$	Vector that contains all the injected currents in the slack source (A).
$\mathbf{J}_{pon}$	Vector that contains all the branch currents ordered by pole (A).
$\mathbf{P}_{d,pn}$	Vector that contains the whole constant power consumption between the poles and the ground in all the demand nodes (W).
$\mathbf{R}_{dd,pon}$	Resistive matrix that relates the demand nodes with each other for the positive, negative, and neutral poles (Ω)
$\mathbf{V}_{d,pn}$	Vector with the voltage variables in the demand nodes for the positive and negative poles (V).
$\mathbf{V}_{d,pon}$	Vector with the voltage variables in the demand nodes for the positive, neutral, and negative poles (V).
$\mathbf{V}_{dk,pn}^{t}$	Vector with the voltage values in the positive and negative poles at node k in iteration t (V).
$\mathbf{V}_{s,pn}$	Vector with the voltage outputs in the generation nodes for the positive and negative poles (V).
$\mathbf{V}_{s,pon}$	Vector with the voltage outputs in the generation nodes for the positive, neutral, and negative poles (V).
$\mathbf{W}_{d,pon}$	Vector that contains the power flow solution for all the demanded nodal voltages in the positive, neutral, and negative poles (V)
ε	Maximum convergence error to finish the iteration procedure.
ζ	Constant parameter associated with the convergence analysis in the Banach fixed-point theorem.
$P_{dk,pn}$	Vector with the constant power demanded at node k in the iteration t between both poles (W).
t	Iterative counter.
$v_{\min}$	Minimum voltage regulation bound for the positive and negative poles (V).
$I_{dk,n}^{t}$	Demanded current at node k in iteration t at the negative pole (A).
$I_{dk,p}^{t}$	Demanded current at node k in iteration t at the positive pole (A).
$P_{dk,n}^{t}$	Constant power demanded at node k in the iteration t at the negative pole (W).
$P_{dk,p}^{t}$	Constant power demanded at node k in the iteration t at the positive pole (W).
$V_{dk,n}^{t}$	Voltage value at node k in iteration t at the negative pole (V).
$V_{dk,p}^{t}$	Voltage value at node k in iteration t at the positive pole (V).

Regarding bipolar DC networks, most recent analyzes have been conducted to solve the OPF problem. [13] proposed an OPF methodology to analyze bipolar DC networks with high imbalances in monopolar loads that produce congestion in the transmission/distribution lines. The authors presented a linearization of the power flow equations to obtain a linear optimization model. This model made it possible to find the local marginal prices per node due to the voltage imbalance levels. The numerical results in two small systems validate the proposed method. However, a comparative analysis with other optimization methods was performed to verify the effectiveness and robustness of the proposed methodology. [12] addressed the problem of the optimal phase-swapping in bipolar DC grids by proposing a multi-objective binary optimization method. However, the constant power terminals were avoided by generating a linear equivalent. This model is over-simplified because the most complicating constraint is the hyperbolic relation between voltages and power in the demand nodes. Ref. [17] presented the application of the classical nodal voltage method to solve the power flow problem in bipolar DC networks considering multiple constant power terminals and grounded and non-grounded neutral wire scenarios. A small test feeder with three nodes was used to analyze the power flow problem. However, the authors only compared their approach with the solution of circuit equations on the PSCAD/EMTDC software. The authors of [18] presented the application of the current

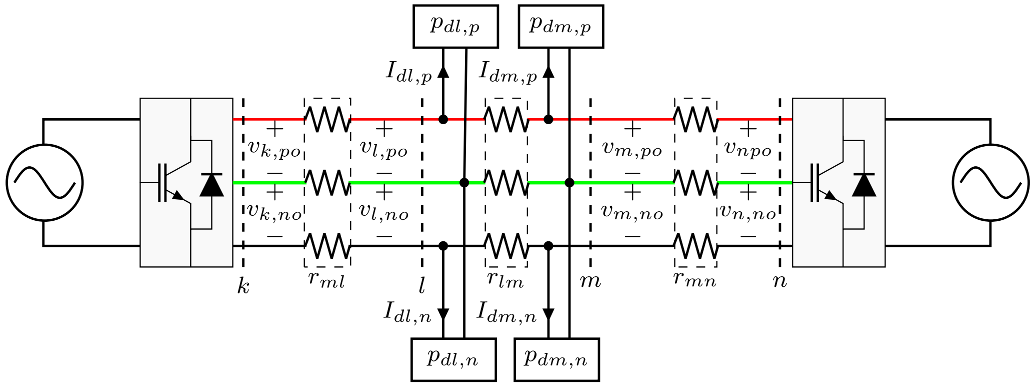

Fig. 1. Simplified diagram of a bipolar DC grid.

injection method to solve the optimal power flow problem in bipolar DC networks. The authors proposed the development of a sensitive index based on the Jacobian matrix in order to reduce the voltage imbalances in the network due to the presence of multiple monopolar power consumptions. A recursive quadratic programming model was implemented to find the solution to the OPF problem considering the penetration of distributed energy resources. Numerical validations in two test feeders composed of 6 and 33 nodes confirmed the efficiency of the proposed OPF method when compared with the solution provided by the PSCAD/EMTDC software.

Bipolar DC networks have mainly been analyzed for OPF studies with simplifications in the power flow constraints. Developing efficient power flow methods for these grids is necessary, considering the high load unbalances and different operational practices concerning the neutral wire. In addition, the developed power flow methods must guarantee convergence for all the possible operative conditions.

This paper proposes a power flow method for unbalanced bipolar DC grids with constant power loads. This method is based on successive approximations reported in [19] for single-phase AC distribution networks. Its main advantage is that it works with radial and meshed distribution configurations while considering solidly grounded and floating neutrals. In addition, the convergence of the proposed method and uniqueness of the solution is demonstrated by using the Banach fixed-point theorem. This theorem ensures that the proposed method guarantees the convergence of the numerical solution of the power flow problem under well-defined load operating conditions. Furthermore, based on the proof presented in [20], we show that the successive approximation method and the classical forward/backward approach are equivalent for DC bipolar networks, *i.e.*, their recursive formulas for solving the power flow problem are completely equivalent.

Note that the main advantages of using the successive approximation method or the matricial backward/forward power flow problem in unbalanced bipolar DC networks are that: (i) they can deal with the operation of the DC network considering the neutral wire solidly grounded or floating without any change in its mathematical formulation; (ii) both methods solve the bipolar power flow problem for radial or meshed distribution networks, which is not possible with the classical backward/forward method [21]; (iii) their implementations do not require inverting matrices at each iteration in contrast with the Newton-based approaches [22], which make their required processing times faster when compared with derivative-based methods.

The remainder of this paper is organized as follows: Section 2 presents the general power flow formulation for bipolar DC grids considering that the neutral is solidly grounded; Section 3 presents the extension of the successive approximation power flow method to bipolar DC grids with non-grounded neutrals; Section 4 presents the convergence analysis of the proposed successive approximation method considering loads connected to the neutral cable by applying the Banach fixed-point theorem; Section 5 presents the equivalence between the backward/forward power flow method and the successive approximation approach, which confirms that both methods have the same recursive power flow formula; Section 6 presents a numerical example of the implementation of the proposed bipolar DC power flow method in a small DC grid composed of 4 nodes; Section 7 describes the numerical implementation of the proposed approach in a DC bipolar network composed of 21 nodes with radial and meshed configurations, considering both possible scenarios in the neutral cable; and finally, Section 8 presents the main concluding remarks derived from this work, as well as some possible future research.

2. Formulation of the power flow problem

Throughout this section, matrices are represented by uppercase bold letters; subscripts represent nodes and superscripts iterations. Variables are defined by the nomenclature presented above.

The power flow problem in bipolar DC networks is developed through the branch-to-line admittance representation of the network [23]. Let us consider the relationship between voltages and currents using the nodal admittance matrix as expressed by Eq. (1).

$$\begin{bmatrix} \mathbf{I}_{s,pn} \\ -\mathbf{I}_{d,pn} \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{ss,pn} & \mathbf{G}_{sd,pn} \\ \mathbf{G}_{ds,pn} & \mathbf{G}_{dd,pn} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{s,pn} \\ \mathbf{V}_{d,pn} \end{bmatrix}, \tag{1}$$

where $\mathbf{I}_{s,pn}$ is a vector that contains the injected currents in the positive and negative poles of the voltage-controlled nodes; $\mathbf{I}_{d,pn}$ contains the demanded currents in both poles of the DC network; $\mathbf{G}_{ss,pn}$, $\mathbf{G}_{sd,pn}$, $\mathbf{G}_{ds,pn}$, and $\mathbf{G}_{dd,pn}$ are the components of the general admittance nodal matrix $\mathbf{G}_{pn}$ that relates voltage-controlled and demand nodes; $\mathbf{V}_{s,pn}$ is a vector that includes the output voltages in voltage-controlled sources; and $\mathbf{V}_{d,pn}$ contains the variables associated with the voltage in the demand nodes. Note that the sign in the second row associated with the demand current corresponds to the direction of the current flow, since the net current injection in electrical networks is positive if the flow arrives at the nodes and it leaves them in the case of the demands.

Notice that the second row in Eq. (1) includes the relation between the demanded currents and the voltage variables. This equation is nonlinear, and its solution requires an iterative methodology.

2.1. Recursive power flow formula

The successive approximation power flow method rearranges the second row of Eq. (1), thus obtaining $\mathbf{V}_{d,pn}$. Then, it computes all the voltages in the demand nodes, as demonstrated in [20]:

$$\mathbf{V}_{d,pn} = -\mathbf{G}_{dd,pn}^{-1} \left[\mathbf{G}_{ds,pn} \mathbf{V}_{s,pn} + \mathbf{I}_{d,pn} \right], \tag{2}$$

The general formulation for power flow analysis in bipolar DC grids with neutral solidly grounded and unbalanced loads can be made by adding an iterative counter to the voltage based on the second Tellegen theorem. The demanded current is a function of the voltage in the demand nodes, *i.e.*, $\mathbf{I}_{d,pn} = f\left(\mathbf{V}_{d,pn}\right)$. Then, the iterative formula to solve the power flow problem takes the following form:

$$\mathbf{V}_{d,pn}^{t+1} = -\mathbf{G}_{dd,pn}^{-1}\left[\mathbf{G}_{ds,pn}\mathbf{V}_{s,pn} + \mathbf{I}_{d,pn}^{t}\right]. \tag{3}$$

The stopping criteria for the iterative formula in Eq. (3) is based on the variation of the voltage magnitude between two consecutive iterations, *i.e.*:

$$\max\left\{\left|\mathbf{V}_{d,pn}^{t+1}|-|\mathbf{V}_{d,pn}^{t}\right|\right\} \leq \varepsilon, \tag{4}$$

where ε is the tolerance.

The main advantage of the recursive formula (3) is that the inverse of the conductance matrix that relates the demand nodes is calculated once and stored. This procedure helps to reduce the processing times required to solve the power flow problem in large-scale bipolar DC grids.

2.2. Calculation of the demanded current

The main difficulty of the power flow problem corresponds to the hyperbolic relation between voltages and constant power loads [19]. Here, we present the current calculation for constant power loads connected between the positive or negative pole and the ground and for when the load is connected between both poles.

In the case of an arbitrary node, k has loads connected between the poles and the ground. The current can be calculated as follows:

$$I_{dk,p}^{t} = \frac{P_{dk,p}}{V_{dk,p}^{t}}, \tag{5}$$

$$I_{dk,n}^{t} = \frac{P_{dk,n}}{V_{dk,n}^{t}}, \tag{6}$$

here $I_{dk,p}^{t}$ and $I_{dk,n}^{t}$ correspond to the current demand in the positive and negative poles; $P_{dk,p}$ and $P_{dk,n}$ represent the values of the power constant power consumption in the positive and negative poles; and $V_{dk,p}^{t}$ and $V_{dk,n}^{t}$ are the voltage values in both poles, respectively.

Note that, if all nodes only have loads connected between the poles and the neutral point, then Eqs. (3) and (5) can be generalized as follows:

$$\mathbf{I}_{dk,pn}^{t} = \mathrm{diag}^{-1}\left(\mathbf{V}_{dk,pn}^{t}\right)\mathbf{P}_{dk,pn}, \tag{7}$$

where $\mathbf{P}_{dk,pn}$ is the vector that contains the constant power demands ordered by a pole at each node; $\mathbf{I}_{dk,pn}^{t}$ is the demanded current in the line-to-neutral (line-to-line) load connected at node k for the positive and negative poles; and $\mathbf{V}_{dk,pn}$ is the vector that contains the positive and negative voltages at node k in iteration t, respectively.

On the other hand, if there is a load connected between both poles at an arbitrary node k, then the demanded current is calculated as follows:

$$I_{dk,p}^{t} = \frac{P_{dk,pn}}{V_{dk,p}^{t} - V_{dk,n}^{t}}, \tag{8}$$

$$I_{dk,n}^{t} = \frac{P_{dk,pn}}{V_{dk,n}^{t} - V_{dk,p}^{t}}, \tag{9}$$

where $P_{dk,pn}$ is the constant power load connected between both poles.

Note that, if all the constant power consumptions are connected between both poles, then Eqs. (8) and (9) can generalized as follows:

$$\mathbf{I}_{dk,pn}^{t} = \mathrm{diag}^{-1}\left(\mathbf{H}\mathbf{V}_{dk,pn}^{t}\right)P_{d,pn}, \tag{10}$$

being $\mathbf{H}$ defined as:

$$\mathbf{H} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

If an arbitrary node has loads connected between the pole and the ground, as well as a load connected between both poles, then the total current demand in this node is the algebraic sum of the currents defined in Eqs. (6) and (7) with the currents in Eqs. (8) and (9), respectively.

2.3. Generalized iterative process

The general implementation of the successive approximation power flow method in bipolar DC networks with asymmetric loads and solidly grounded neutrals is presented in Algorithm 1.

```
Data: Define the bipolar DC grid under study.
Obtain the per-unit equivalent of the network;
Calculate the nodal conductance nodal matrix G_pn and split its
  components G_dd,pn and G_ds,pn;
Calculate and store the inverse of G_dd,pn ans Z_dd,pn = G^-1_dd,pn;
Select the maximum number of iterations t_max;
Chose the convergence error ϵ;
Define the substation voltages: V_s,pn = [1, −1]^T;
Make t = 0;
Define the initial voltage as V^t_d,pn = 1_d,pn V_s,pn;
for t ≤ t_max do
    for k = 2 : n do
        if Load in node k is phase-to-ground then
            Calculate the demanded current I^{t,Y}_dk,pn using Eq. (7) ;
        else
            Calculate the demanded current I^{t,D}_dk,pn using Eq. (10) ;
        end
        Sum both currents I^{t,Y}_dk,pn and I^{t,D}_dk,pn to obtain I^t_dk,pn;
    end
    Calculate the new demanded V^{t+1}_d,pn using Eq. (3);
    if max{||V^{t+1}_d,pn| − |V^t_d,pn||} < ϵ then
        Report the nodal voltages as 𝕍_pn = [V_s,pn; V^{t+1}_d,pn];
        break;
    else
        Make V^t_d,pn = V^{t+1}_d,pn;
    end
end
```

Algorithm 1: Pseudo-code for the solution of the power flow problem in bipolar DC networks with asymmetric loads

In Algorithm 1 the $\mathbf{I}_{dk,pn}^{t,Y}$ and $\mathbf{I}_{dk,pn}^{t,D}$ are the demanded currents for constant power loads connected phase-to-ground and phase-to-phase, respectively at node k.

3. Extension to non-grounded neutral

In the design of bipolar DC networks, especially in low-voltage grids, there is the possibility that the neutral has no grounded connection at each node except to the power converter terminal. This situation implies that the previously presented formulation must explicitly include the neutral effect. A subindex is included to explicitly represent the neutral cable and extend the recursive power flow formula to non-solidly grounded neutrals, as expressed by Eq. (3):

$$\mathbf{V}_{d,pon}^{t+1} = -\mathbf{G}_{dd,pon}^{-1}\left[\mathbf{G}_{ds,pon}\mathbf{V}_{s,pon} + \mathbf{I}_{d,pon}^{t}\right], \tag{11}$$

where $\mathbf{V}_{d,pon}$ contains nodal voltages of the demand; $\mathbf{I}_{d,pon}$ is a vector that contains all the demanded currents in the constant power loads; $\mathbf{V}_{s,pon}$ contains the voltage outputs at the voltage–voltage controlled sources; and all vectors are ordered from positive, neutral, and negative terminals, respectively. The components $\mathbf{G}_{dd,pon}$ and $\mathbf{G}_{ds,pon}$ are the components of the conductance matrix that include the conductance of the neutral conductor.

The main effect of including the neutral in the bipolar power flow formulation corresponds to the calculation of the demanded currents. Thus, in any arbitrary node k, a load is connected from the positive and negative poles to the neutral cable; the demanded currents are calculated as follows:

$$I^t_{dk,p} = \frac{P_{dk,po}}{V^t_{dk,p} - V^t_{dk,o}}, \quad (12)$$

$$I^t_{dk,o} = \frac{P_{dk,po}}{V^t_{dk,o} - V^t_{dk,p}} + \frac{P_{dk,no}}{V^t_{dk,o} - V^t_{dk,n}}, \quad (13)$$

$$I^t_{dk,n} = \frac{P_{dk,no}}{V^t_{dk,n} - V^t_{dk,o}}, \quad (14)$$

where $P_{dk,po}$ and $P_{dk,no}$ are the constant power loads connected between the positive and negative poles with respect of the neutral cable, and $V_{dk,o}$ corresponds to the voltage of the neutral point at node k.

If node k has a load connected between the positive and negative poles (phase-to-phase load), the positive and negative currents are determined with Eqs. (8) and (9), while the current from the neutral cable is assigned as zero in this node, if and only if there are no phase-to-neutral loads on it.

4. Convergence analysis

The main advantage of the proposed successive approximation method for power flow analysis is that it is possible to demonstrate its convergence through the Banach fixed-point theorem when all the loads are connected phase-to-ground [24]. To demonstrate the convergence of the successive approximation approach for bipolar DC grids, let us consider the following assumptions [23]:

Assumption 1. The graph that represents the network is connected.

Assumption 2. The system operates in normal operation in the following region:

$$\mathcal{B} = \left\{ v \in \mathbb{R}^n,\ v \geq v_{\min} > 0 \right\}. \quad (15)$$

Assumption 3. There is a real ζ such that

$$\zeta = \frac{\left\| \mathbf{G}^{-1}_{dd,pon} \mathbf{P}_{d,pon} \right\|}{\left(v_{\min}\right)^2} < 1. \quad (16)$$

To demonstrate the algorithm's convergence, we consider a weaker version of the Banach fixed-point theorem in the real domain. Some definitions are presented here for the sake of completeness:

Definition 1. Let f be a map of $\mathbb{R}^n$ into $\mathbb{R}^n$. A point v is called a fixed point for f if $v = f(v)$.

In simple terms, a fixed-point is a point that does not change under the application of the map f. Therefore, it is a solution of the set of algebraic equations $v - f(v) = 0$ hence its value as a tool to solve and analyze this type of system.[1]

Definition 2. Let $\mathcal{B} = \left\{ v : \left\| v - v_0 \right\| \leq \delta \right\}$ be a closed ball of $\mathbb{R}^n$, and let $f : \mathcal{B} \rightarrow \mathbb{R}^n$. Then f is said to be a contraction mapping if there is a β such that $\| f(v) - f(u) \| \leq \beta \| v - u \|$, with $0 \leq \beta < 1$, $\forall\ v, u \in \mathcal{B}$.

A contraction mapping has a clear relation with fixed-point theory, which is given by the following theorem:

Theorem 1 (*Fixed-point Theorem [25]*). *If f is a contraction mapping, then there is a unique $v \in \mathcal{B}$ satisfying $v = f(v)$, which can be obtained by applying the iteration $v \leftarrow f(v)$ starting from an initial point in $\mathcal{B}$.*

This theorem allows a simple convergence proof for the case of the power flow problem, as demonstrated in [23]. Let us start by defining some basic properties of the power flow equations:

[1] Unless otherwise indicated, $\|\cdot\|$ is the conventional 2-norm.

Definition 3. Let $N = \{1, 2, \ldots n\}$, and A an $n \times n$ matrix. We can say that A is weakly chained diagonally dominant if the following three conditions are met:

1. $|a_{kk}| \geq \sum_{k \neq m} |a_{km}|, \forall k \in N$
2. The set $\mathcal{J}(A) = \left\{ k \in N : |a_{kk}| > \sum_{k \neq m} |a_{km}| \right\} \neq \emptyset$
3. For each $m \in N$ with $m \notin \mathcal{J}(A)$, there is a walk in the directed graph of A starting in m which ends in a $k \in \mathcal{J}(A)$.

Lemma 1 (*See [26]*). *We denote that A_{n-1} is a principal submatrix of A formed from all rows and columns with indices from 2 to n. This matrix is $(n-1) \times (n-1)$ and weakly chained diagonally dominant.*

Lemma 2 (*See [26]*). *A weakly chained diagonally dominant matrix is non-singular. In addition, its inverse is bounded as follows:*

$$\left\| A^{-1} \right\|_\infty \leq \frac{1}{a_{11}(1-\rho_1)} + \frac{\left\| A_{n-1} \right\|_\infty}{1-\rho_1}, \quad (17)$$

where

$$\rho_1 = \frac{1}{|a_{11}|} \sum_{m>1} |a_{km}|. \quad (18)$$

Lemma 3. *The reduced nodal-admittance matrix $\mathbf{G}_{dd,pon}$, is weakly chained diagonally dominant.*

Proof. The first condition for a matrix to be weakly diagonally dominant is trivially satisfied considering the construction of the nodal admittance matrix, namely:

$$g_{km} = \begin{Bmatrix} g_k + \sum_{k \in l} g_l & k = m \\ -g_l & k \neq m \end{Bmatrix}, \quad (19)$$

where g_l indicates the line admittance, and $g_k \geq 0$ is a constant admittance load (it exists). The second condition is guaranteed provided that the graph is connected (Assumption 1); hence, there is at least one g_l that connects a node m with node 1. Obviously, $m \in \mathcal{J}(G) \neq \emptyset$. Finally, the third condition is clearly satisfied due to Assumption 1. □

Theorem 2. *Under Assumptions 1 to 3, the map (11) for ground-connected loads is well defined and Lipschitz-continuous in the set $\mathcal{B}$ given by (15). It corresponds to a contraction mapping. Therefore, Algorithm 1 converges to a fixed point. This point is the solution of the power flow equation and is unique in $\mathcal{B}$.*

Proof. Lemma 3 guarantees that $\mathbf{G}_{dd,pon}$ is weakly chained diagonally dominant. Moreover, $\mathbf{I}_{d,pon}$ is continuous in Eq. (15) due to Assumption 2. Then, (11) is well defined and continuous in $\mathcal{B}$. Moreover, Lemma 2 allows defining an upper bound for $\mathbf{G}^{-1}_{dd,pon}$.

On the other hand, the recursive power flow formula in Eq. (11), considering that all the loads are connected to the ground, can be rewritten as follows:

$$\begin{aligned} \mathbf{V}^{t+1}_{d,pon} &= f\left(\mathbf{V}^t_{d,pon}\right) \quad (20) \\ &= -\mathbf{G}^{-1}_{dd,pon}\left(\mathbf{G}_{ds,pon}\mathbf{V}_{s,pon} + \mathbf{I}^t_{d,pon}\right), \\ &= -\mathbf{G}^{-1}_{dd,pon}\left(\mathbf{G}_{ds,pon}\mathbf{V}_{s,pon} + \text{diag}^{-1}\left(\mathbf{V}^t_{d,pon}\right)\mathbf{P}_{d,pon}\right). \quad (21) \end{aligned}$$

With a slight abuse of notation, the following iteration is obtained:

$$\mathbf{V}^{t+1}_{d,pon} = -\mathbf{G}^{-1}_{dd,pon}\left(\mathbf{G}_{ds,pon}\mathbf{V}_{s,pon} + \left(\frac{\mathbf{P}_{dk,pon}}{\mathbf{V}^t_{dk,pon}}\right)^\top_{k \in \mathcal{D}}\right), \quad (22)$$

with $\mathcal{D}$ being the set that contains all the demand nodes ordered per bus and phase, respectively.

Now, consider the right hand side of (22) and two vector of voltages $\mathbf{V}_{dk,pon}, \mathbf{U}_{dk,pon} \in \mathcal{B}$. Then, we have the following:

$$\begin{aligned} &\left\| f(\mathbf{V}_{dk,pon}) - f(\mathbf{U}_{dk,pon}) \right\| \\ = &\left\| -\mathbf{G}_{dd,pon}^{-1} \left(\left(\frac{\mathbf{P}_{dk,pon}}{\mathbf{V}_{dk,pon}} \right)_{k\in D}^{\top} - \left(\frac{\mathbf{P}_{dk,pon}}{\mathbf{U}_{dk,pon}} \right)_{k\in D}^{\top} \right) \right\| \\ &\leq \frac{\left\| \mathbf{G}_{dd,pon}^{-1} \mathbf{P}_{d,pon} \right\|}{(v_{\min})^2}. \end{aligned} \quad (23)$$

The last part of (23) is equal to ζ in Eq. (15) and constitutes a Lipschitz constant of f. Moreover, due to Assumption 3, $\zeta < 1$ –and hence f– is a contraction mapping. Using Theorem 1, it is concluded that the solution of the power flow equation is unique and can be obtained by Algorithm 1. □

If we contemplate Consideration 3, which is associated with the nature of the resistance-like matrix, the condition in Eq. (16) can be rewritten as follows:

$$\zeta = \max_{k\in D} \left\{ \frac{\left| \mathbf{G}_{dd,pon_{kk}} \right| \left| \mathbf{P}_{dk,pon} \right|}{(V_{\min})^2} \right\}. \quad (24)$$

Note that, in (24), if we consider that the resistance-like parameter $\mathbf{G}_{dd,pon_{kk}}$ is the Thëvenin equivalent resistance at node k, then Eq. (24) can be rewritten as follows:

$$\zeta = \max_{k\in D} \left\{ \frac{\mathbf{P}_{dk,pon}/V_{\min}}{|V_{\min}|/\mathbf{G}_{dd,pon_{kk}}} \right\}. \quad (25)$$

Remark 1. From Eq. (25), we can observe that the condition $0 < \zeta < 1$ is ensured since the denominator of Eq. (25) can be understood as the lowest short-circuit current of the system. At the same time, its numerator defines the highest load current. This is important because, for any operating load condition covered by Consideration 2, the short-circuit current is always higher than the load current.

Remark 2. Note that with the confirmation of the parameter ζ in its bounds, as required by the Banach fixed-point theorem, we can ensure that the recursive power flow formula in Eq. (11), which is obtained based on applying the successive approximation power flow method, is stable, and converges to the solution.

5. Equivalence backward/forward power flow formulation

The main characteristic of the successive approximation studied in Sections 2 and 3 is that its formulation is entirely equivalent to the backward/forward power flow method in its matrix form [20]. Furthermore, both formulations can work with radial and meshed bipolar DC networks without modifying their power flow formulations. Here, we explore the equivalence between both formulations, which are new for solving the power flow problem in bipolar DC grids with grounded and non-grounded neutral conductors.

To obtain the recursive formula as expressed by Eq. (11) using the backward/forward power flow formulation, let us use the branch-to-node incidence matrix $\mathbf{A}_{pon}$ to relate branch voltage and currents with nodal voltages and currents [27,28].

The branch-to-node incidence matrix can split into two submatrices as follows:

$$\mathbf{A}_{pon} = \left[\mathbf{A}_{s,pon} \; \mathbf{A}_{d,pon} \right], \quad (26)$$

where $\mathbf{A}_{s,pon}$ and $\mathbf{A}_{d,pon}$ represent the components of the branch-to-node incidence matrix that relates the branches to the slack and demand nodes, respectively.

Now, with the submatrices $\mathbf{A}_{s,pon}$ and $\mathbf{A}_{d,pon}$, it is possible to obtain a general formula for the branch voltages as follows:

$$\mathbf{E}_{pon} = \mathbf{A}_{s,pon}\mathbf{V}_{s,pon} + \mathbf{A}_{d,pon}\mathbf{V}_{d,pon}, \quad (27)$$

where $\mathbf{E}_{pon}$ is the voltage drop in all the lines.

On the other hand, the net injected currents can be related to the branch currents as follows [20]:

$$\begin{bmatrix} \mathbf{I}_{s,pon} \\ \mathbf{I}_{d,pon} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{s,pon}^{\top} \\ -\mathbf{A}_{d,pon}^{\top} \end{bmatrix} \mathbf{J}_{pon}, \quad (28)$$

where $\mathbf{I}_{s,pon}$ is the net injected current in the slack source, and $\mathbf{J}_{pon}$ is the vector that contains all the currents in the branches ordered by pole.

Now, by using Ohm's law, it is possible to relate the branch currents to their voltage drops as follows:

$$\mathbf{J}_{pon} = \mathbf{G}_{pon}^{\text{prim}}\mathbf{E}_{pon}, \quad (29)$$

where $\mathbf{G}_{pon}^{\text{prim}}$ is the primitive conductance matrix that contains all the inverses of the resistive parameter of each branch per pole (note that this is a purely diagonal matrix).

Note that, if Eq. (27) and the second row of Eq. (28) are combined with Eq. (29), the following result is obtained:

$$\mathbf{I}_{d,pon} = -\mathbf{A}_{d,pon}^{\top}\mathbf{G}_{pon}^{\text{prim}} \left[\mathbf{A}_{s,pon}\mathbf{V}_{s,pon} + \mathbf{A}_{d,pon}\mathbf{V}_{d,pon} \right], \quad (30)$$

which can be easily rewritten as follows:

$$\mathbf{V}_{d,pon} = -\left[\mathbf{A}_{d,pon}^{\top}\mathbf{G}_{pon}^{\text{prim}}\mathbf{A}_{d,pon} \right]^{-1} \left[\begin{matrix} \mathbf{A}_{d,pon}^{\top}\mathbf{G}_{pon}^{\text{prim}}\mathbf{A}_{s,pon}\mathbf{V}_{s,pon} \\ +\mathbf{I}_{d,pon} \end{matrix} \right]. \quad (31)$$

Remark 3. Note that, if the iterative counter t is used to recursively solve (31) and the following definitions are made, namely

$$\mathbf{G}_{dd,pon} = \mathbf{A}_{d,pon}^{\top}\mathbf{G}_{pon}^{\text{prim}}\mathbf{A}_{d,pon},$$
$$\mathbf{G}_{ds,pon} = \mathbf{A}_{d,pon}^{\top}\mathbf{G}_{pon}^{\text{prim}}\mathbf{A}_{s,pon},$$

then the matricial backward/forward and the successive approximation methods are completely equivalent for solving the power flow problem in bipolar DC grids [20].

6. Numerical example

To illustrate the main characteristics of the bipolar DC power flow problem, in this section, we present a numerical example composed of 4 nodes. The information of the lines and the constant power loads are reported in Table 1.

Table 1
Branch and load information for the 4-bus example.

Node j	Node k	R_{jk} (Ω)	$P_{dk,po}$ (W)	$P_{dk,no}$ (W)	$P_{dk,pn}$ (W)
1	2	0.25	500	700	950
2	3	0.50	750	350	0
3	4	0.45	250	600	700

For this test system, we consider that all the positive, negative, and neutral terminals are assigned to the same conductor size [12]. In addition, the voltage in the voltage-controlled node, *i.e.*, node 1, is assigned as ±220 V, with 0 V in the neutral point.

For this system, the first case considers that the neutral point is solidly grounded in each load connection, and the second case assumes that the neutral is only grounded at the substation point.

Table 2 presents the voltage in the positive, neutral, and negative poles when both cases are simulated.

The results in Table 2 show that (i) the positive and negative poles in both simulation cases exhibit different voltage magnitudes, which are independent from the type of connection of the neutral terminal, a situation caused by the asymmetric nature of the loads, since the positive pole has a total demand (line-to-neutral) of 1500 , while the negative pole accumulates a total demand of 1650 W; (ii) the main effect of the non-grounding of the neutral point is the voltages that appear at each node (however, for this example, these values not exceed

Table 2
Voltage for both simulation cases.

Node	+ pole (V)	0 pole (V)	− pole (V)
Grounded neutral			
1	220	0	−220
2	217.2970	0	−217.1193
3	214.1348	0	−214.0633
4	212.8650	0	−212.0487
Non-grounded neutral			
1	220	0	−220
2	217.2990	−0.1834	−217.1155
3	214.1399	−0.0864	−214.0535
4	212.8721	−0.8384	−212.0337

Table 3
Branch and load information for the 21-bus system.

Node j	Node k	R_{jk} (Ω)	$P_{dk,po}$ (kW)	$P_{dk,no}$ (kW)	$P_{dk,pn}$ (kW)
1	2	0.053	70	100	0
1	3	0.054	0	0	0
3	4	0.054	36	40	120
4	5	0.063	4	0	0
4	6	0.051	36	0	0
3	7	0.037	0	0	0
7	8	0.079	32	50	0
7	9	0.072	80	0	100
3	10	0.053	0	10	0
10	11	0.038	45	30	0
11	12	0.079	68	70	0
11	13	0.078	10	0	75
10	14	0.083	0	0	0
14	15	0.065	22	30	0
15	16	0.064	23	10	0
16	17	0.074	43	0	60
16	18	0.081	34	60	0
14	19	0.078	9	15	0
19	20	0.084	21	10	50
19	21	0.082	21	20	0

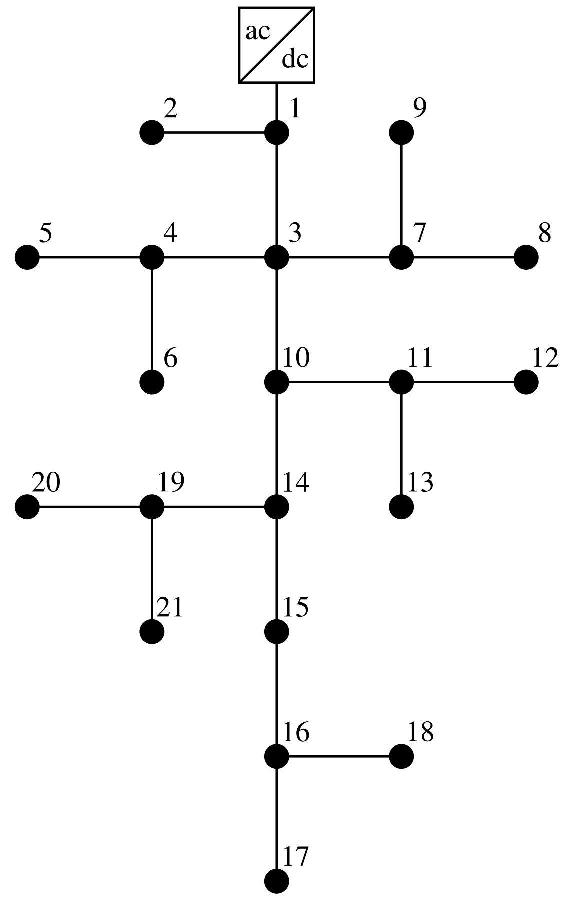

Fig. 2. Electrical connections between nodes in the 21-bus system.

1 V, which is due to the small power consumption in this low-voltage bipolar DC grid); and (iii) when power losses are calculated in for both simulation cases, case 1 presents a total power loss of 113.6987 W, while the second simulation case has a total power losses of 115.2223 W, which implies that the presence of a non-solidly grounded neutral increases the power losses due to the currents that flow through the conductor.

7. Numerical experiments

The computational validation of the proposed power flow approach for asymmetric bipolar DC networks considers a medium voltage distribution network, which is illustrated in Fig. 2. The test system is composed of 21 nodes and 21 constant power loads originally used for balanced DC power flow studies [29]. This is adapted in this research to include unbalanced phase-to-ground loads. The electrical configuration of this test feeder is presented below (see Fig. 2).

In the 21-bus system, the slack bus is operated in both poles with ±1 kV. The electrical parameters for this test feeder are reported in Table 3.

For this test feeder, we consider having all the nodes concerning the solidly grounded neutral point in simulation scenario 1. The effect of the non-grounded neutral is studied in simulation scenario 2.

Table 4 presents the voltage in the positive and negative poles when both cases are simulated.

Numerical results in Table 4 allow noting that, in simulation case 1 (*i.e.*, solidly grounded neutral), the positive pole has some nodes with voltage regulations higher than 10%, with the minimum voltage at node 17 having a magnitude of 890.1027 V, *i.e.*, a regulation of 10.99%. In contrast, the worst voltage regulation for the negative pole occurs at node 18, with a value of 9.14%. This happens because the total demand between the positive pole and the neutral conductor is 554 kW. In contrast, the negative pole has a total phase-to-ground demand of 445 kW, which implies that the positive pole has more than 100 kW of demand compared to the negative pole. This difference translates into a drastic effect on the voltage along the grid. Furthermore, the neutral pole in nodes 16 and 17 has voltages higher than 20 V, with magnitudes of 20.6576 and 24.3408, respectively. These increments in the neutral voltage can affect some sensitivity devices connected between the positive or negative poles to the neutral since the reference value is far from zero. The effective voltage between both terminals can be significantly reduced (notice that node 17 is between the positive and the neutral poles).

To demonstrate the proposed successive approximation power flow method's effectiveness at solving power flow in bipolar DC grids, we include a simulation scenario where two meshes are added to the 21-bus system. The added lines are in corridors 7–19 and 11–16 with resistances of 0.082 Ω and 0.037 Ω, respectively.

Numerical results for the meshed grid allow observing that (i) the power loss in the case of the solidly grounded neutral is 75.1832 kW, which increases to 78.7372 kW for the non-grounded neutral, a difference of about 3.5540 kW. This is associated with the currents that flow through the neutral cable product of the load unbalances between the positive and negative poles; and (ii) the behavior of the voltage profiles in Fig. 3 for the meshed network in the grounded and non-grounded cases follow pretty similar profiles. However, the main difference is observed while comparing the positive and negative poles; in the first case, the voltages at some nodes fall to 0.9252 pu (node 17), while the negative pole has a minimum value of 0.9392 pu (node 18). The

Table 4
Voltage for both simulation cases in the 21-bus system.

Node	+ pole (V)	0 pole (V)	− pole (V)
Grounded neutral			
1	1000	0	−1000
2	996.2761	0	−994.6716
3	960.0683	0	−968.4100
4	952.3714	0	−962.7830
5	952.1067	0	−962.7830
6	950.4396	0	−962.7830
7	953.7448	0	−964.5412
8	951.0867	0	−960.4284
9	943.8619	0	−960.7609
10	937.4888	0	−948.4698
11	930.9220	0	−942.8952
12	925.1152	0	−936.9933
13	926.9467	0	−939.7613
14	916.4715	0	−930.2940
15	905.4444	0	−921.0085
16	896.1420	0	−913.9505
17	890.1027	0	−911.4861
18	893.0582	0	−908.6017
19	909.9528	0	−924.3558
20	905.7061	0	−921.1448
21	908.0565	0	−922.5781
Node	+ pole (V)	0 pole (V)	− pole (V)
Non-grounded neutral			
1	1000	0	−1000
2	996.2821	−1.6193	−994.6628
3	959.5205	9.2157	−968.7363
4	951.7636	11.3722	−963.1358
5	951.4955	11.6403	−963.1358
6	949.8030	13.3327	−963.1358
7	953.1183	11.7700	−964.8884
8	950.4291	10.3922	−960.8213
9	943.1110	17.9963	−961.1073
10	936.5746	12.4855	−949.0602
11	929.9259	13.6204	−943.5464
12	924.0247	13.7095	−937.7342
13	925.9357	14.4762	−940.4119
14	915.1628	15.9904	−931.1532
15	903.9040	18.1140	−922.0181
16	894.4081	20.6576	−915.0657
17	888.2594	24.3408	−912.6002
18	891.2522	18.5786	−909.8309
19	908.5513	16.7359	−925.2872
20	904.2616	17.8323	−922.0939
21	906.6158	16.9276	−923.5434

difference between both poles can be attributed to the presence of unbalanced phase-to-ground loads that cause the positive pole to have a higher voltage drop when compared with the negative pole.

Finally, in the case of the radial configuration, the proposed bipolar power flow method takes about 10 iterations (0.4911 ms) for the grounded simulation case and 13 iterations (0.5275 ms) for the non-grounded scenario. In the case of the meshed network, the grounded simulation takes 10 iterations (0.4780 ms), and the non-grounded scenario takes about 11 iterations (0.5262 ms). These values confirm that the meshed case requires fewer iterations to stabilize the power flow solution, which is expected due to the better distribution of the voltage profiles along the grid. In addition, the average processing time shows a linear tendency regarding the number of iterations and the required processing times. However, for all the simulation cases, the total processing time was about half a millisecond, which is an adequate processing time for any real-time operation in steady-state conditions.

8. Conclusions

This paper addressed the problem of the power flow analysis in bipolar DC networks considering the possibility of grounding or not grounding the neutral. It was demonstrated that the classical backward/forward and the successive approximation power flow methods have the same recursive iterative formula, *i.e.*, these methods are entirely equivalent and apply to radial and meshed DC distribution grids. The Banach fixed-point theorem demonstrated that the proposed successive approximation method converges to the power flow solution since its recursive formula is indeed a contraction map.

Future works will cover the following topics: (i) using the proposed successive approximations embedded in a master–slave optimization methodology to address the problem of phase-balancing in bipolar DC grids; and (ii) formulating the optimal power flow problem in bipolar DC grids using a convex quadratic approximation that guarantees that a global optimum is found via interior point methods.

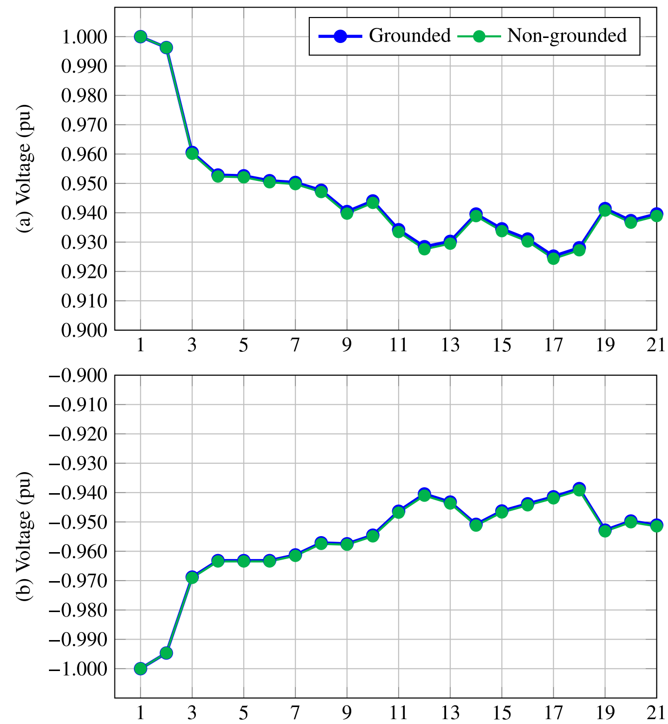

Fig. 3. Voltage profiles in the meshed network with and without grounded-neutral.

CRediT authorship contribution statement

Oscar Danilo Montoya: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Acquisition of funds. **Walter Gil-González:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Alejandro Garcés:** Conceptualization, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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