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On the mathematical modeling for optimal selecting of calibers of conductors in DC radial distribution networks: An MINLP approach

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ABSTRACT

This paper addresses the problem of optimal conductor selection in direct current (DC) distribution networks with radial topology. A nonlinear mixed-integer programming model (MINLP) is developed through a branch-to-node incidence matrix. An important contribution is that the proposed MINLP model integrates a set of constraints related to the telescopic structure of the network, which allows reducing installation costs. The proposed model also includes a time-domain dependency that helps analyze the DC network under different load conditions, including renewable generation and battery energy storage systems, and different voltage regulation operative consigns. The objective function of the proposed model is made up of the total investment in conductors and the total cost of energy losses in one year of operation. These components of the objective function show multi-objective behavior. For this reason, different simulation scenarios are performed to identify their effects on the final grid configuration. An illustrative 10-nodes medium-voltage DC grid with 9 lines is used to carry out all the simulations through the General Algebraic Modeling System known as GAMS.

Acronyms

AC	Alternating current
DC	Direct current
COP	Colombian pesos
GAMS	General algebraic modeling system
MCOP	Millions of Colombian pesos
MINLP	Mixed-integer nonlinear programming
MST	Minimum spanning tree

1. Introduction

1.1. General context

The optimal selection of conductors in electrical networks is an important subject of study since a poor choice can compromise the network operation. Because if the conductors are incorrectly selected, the electrical system may not meet user demand or would have low reliability and efficiency [1]. The problem of optimal selection of size conductors is a sub-field of efficient expansion for electrical networks [2]. Additionally, the efficient expansion plan for electricity networks should improve the economic viability of utilities and an efficient, safe, and reliable providing of the service to end-users, satisfying the quality problems required by service regulatory entities [3].

Optimal selection of conductor calibers in electrical networks is a complex problem since it must consider different constraints, such as the voltage profile bounds in all buses, the current flow capacity of the feeders, and variation in the demand profile, and its growth [4]. Additionally, other economic aspects must also consider considerations related to the cost of losses and conductors and interest and depreciation rates [5].

In general, this problem is interesting because it involves all the classical complications of mixed-integer optimization problems. In addition, the size of the solution space is an exponential function of the number of calibers available and the entire lines of the grid, which implies that powerful optimizations techniques must be necessary to solve in polynomial times, as reported in alternating currents.

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1.2. Motivation

The study and analysis of direct current (DC) networks have increased in recent years, thanks to the deployments of power electronics, which have permitted direct integration between electrical systems such as energy storage devices, photovoltaic solar systems, and demands, among others [6]. Furthermore, these studies and analyzes have also increased because DC networks have advantages over alternating current (AC) networks since they are more efficient in energy and voltage loss and less complex in operation and control than AC networks [7,8].

In the same form that AC grids, DC networks' planning and operation must comply with operational standards and develop efficient plans and methodologies for their expansion. Therefore, few works have been reported in specialized literature for addressing planning problems in DC electrical networks [9]. The problems that have not been explored until the revision of the state-of-the-art are the problem of optimal selection of the conductors' calibers for radial DC distribution networks, which is identified in this paper as an important opportunity contributing to the literature. For this reason, here we propose a new mathematical formulation for addressing the problem of selecting the best calibers for conductors in DC grids that can be used in expansion and operation plans by utilities for providing high-quality energy service in rural remote and urban areas.

1.3. Brief state-of-the-art

Optimal conductor selection has generally been studied for AC networks, where different approaches have been proposed to solve this problem considering different objectives. The most outstanding works in the efficient solutions to this problem are the application of heuristic algorithms. In [10], an economical current density method and a heuristic algorithm were combined to select the conductor size optimal in planning radial distribution systems. In [11], a genetic algorithm method was implemented for the optimal conductor selection. This method focused on minimizing real and reactive power losses in the network and maximizing the total saving in installation costs. In [12], the optimal conductor selection in the radial distribution network was carried out using the harmony search algorithm. The objective function proposed in [12] minimized the sum of capital investment and the energy loss cost. In [13], a selective particle swarm optimization algorithm was presented to choose the optimal capacitor placement and conductor sizing in a radial distribution system. The objective function has two objectives, which were to minimize power losses and improve voltage profiles. In [14], a particle swarm optimization modified with a differential evolution algorithm was performed to select the optimal conductor size in the radial distribution networks. It is also possible to find linear [15] and non-linear [3] approaches to formulate the problem. In the case of DC networks, no models have been found for this problem. Only optimal planning and operation have been studied, such as, in [9], a framework for optimal planning and design of low-power low-voltage dc microgrids for minimum upfront cost was developed. In [16], the planning and optimization of DC microgrids for the Indian context were looked. In [17], optimal operational planning of DC microgrid with demand response was presented by considering battery degradation cost. Observe that optimal conductor selection has not yet been analyzed and modeled for DC networks, which is a gap in the literature that this research tries to fulfill.

1.4. Contribution and scope

There are no reports about the problem of the optimal selection of the calibers of conductors in DC networks in the specialized literature. The main contribution of this paper is regarding the proposition of a mixed-integer nonlinear programming model (MINLP) model to represent this problem. The most important aspects of this new model are: *i)* the formulation of two objective functions related to the total inversion costs in conductors and the minimization of the total cost of the energy during the planning horizon. *ii)* the explicit representation of the resistive value of the conductor as a function of the binary decision variable and its effect on the current flow at each branch. *iii)* the usage of the conventional node-to-branch matrix for determining the voltage drops in all the branches and the effect of branch current into the power balance constraints, and *iv)* the explicit formulation of the telescopic constraint that is rarely used in distribution system planning problems.

It is worth mentioning regarding engineering education, we consider as an additional contribution the proposed methodology for solving the proposed MINLP model is presented in a tutorial form by using the General Algebraic Modeling System (GAMS). This software has been previously employed in [18] and [19] in relation to the optimal location and sizing of distributed generators, as in general form for power system engineering in [20]. To introduce the GAMS software, we give a simple and classical example in the optimization area, which is the minimum spanning tree (MST) problem. This problem has binary variables, and it can also be modeled with incidence matrices, which is perfect for introducing engineering students to optimization problems using compact models that involve discrete variables as the case studied in this research.

1.5. Paper organization

The remainder of this paper is organized as follows: Section 2 presents the complete mathematical formulation of the problem of the optimal selection of the calibers for conductors in radial DC distribution networks. Section 3 presents the proposed solution methodology by using GAMS software. An illustrative example based on the classical MST problem introduces students and researchers to discrete optimization. Section 4 presents the information of the 10-node test feeder and the numerical implementation and results by scenarios of simulation. Section 5 presents the main concluding remarks derived from this research as well as possible future works.

2. General MINLP formulation

The problem of the optimal selection of the calibers for conductors in DC distribution networks has an MINLP structure due to the following aspects:

- ✓ the presence of binary variables regarding the conductor type-c selected for the line-route j, i.e., variable x_{jc}.
- ✓ the product between continuous variables in power balance equations, i.e., voltages in nodes v_{it} and currents through lines I_{jt} at each period of time.

To reach the general MINLP formulation of the optimal selection of conductors in DC distribution networks, let us define some important matrices based on the radial DC network example depicted in Fig. 1.

Definition 1. (*Node-to-branch incidence matrix*) The node-to-branch incidence matrix $\mathbf{A}$ is composed by n rows (nodes) and b columns (branches) that contains the information about connections between lines and nodes, and each components are determined as follows:

$$\mathbf{A}_{ij} = \begin{cases} +1 & \text{if } I_j \text{ leaves from node } i. \\ -1 & \text{if } I_j \text{ arrives to node } i. \\ 0 & \text{otherwise.} \end{cases}$$

Now, considering Definition 1, the branch-to-node incidence matrix $\mathbf{A}$ takes the following form:

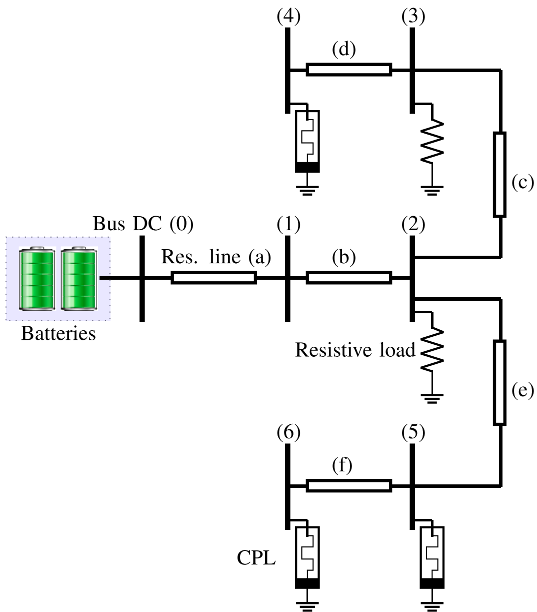

Fig. 1. Example of a possible configuration of a radial DC network.

$$\mathbf{A} = \begin{bmatrix} i/j & a & b & c & d & e & f \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & -1 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & -1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & -1 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & -1 & 1 \\ 6 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Remark 1. Note that the signs at each column (branch) determine the sending and receiving nodes, and they will help define the voltage drops for determining the magnitude of the current as a function of the caliber selected (resistance).

It is worth mentioning that the incidence matrix contains the topological information of the electrical network regarding nodal and branch connections, being it the essential matrix in power flow analyses via admittance formulations [21].

Definition 2. (*Line-to-line incidence matrix*) The line-to-line incidence matrix **T** is a square matrix with dimensions $b \times b$ that determines the scale of connections between lines as follows

$$\mathbf{T}_{jk} = \begin{cases} 1 & \text{if line } k \text{ is connected directly upstream of the line } j. \\ 0 & \text{otherwise.} \end{cases}$$

Now, considering Definition 2, the line-to-line incidence matrix **T** takes the following form:

$$\mathbf{T} = \begin{bmatrix} j/k & a & b & c & d & e & f \\ a & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 1 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 0 & 0 \\ e & 0 & 1 & 0 & 0 & 0 & 0 \\ f & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Remark 2. Note that the **T** matrix will help to guarantee telescopic configurations in radial DC distribution networks, i.e., reduction of calibers in conductors and the branch, is far from the primary substation.

Now, we present the general MINLP model for selecting calibers for radial DC distribution networks. Regarding the objective function, it considers the energy cost during a year of operation, as well as the total investment in conductors as follows:

Objective function:

$$\min z = z_1 + z_2, \tag{1}$$

$$z_1 = \sum_{j \in \mathscr{L}} L_j \left(\sum_{c \in \mathscr{C}} C_c^{\text{inv}} x_{jc} \right), \tag{2}$$

$$z_2 = \alpha \sum_{j \in \mathscr{L}} L_j \left(\sum_{c \in \mathscr{C}} x_{jc} r_c \left(\sum_{t \in \mathscr{T}} C_t^{Wh} I_{jt}^2 \Delta T \right) \right), \tag{3}$$

where z_1 is the objective function related to the installation costs of conductors in the DC network, and z_2 corresponds to the objective function related to the costs of the energy losses during the planning horizon (typically a year, i.e., 8760 h). In addition, L_j is the length of the branch j in kilometers, C_c^{inv} is the investment cost of a conductor with caliber type-c. α is constant parameter related to the length of the planning horizon r_c is the resistance value of the conductor with caliber type-c; C_t^{Wh} is the cost of the energy in watts-hour, and I_{jt} is a continuous variable regarding the current that flows in the branch j during the period of time t, and ΔT is the fraction of the time where power losses are evaluated (i.e., $\Delta T = 1$ h). Note that $\mathscr{C}$, $\mathscr{L}$ and $\mathscr{T}$ are the sets that contains all the calibers, branches and periods of time, respectively.

In the case of the constraints for the optimal selection of conductors in DC networks, they are related to power balance, current capabilities, voltage regulation bounds, and radiality. All the set of constraints for this optimization problem are listed below:

Set of Constraints:

$$I_{jt} = \left(\sum_{c \in \mathscr{C}} \frac{x_{jc}}{r_c L_j} \right) \left(\sum_{i \in \mathscr{N}} A_{ij} v_{it} \right), \quad \{\forall j \in \mathscr{L}, \forall t \in \mathscr{T}\} \tag{4}$$

$$p_{it}^g - p_{it}^d = v_{it} \sum_{j \in \mathscr{L}} A_{ij} I_{jt}, \quad \{\forall i \in \mathscr{N}, \forall t \in \mathscr{T},\} \tag{5}$$

$$\sum_{c \in \mathscr{C}} \sum_{j \in \mathscr{L}} x_{jc} = b, \tag{6}$$

$$\sum_{c \in \mathscr{C}} x_{jc} = 1, \quad \{\forall j \in \mathscr{L}\} \tag{7}$$

$$\sum_{k \in \mathscr{L}} \sum_{c \in \mathscr{C}} \beta_c T_{jk} x_{kc} \geq \left(\sum_{c \in \mathscr{C}} \beta_c x_{jc} \right) \left(\sum_{k \in \mathscr{L}} T_{jk} \right), \quad \{\forall j \in \mathscr{L}\} \tag{8}$$

$$-\sum_{c \in \mathscr{C}} x_{jc} I_c^{\max} \leq I_{jt} \leq \sum_{c \in \mathscr{C}} x_{jc} I_c^{\max}, \quad \{\forall j \in \mathscr{L}, \forall t \in \mathscr{T},\} \tag{9}$$

$$v_i^{\min} \leq v_{jt} \leq v_i^{\max}, \quad \{\forall i \in \mathscr{N}, \forall t \in \mathscr{T}\} \tag{10}$$

where p_{it}^g and p_{it}^d are the power generation and demands at node i during the period of time t; β_c corresponds to the type of conductor c (numerical designation of the caliber, i.e., natural number, where this is assigned in increasing way with the current capability of the conductor). $I_c^{\max}$ is the maximum thermal current allowed in the branch j with caliber type-c; and $v_i^{\min}$ and $v_i^{\max}$ are the minimum and maximum voltage bounds allowed at each node i at any period of time. Note that two important components of the proposed mathematical model correspond to the branch-to-node and the line-to-line incidence matrices, which allow defining power balance and radiality constraints.

Remark 3. The main consideration in the proposed optimization model is that the capability of the distribution substation, i.e., p_{it}^g, is such that the total power consumption is attended by this substation (e.g., AC/DC converter) including power losses. This can be formulated as follows:

$$\sum_{i\in\mathcal{N}} p_{it}^{g} \geq \sum_{i\in\mathcal{N}} p_{it}^{d} + \sum_{j\in\mathcal{L}} L_j \left(\sum_{c\in\mathcal{C}} x_{jc} r_c I_{jt}^2 \right), \quad \forall t \in \mathcal{T}.$$

It is worth mentioning that, in a practical sense, the expression above is not implemented in the optimization since p_{it}^{g} is a free variable in the case of the slack node.

The interpretation of the mathematical model described from (1) to (10) is as follows: Equation (1) corresponds to the total cost of investment in conductors and the total cost of the energy losses during the period. Both components of the objective function are defined in Expressions (2) and (3), respectively. Equation (4) presents the calculation of the current in-branch j as a function of the resistance selected (caliber of a conductor) and the voltage difference between nodes associated with this branch. In Equation (5) presents the power balance constraint as each node (essential in power flow analysis defined in the set of nodes, i.e., $\mathcal{N}$); Expressions (6) and (7) show the maximum number of cables allowed per line, as well as the maximum of conductors selected into the network for maintaining radiality. Equation (8) is the constraint that guarantees that all the conductors are selected for a telescopic configuration if necessary. In Expressions (9) and (10) define the thermal-current capabilities in all the branches and the voltage regulation bounds in all the nodes of the network, respectively.

Remark 4. The proposed mathematical model for the optimal selection of conductors (1)-(10) can be extended to mesh networks by eliminating the telescopic constraint (8) as well as by relaxing the radiality constraint defined in (7).

Due to the main contribution in this paper is the mathematical modeling for the problem of the optimal selection of conductors in DC radial distribution networks as defined from (1) to (10), by considering telescopic constraints, we select as solution technique the general algebraic modeling system (GAMS) optimization package to solve considering different combinations of z_1 and z_2 in the objective function. To illustrate it, we select a small test feeder composed of 10 nodes, 9 lines, and 6 possible conductors to be installed, and the period of analysis will be a year, i.e., 8760 h divided into periods of 24 hours.

3. Solution methodology

To solve the mathematical model presented from (1) to (10), we select the GAMS package since this has been widely used in specialized literature for large-scale and complex optimization problems. Some of these works are: optimal location of distributed generators in AC and DC distribution networks [18,19], optimal design of water distribution grids [22], optimal operation and dispatch of battery energy storage systems [23], optimal design and operation of thermoacoustic engines [24], optimal sizing and dimensioning of osmotic power plants [25], and optimal planning and operation of power systems [20,26].

Here, to present the main aspects of using GAMS optimization software, let us solve the classical minimum spanning tree problem, as depicted in Fig. 2. The main idea is to find the subset of branches that interconnects all the nodes (vertices) with the minimum length possible.

The mathematical model of this problem considers all the branches as variables, as presented in the following formulation:

$$\min w = \sum_{j\in\mathcal{L}} L_j x_j, \tag{11a}$$

$$\text{s.t:} \sum_{j\in\mathcal{L}} \mathcal{A}_{ij} x_j \geq 1, \quad \forall i \in \mathcal{N} \tag{11b}$$

$$\sum_{j\in\mathcal{L}} x_j = n - 1, \tag{11c}$$

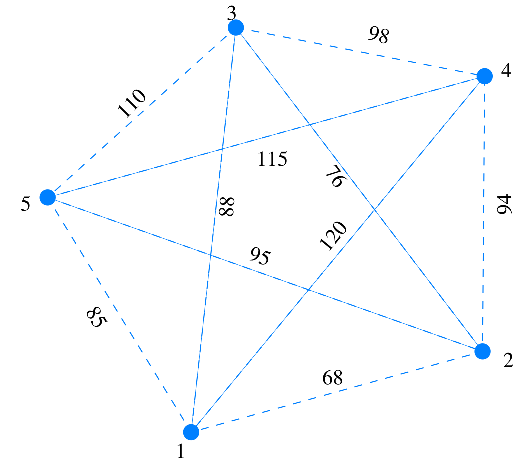

Fig. 2. Possible routes in the minimum spanning tree problem.

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{L} \tag{11d}$$

where w is the objective function value, $\mathcal{A}_{ij}$ has the same interpretation of the node-to-branch defined in 1 considering that all of its components can be zero or positive. Note that Expression (11a) presents the objective function, which is the algebraic sum of the different lengths of the branches; Equation (11b) is a linear constraint that guarantee that each node be connected at least for one branch; Expression (11c) presents the radiality constraint regarding the fact a tree with n nodes can be completely connected with $n-1$ branches [2]; and Expression (11d) shows the binary nature of the decision variables.

Remark 5. The optimization model (11) represents the minimum spanning tree problem using branches as variables, and it corresponds to a mixed-integer programming (MIP) problem that requires discrete methods to be solved.

The implementation of the simple optimization problem defined in (11) is made in the GAMS package, as presented in Fig. 3 [20].

As presented in Algorithm 3, the implementation of any optimization problem in GAMS is composed of five main aspects to know: *i)* it uses a compact formulation based on sets that allow changing the dimension of the problem without altering the mathematical structure; *ii)* all the constant values are defined as scalars, parameters, and tables, and these have the numerical information to feed the model; *iii)* the variables of the model can be continuous (i.e., w), binary or integer depending on the nature of the optimization problem; *iv)* the implementation of the equations allows writing them in symbolic form preserving the same mathematical form defined in (11); and *v)* To solve the model is required to select the nature of it, i.e., MIP, and the sense of the optimization search, i.e., minimization or maximization. To summarize, the implementation of any mathematical optimization model in the GAMS software in Fig. 4 is presented in the flow diagram employed in this research.

Once the mathematical model of the minimum spanning three is solved by GAMS, the following solution is reached:

```
GAMS 25.1.3 Execution
VARIABLE z.L = 323.000  Objective fun. value
VARIABLE x.L  Selection of the branch j
a 1.000,   d 1.000,   e 1.000,   f 1.000
```

Note that the objective function value is 323 units of length and this solution can be presented in graphic form as presented in Fig. 5.

Remark 6. The solution reported in Fig. 5 is the same reached by the Kruskal algorithm when it is implemented the minimum spanning three problems in MATLAB software [27].

```
*% Definition of the sets
SET V vertices of the tree /v1*v5/
SET L possible lines /a,b,c,d,e,f,g,h,i,j/;
*% Definition of the parameters (vectors)
PARAMETERS
Len(L)
/a 68,b 88,c 120,d 85,e 76,f 94,g 95,h 98,i 110,j
    115/;
*% Definition of the tables (matrices)
TABLE A(V,L)
a b c d e f g h i j
v1 1 1 1 1 0 0 0 0 0 0
v2 1 0 0 0 1 1 1 0 0 0
v3 0 1 0 0 1 0 0 1 1 0
v4 0 0 1 0 0 1 0 1 0 1
v5 0 0 0 1 0 0 1 0 1 1;
*% Definition of variables
VARIABLES
z Objetive function value
BINARY VARIABLE
x(L) Selection of the branch j;
*% Defintion of Equations
EQUATIONS
ObjF Objective function
Vert(V) Equation related to vertices
Tbranch Total branches;
*% Declaration of the equations
ObjF.. z =E= sum(L,Len(L)*x(L));
Vert(v).. sum(L,A(V,L)*x(L)) =G= 1;
Tbranch.. sum(L,x(L)) =E= card(V)-1;
*% Solution of the model
Model MST /all/;
solve MST us MIP min z;
display z.l,x.l;
```

Fig. 3. GAMS implementation of the minimum spanning tree problem.

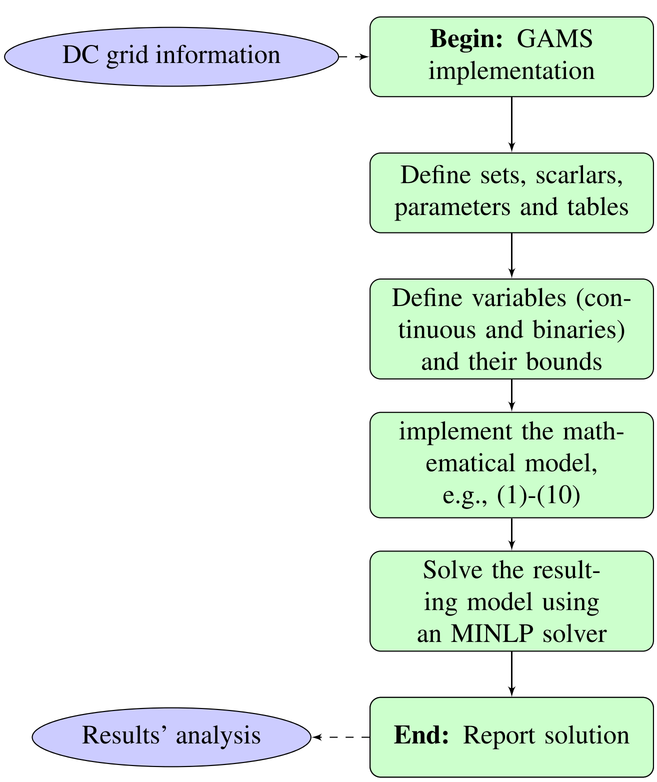

Fig. 4. Flow chart of the proposed optimization approach for optimal selection of conductors in dc networks.

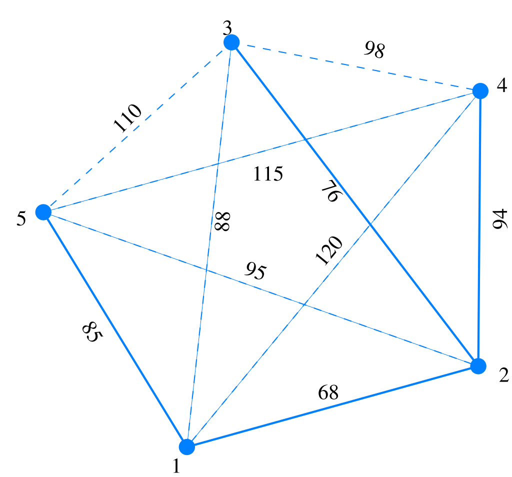

Fig. 5. Final routes for the minimum spanning tree problem.

4. Numerical validation

In this section, we present the numerical results for a 10-node test feeder, which are simulated in a desktop computer running on INTEL(R) Core(TM) i7-7700, 3.60 GHz, 8 GB RAM with 64-bit Windows 10 Prousing GAMS 25.1.3 with the large-scale nonlinear solver BONMIN.

4.1. Test system information

To deal with the problem of optimal selection of conductors in DC distribution networks with radial structure, let us consider a small DC network composed of 10 nodes, which are distributed in 9 constant power loads and 1 slack source. This test feeder will operate in low-voltage levels (i.e., 380 V) to feed a total demand of about 70 kW. The grid topology of this test feeder is presented in Fig. 6.

The demand information of this test feeder is presented in Table 1, and the possible calibers used in the grid design are provided in Table 2.

Note that the information of the cost conductors was updated from [3]. This information was reported Rs$/km using a rate of interchange between Brazilian reals and Colombian pesos about 749.35.

To analyze this test feeder we consider four different simulation scenarios to identify the effect of the objective function in the final selection of conductors.

- **Scenario 1 (S_1):** It is considered as objective function the total investment cost in conductors, i.e., $z = z_1$.
- **Scenario 2 (S_2):**It is considered as objective function the total cost of energy losses during a year of operation, i.e., $z = z_2$. The cost of the kilowatt-hour is assumed as COP$/kWh 479.3389, which corresponds to a value of the energy for a Colombian electricity company in 2019 [23]. In addition, we consider the percentage of variation of the demand in intervals of one hour as reported in Table 3, and the constant α is assumed as 365 days (normal duration of a year).
- **Scenario 3 (S_3):** In this scenario is considered a linear combination of both objective functions, i.e., $z = z_1 + z_2$.
- **Scenario 1 (S_1):** It is constructed a Pareto from by using one of the objective functions as constraint [28].

4.2. Simulation results

In this simulation, we present the main effects of using different combinations of the objective function in analyzing the optimal selection of conductor calibers in radial DC distribution networks.

4.2.1. Single-objective function analysis

Fig. 7 presents the numerical results achieved in GAMS for the first three simulation scenarios. From these results, we can observe that: *i)* when the objective function is minimizing the total investment in conductors for the distribution test feeder, the set of calibers selected are the minimum possible (see Table 4), i.e., the system will present higher chargeability factors in the peak hour since currents through the lines will come near to the nominal rate. Note that in S_1, the total investment is about MCOP$ 4.666, and the yearly power losses have a cost of about MCOP$ 11.304. With these results, the total cost in the first year is about MCOP$ 15.970, which the power losses takes about 70.78 % of the total cost. *ii)* when the goal is minimizing the total cost of the energy losses, GAMS finds an intuitive solution, which corresponds to assign to all the lines the highest caliber since this has the minimum resistance that helps with minimum losses due to these are proportional to this factor (see Table 4). Note that in this scenario, i.e., S_2 the function z_1 takes a value of MCOP$ 18.409 and the function z_2 takes a value of MCOP$ 6.538, which implies that the yearly power losses take about 26.21 % of the total cost. *iii)* regarding the linear combination of both objective functions it is observed that the total costs of the year in S_3 are lower when compared to scenarios S_1 and S_2, respectively. In this scenario, the selected calibers are uniformly distributed to help with minimizing both objective functions, as can be seen in Table 4. Note that the total cost of the year is about MCOP$ 15.201, where the 59.10 % represents the cost of the energy losses, and the resting 40.90 % is related to the inversion in conductors.

In conclusion, considering as simulation cases the three first simulation scenarios, the best model corresponds to the linear combination of

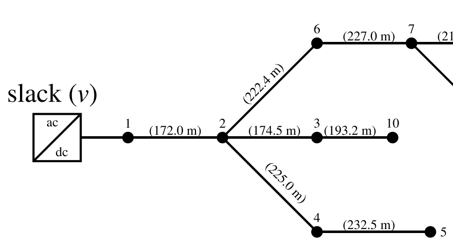

Fig. 6. Electrical configuration for the 10-node test system for selecting the calibers of conductors.

Table 1
Demand information.

Node	**Power** [kW]	Node	**Power** [kW]
1	0	6	13
2	0	7	0
3	7.50	8	11.50
4	10	9	12
5	8.25	10	7.75

Table 2
Information about conductors.

β_c	r_c [Ω/km]	C_c^{inv} [COP$/km]	I_c^{max} [A]
1	0.8763	1488402.67	180
2	0.6960	2090958.43	200
3	0.5518	2859142.08	230
4	0.4387	3814687.60	270
5	0.3480	6045792.70	300
6	0.2765	9497747.72	340

Table 3
Demand variation during the day.

t	Var. [p.u.]	t	Var. [p.u.]	t	Var. [p.u.]	t	Var. [p.u.]
1	0.4240	7	0.5669	13	0.8013	19	1.0000
2	0.4108	8	0.6326	14	0.7899	20	0.9682
3	0.3999	9	0.7202	15	0.7774	21	0.8890
4	0.4083	10	0.7805	16	0.7704	22	0.7832
5	0.4744	11	0.8268	17	0.8022	23	0.6175
6	0.5301	12	0.8369	18	0.8926	24	0.5212

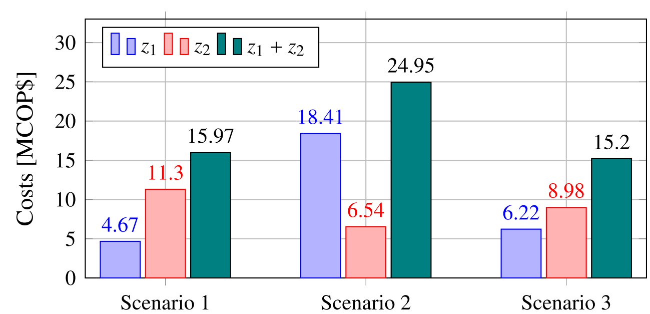

Fig. 7. Behavior of the costs of installation and energy losses.

Table 4
Type of conductors selected at each simulation case.

Line	Nodes	**Calibers S_1**	**Calibers S_2**	**Calibers S_3**
1	1–2	5	6	6
2	2–3	1	6	2
3	2–4	1	6	3
4	2–6	4	6	5
5	3–10	1	6	1
6	4–5	1	6	1
7	6–7	3	6	4
8	7–8	1	6	1
9	7–9	2	6	1

both objective functions, i.e., $z_1 + z_2$, since this is 4.82 % and 39.07 % cheapest than $\mathbf{S}_1$ and $\mathbf{S}_2$, respectively.

The chargeability factor in the lines in Fig. 8 has presented these values for each simulation scenario.

As mentioned before, the chargeability in all the conductors is lower when the objective function is minimizing the total energy losses since big calibers are selected (see scenario $\mathbf{S}_2$ in Table 4 and Fig. 8); while in the case of total investment costs minimization the chargeability is higher in all the lines since the smallest possibles calibers are selected (see scenario $\mathbf{S}_1$ in Table 4 and in Fig. 8). Finally, in the case of the linear combination of both objective functions, the chargeability factor in all the lines is in the middle of both previous cases, since calibers are distributed between the maximum and minimum possibilities as can bee observed in Fig. 8 and Table 4 for the third scenario.

To verify that all the solutions found in the evaluation of each simulation scenario fulfill with the voltage profile bounds, i.e., ± 10, in Fig. 9 are presented the voltage profile at the peak load condition, i.e., hour 19th in Table 3.

From Fig. 9, we can confirm that the third simulation scenario presents the average behavior regarding voltage profile since this balances the total power losses with the total investment costs. In the case of **S2**, it selects the highest possible calibers, which implies that the best voltage profiles can exhibit by this solution. At the same time, **S1** presents the worst voltage performance since this corresponds to the most economical solution in terms of calibers investment, which implies the highest resistances and lower voltage profiles. However, in all the simulation scenarios, the minimum voltage profile is always fulfilled. This implies that all the solutions are 100 % feasible.

4.2.2. *Multi-objective function analysis*

To identify the multi-objective compromise between both objective functions, we construct a Pareto from by using the ϵ − constraint method, as depicted in Fig. 10. From this picture, we can observe the following facts:

- ✓ The extremes of the Pareto from, i.e., solutions 1 and 10, corresponding to the minimum investment costs (maximum costs of the energy losses) and the maximum investment costs (minimum cost of the energy losses). Note that these results are the solutions reported in Fig. 7 for simulation scenarios 1 and 2, respectively.
- ✓ Solutions located in the middle of the Pareto from, i.e., z_1 about MCOP$ 6 and z_2 about MCOP$ 9, presents the best trade-off between investments and total costs of the energy losses, being both the most attractive alternatives for a utility.

It is important to mention that this Pareto front can be found using the weighting method for objective function, i.e., $z = \omega z_1 + (1 - \omega) z_2$ being ω contained between 0 and 1 [29].

Regarding the total operating costs of the network during the planning horizon, if we added the values of the objective functions reported in the Pareto front in Fig. 10, then, we find the results reported in Fig. 11.

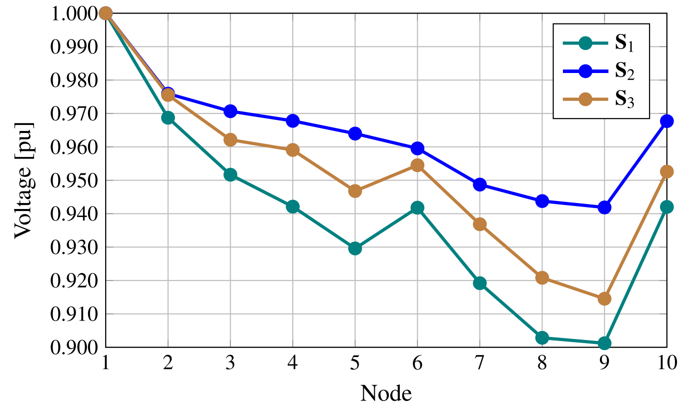

Fig. 9. Voltage profile for each simulation scenario at the peak load condition.

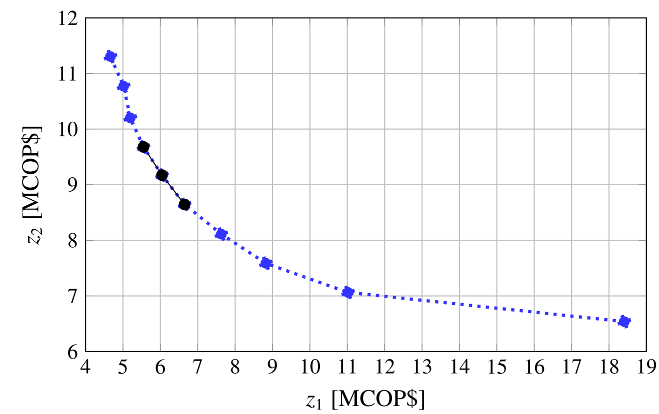

Fig. 10. Pareto front of the economic–environmental dispatch problem.

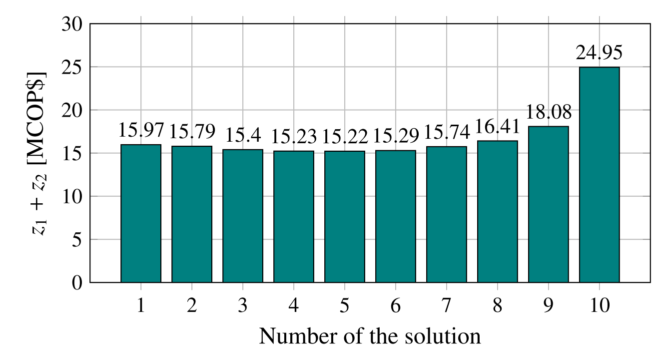

Fig. 11. Pareto front of the for the multi-objective evaluation in the problem of optimal selection of calibers.

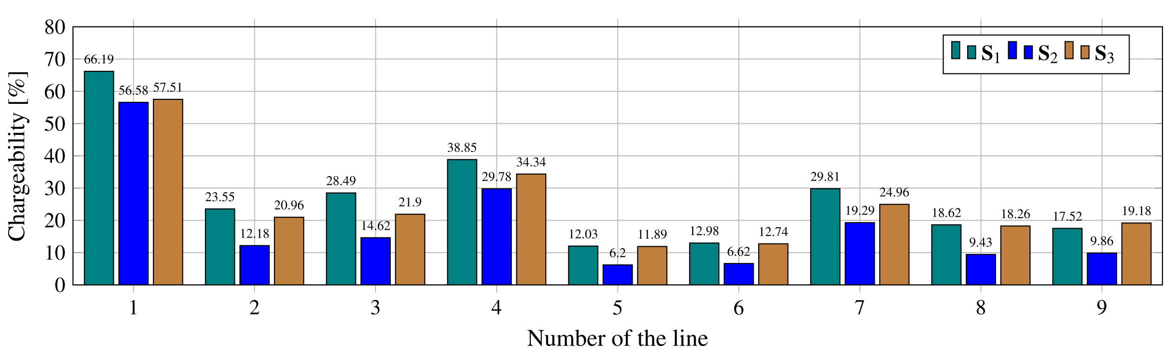

Fig. 8. Chargeability in conductors for scenarios $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ at the peak hour.

From this results it is important to mention that: *i)* the extreme solutions, i.e., simulation cases $\mathbf{S}_1$ and $\mathbf{S}_2$, are non-attractive alternatives, since both present costs upper than MCOP$ 15.90 (compared to Fig. 7); and *ii)* the most attractive solution in terms of total costs in the solution labeled 5, since it has a value about MCOP$ 15.22 divided in investment costs about MCOP$ 6.05 and energy losses cost about MCOP$ 9.17, respectively.

An important fact observed in Fig. 11, it is that the solutions in the middle of the front (see solutions 3 to 7) are similar to the solution reported in the third scenario, were both objective functions are added. This implies that an alternative for the utility is to make scenarios that combine both objective functions in linear form as follows $z = \omega_1 z_1 + \omega_2 z_2$, where ω_1 and ω_2 factors related to the importance of each objective function for the electricity company as a function of its budget priorities.

Remark 7. The problem of the optimal selection of calibers in dc grids is highly complex due to its non-convexity (discrete nature) and the solution space's dimension, which complicates the possibility of finding the global optimum solution. For this reason, it requires to reformulate this problem with convex approximations, including binary variables, in order to ensure the global optimum finding via branch & bound methods.

Finally, regarding the computational performance of the proposed MINLP model solved in GAMS with the BONMIN solver, it is important to mention that: *i)* the average processing time for solving each simulation scenario was about 10 s, while for constructing the Pareto from was about 3 minutes with ten solution points in the final report; and *ii)* the dimension of the solution space in relation with the discrete variables is about 6^9, where 6 is the number of conductor sizes available and 9 the number of lines, i.e., 10077696 possible solutions. This dimension of the solution space permits remarking that the proposed MINLP model is speedily solved with excellent performance regarding the objective function values demonstrating its efficiency to address the studied problem.

4.3. Additional simulation cases

In this part, it is evaluated two important simulation cases regarding the expected behavior of the DC distribution network in the presence of renewable energy resources and battery energy storage systems. Furthermore, the proposed model is evaluated to different voltage regulation policies since the voltage profile allows observing the grid's quality service. This can be considered a key factor that must be considered in the planning of the electrical infrastructure.

4.3.1. Voltage profile performance

In Fig. 9 was presented the voltage profile performance for the DC network considering three different combinations of the objective functions, i.e., $\mathbf{S}_1$ to $\mathbf{S}_3$. However, it can observe that in the $\mathbf{S}_2$ when the highest caliber was used in all the distribution lines, i.e., caliber 6, the voltage regulation was about 5.91 % with the worst voltage at node 9 with a magnitude of 0.9419. For this reason, to demonstrate the effect of restricted voltage regulation bounds, we added to the caliber options in Table 2 three additional caliber options as reported in Table 5.

In this simulation case, we evaluate the nine possible conductors in the 10-node test feeders considering that the voltage bounds are restricted from ±10 % to ±4 % using steps of 2 %, when is considered the third simulation scenario, i.e., the minimization of the algebraic sum of the energy and investment costs. In Table 6 is reported the selection of the different calibers considering variations in the voltage constraint as well as the value of each objective function and the worst voltage profile.

Numerical results in Table 6 allow observing that:

- ✓ When the voltage constraint decreases from ±10 % to ±4 % the investment costs in calibers of conductors increases from MCOP $6.074 to MCOP$ 13.001 since to improve the quality of the voltage in the DC network to fulfill the voltage regulation impositions are required conductors with low resistance values in order to reduce the voltage drop in all the lines, which is evidenced in the reduced annual energy costs which decrease from MCOP$7.954 to MCOP$ 5.076.
- ✓ Regarding voltage profiles, it is observed that the minimum voltage appears at node 9 for voltage regulations from ±10 % to ±6 % always fulfilling the regulation imposition. When the voltage profile is restricted to ±4 %, the worst voltage magnitude appears at node 8 (0.9604 p.u), which confirms that the regulation is also fulfilled.
- ✓ The total annual cost of the DC network for the cases where regulation voltage is upper or equal to ±6 % is lower than the simulations presented in the single-objective analysis depicted in Fig. 7, which occur since, in this new context, it has three additional options regarding calibers that increase the possibilities of selection in the solution space. This enlarged solution space, i.e., 387,420,489 potential solutions, allows reaching high-quality solutions compared to the initial solution space conformed by 10,077,696 potential options.

In Fig. 12 is reported the voltage profile in all the nodes which allow confirming that for each simulation case, the voltage regulation is fulfilled as presented in Table 6.

4.3.2. Effect of the renewable generation and battery energy storage systems

This part presents the effect of renewable energy and battery energy storage systems in the optimal selection of calibers in the DC network. For this purpose, it is considered the presence of a photovoltaic generator at node 7 with a nominal capacity of 35 kW and a battery energy storage system at node 3 with an energy storage capability of 60 kWh, which takes 4 hours to be fully charged/discharged. In Table 7 is presented the renewable generation profile and the battery energy storage behavior for a typical sunny day in Colombia.

Note that the negative sign in Table 7 for batteries implies that the battery is absorbing energy from the grid and the positive sign implies power injection from the battery.

In Table 8 are presented the solutions reached by the solution of the MINLP proposed model in GAMS with the BONMIN solver considering two possible regulation bounds defined as ±10 % and ±5 %, respectively.

Table 5
Additional conductors for voltage regulation bounds evaluation .

β_c	r_c [Ω/km]	C_c^{inv} [COP$/km]	$I_c^{\max}$ [A]
7	0.2560	9895680.60	370
8	0.2136	10654221.50	400
9	0.1745	11546524.35	440

Table 6
Selection of calibers as function of the voltage regulation constraint .

Branches		Voltage regulation ΔV			
Line	**Nodes**	±10 %	±8 %	±6 %	±4 %
1	1–2	9	9	9	9
2	2–3	2	2	2	2
3	2–4	3	3	3	5
4	2–6	4	5	9	9
5	3–10	1	1	1	1
6	4–5	1	1	1	1
7	6–7	4	4	4	9
8	7–8	1	1	1	4
9	7–9	1	1	2	6
z_1	[MCOP$]	6.074	6.570	7.962	13.001
z_2	[MCOP$]	7.954	7.463	6.417	5.076
$z = z_1 + z_2$	[MCOP$]	14.027	14.033	14.379	18.077
$\min v$	[p.u]	0.9185(9)	0.9243(9)	0.9401(9)	0.9604(8)

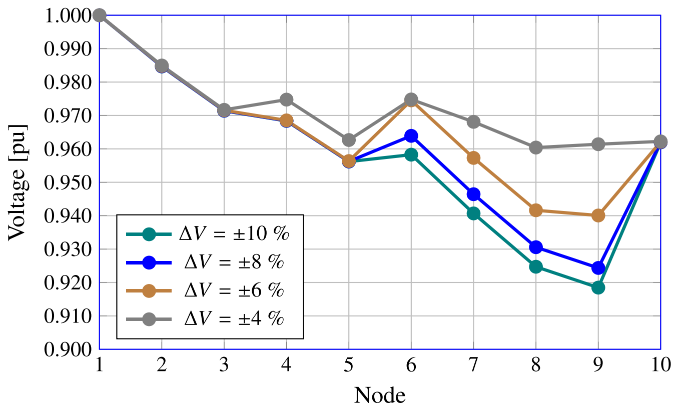

Fig. 12. Voltage profile for voltage regulation case at the peak load condition.

Table 7
Daily behavior of the photovoltaic generation and battery charging/discharging behavior.

Time [h]	**PV** [kW]	**Bat.** [kW]	**Time** [h]	**PV** [kW]	**Bat.** [kW]
1	0	–10	13	35	0
2	0	–12	14	34	–15
3	0	0	15	27	–15
4	0	0	16	20	–15
5	0	0	17	10	0
6	0	5	18	5	0
7	7	5	19	0	10
8	10	15	20	0	15
9	17	10	21	0	15
10	19	5	22	0	0
11	25	4	23	0	–10
12	33	0	24	0	–10

Table 8
Selection of calibers as function of the voltage regulation constraint.

Branches		**Voltage regulation** ΔV	
Line	**Nodes**	± 10 %	± 5 %
1	1–2	9	9
2	2–3	3	3
3	2–4	3	3
4	2–6	4	9
5	3–10	1	1
6	4–5	1	1
7	6–7	3	9
8	7–8	1	1
9	7–9	1	2
z_1	[MCOP$]	5.991	9.851
z_2	[MCOP$]	5.917	4.569
$z = z_1 + z_2$	[MCOP$]	11.908	14.420

From Table 8, we can observe that: *i)* the modification of the voltage regulation bounds from ± 10 % to ± 5 % make necessary increase the calibers in some branches to guarantee the voltage profile exigency. *ii)* note that when voltage regulation is more exigent, i.e., ± 5 %, the annual energy losses are about 22.78 % cheaper than the more flexible voltage regulation condition, i.e., ± 5 %; *iii)* the difference between the annual costs under both voltage conditions are mainly caused by the initial inversion in the calibers of the conductors; however, after the first year of operation will be more attractive the scenario with ± 5 % of voltage regulation, since the costs of the energy losses are reduced in comparison with the ± 10 % voltage regulation case.

Finally, it is worth mentioning that the inclusion of the renewable energy and battery energy storage systems in the problem of the optimal selection of calibers in DC networks has positive effects over the grid performance since power losses can be reduced with an adequate operation scheme of these distributed energy resources (compare information regarding annual energy costs in Tables 6 and 8). In addition, these distributed energy resources will help to deal with load increments without additional investments.

5. Conclusions and future works

A new mathematical model for the optimal selection of calibers in DC distribution networks was proposed in this paper based on an MINLP formulation. This model has two objective functions regarding investment and operation costs. These functions exhibit a multi-objective compromise since the improvement of one implies a deterioration of the other one. Different simulation scenarios are proposed to evaluate the proposed MINLP model's main aspects. They considered single-objective optimization, linear combination, and $\epsilon-$ constraint method for multi-objective optimization. Numerical results show that the solutions in the middle of the Pareto front are pretty similar to the linear combination of both objective functions since these present the best trade-off between investment and operational costs. The behavior of the voltage profiles in all the simulation scenarios has confirmed the great influence that has the caliber of the conductor selected for each branch and its corresponding voltage drop. This entails that will need to include some quality aspects regarding voltage regulation to have the best voltage profiles in the worst load scenario, i.e., the peak hour; however, this will increase the required investments on the DC network plan.

Regarding processing times taken by GAMS and the BONMIM solver, numerical results confirmed that the proposed model is easily solved even if this is highly nonlinear and non-convex since, for all the simulation scenarios, the average processing time does not overpass 10 s. This time can be considered significantly faster for an optimization problem with a solution space with a dimension higher than 10 million of a possible solution, demonstrating the effectiveness of the proposed MINLP model to select the type of calibers for the conductors in DC distribution applications.

Numerical simulations, including different voltage bounds and distributed energy resources, allowed the capability to adapt the proposed MINLP model to exigent operative scenarios. Furthermore, the proposed model has reached optimal solutions in both scenarios taking about 10 s, which implies that this simulation's additional information no affected the model performance. In the voltage profile, it was possible to observe that exigent voltage regulation bounds increase the necessary investments in the conductors' calibers while the annual grid power losses are reduced. This situation happens due to best voltage profiles that imply low voltage drops in branches, directly associated with reductions in energy dissipation.

Some of the future works derived from this research can be:

- ✓ To modify the proposed model for including renewable energy resources and batteries in the planning model considering the possibility of convexification of the products between continuous variables.
- ✓ To transform the problem of optimal selection of calibers into a problem of optimal selection of routes and calibers at the same time by combining the proposed model with the MST problem.
- ✓ To solve the proposed mathematical model via metaheuristic techniques such as genetic algorithms or tabu search to deal with nonconvexities and discrete variables via sequential programming
- ✓ To evaluate the effect of the planning horizon in the final result about calibers since the present value of the cost of the energy losses can have important influences on this solution.

CRediT authorship contribution statement

Oscar Danilo Montoya: Conceptualization, Formal analysis, Writing - review & editing, Validation. **Walter Gil-González:**

Conceptualization, Formal analysis, Writing - review & editing, Validation. **Luis F. Grisales-Noreña:** Conceptualization, Formal analysis, Writing - review & editing, Validation.

Declaration of Competing Interest

Authors affirm that manuscript has not been submitted to, nor is under review at, another journal or other publishing venue. Lastly, they affirm that they do not have any conflicts of interest.

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